QUALITATIVE BEHAVIOUR OF BIOCHEMICAL-OXYGEN DEMAND AND DISSOLVED-OXYGEN INTERACTIONS FOR A MILD ENVIRONMENTAL PERTURBATION ON DODO RIVER.

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ABSTRACT:
In studying the qualitative behaviour of biochemical-oxygen demand (BOD) and dissolved-oxygen (DO) interactions for a mild environmental perturbation on the BOD, we have used a computational method of the Runge-Kutta ODE45 numerical scheme for the analysis. The results outputs shows that due to a slight change in the initial condition of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) portrays a threat to the aquatic environment as the grey areas indicates a depletion in the coordinates of the DO down the trend, based on the monotonic decrease in the relative abundance of the DO coordinate with a critical $DO_{mc} = 2.0295$ against the actual $DO_c = 2.6357$ at the 21st day and recovery at $DO_m = 1.8670$ against actual recovery DO = 2.6677 on the 26th day of our experimental time before the control. In the same scenario, due to the inclusion of a mild environmental perturbation on the BOD coordinates for a random noise intensity value of 0.1, we observed a slight gain in the coordinate of the BOD with BOD_0 (IC = 2.08) on the base day which depleted to $BOD_m = 1.3714$ against the actual BOD = 1.5171 on the Sixth (6th) day. The BOD_m coordinates due to the modification, fluctuates drastically from $BOD_m = 1.3714$ on the sixth (6th) day to a converging (saturating) value of $BOD_m = 0.4248$ on the 361st day of our experimental time. The detailed results of this study is fully presented and discussed accordindly.

Keywords: Random Noise Intensity, Environmental Perturbation Biochemical-Oxygen Demand, Dissolved Oxygen, Numerical Simulation, Biological Extinction, Model Parameters.

INTRODUCTION

The qualitative behaviour of biochemical-oxygen demand (BOD) and dissolved-oxygen (DO) interactions for a mild environmental perturbation on the BOD is a challenging environmental problem that will be solved computationally using numerical simulation approach (**Akpodee & Ekaka-a, 2019**) on an interacting DO and BOD mathematical model formulation on a polluted river for water quality and aquatic species survival. Sewage disposal, municipal dumps, artisanal refineries, effluent discharge and several other unchecked activities aid in polluting our water bodies thereby making the aquatic environment vulnerable to BOD as the toxic pollutants outweighs the DO meant for the usage and survival of the aquatic species - a situation that can lead to the biological extinction of the aquatic species within the highly river. Research has shown that the river of high-profile cities like Port-Harcourt, Warri and Yenagoa in Nigeria are highly polluted due the regular discharge of toxic effluent water. In this study, pollution will be analysed at the highly polluted study area and a dynamical mathematical model will be use to solve this complex environmental problem. This work is intended to summarise the harmful effects of water pollution and the threat to aquatic species in order to provide a solution using mathematical modelling that will useful in predicting the changes in water quality and stop further damage. In this research work, it has been closely observed that the water bodies become polluted due to the release of untreated waste, sewage, dissolved oxygen, bacteria and toxic chemical substances from industries which is worsening the physical, chemical and biological properties of water. The mathematical concepts used for one-dimensional habitat to solve non-linear differential equations will be applied to solve this complex problem.

MATHEMATICAL FORMULATION

When water of river is polluted then the river has an ability to purify itself using some chemical and biological actions. This is known as self-purification. Self-purification can be proved as a good indicator for the status of a river, whenever pollutants is there in a river; 2 process takes place simultaneously.

1. De-oxygen
2. Re-aeration

The interaction of these two processes gives the river its self-cleansing property.
The first-order decay equation of industrial wastes for the two processes is as follows **(Kaushik, 2015)**:

$$\dot{X} = -DC^{\tau}Y + KT_c^{\tau}(S - X)$$ (1)

$$\dot{Y} = -DC^{\tau}Y \qquad (2)$$

D = decay rate/day

K = proportional coefficient

C = correction coefficient

$\tau = T - 20$

X =Dissolved Oxygen (D.O); Y =Biochemical-Oxygen Demand (B.O.D).

T_c = temperature correction coefficient

S = saturation concentration

Analytical Solution

$$\frac{dX}{dt} = -DC^{\tau}Y + (S - X), \qquad X(0) = X_0 > 0 \qquad (3)$$

$$\frac{dY}{dt} = -DC^{\tau}Y, \qquad Y(0) = Y_0 > 0 \qquad (4)$$

Solving equation (3.4) by separation of the variable

$$\frac{dY}{dt} = -DC^{\tau}Y$$

$$\frac{dY}{Y} = -DC^{\tau}dt$$

Integrating both sides,

$$\int \frac{1}{Y} dY = -DC^{\tau} \int dt$$

$$\log_e Y(t) = -DC^{\tau}t + h$$

Taking exponential of both side

$$Y(t) = e^{(-DC^{\tau}t+h)} = e^{-DC^{\tau}}.e^{h} = Ae^{-DC^{\tau}t} \qquad \text{Where } A = e^{h} \qquad (5)$$

$$y(0) = y_0$$

$$y(0) = y_0 = e^{-DC^{\tau}(0)}.A = A$$

$$y_0 = A$$

Thus

$$y(t) = y_0 e^{-DC^{\tau}t} \qquad (6)$$

Substituting equation (3.6) into equation (3.3)

$$\frac{dX}{dt} = -DC^{\tau}y_0 e^{-DC^{\tau}t} + KT_c^{\tau}(S - X)$$

$$\frac{dX}{dt} = -DC^{\tau}y_0 e^{-DC^{\tau}t} + KT_c^{\tau}S - XKT_c^{\tau}$$

$$\frac{dX}{dt} + XKT_c^{\tau} = KT_c^{\tau}S - DC^{\tau}y_0 e^{-DC^{\tau}t} \qquad (7)$$

The equation is a linear inhomogeneous first order ODE with integrating factor (I.F) of the form

$$\frac{dy}{dx}+p(x)y=Q(x)\equiv\frac{dX}{dt}+p(t)X=Q(t) \tag{8}$$

where $I.F=e^{\int p(x)dx}\equiv e^{\int p(t)dt}$

hence, $I.F=e^{\int KT_c^\tau dt}=e^{KT_c^\tau t}$

multiplying I.F with equation (3.7) we have

$$e^{KT_c^\tau t}\frac{dX}{dt}+e^{KT_c^\tau t}XKT_c^\tau=(KT_c^\tau S-DC^\tau y_0e^{-DC^\tau t})e^{KT_c^\tau t} \tag{9}$$

$$\frac{d}{dt}(Xe^{KT_c^\tau t})=KT_c^\tau Se^{KT_c^\tau t}-DC^\tau y_0e^{-DC^\tau t}.e^{KT_c^\tau t}$$

$$\frac{d}{dt}\left(Xe^{KT_c^\tau t}\right)=KT_c^\tau Se^{KT_c^\tau t}-DC^\tau y_0e^{-(DC^\tau-KT_c^\tau)t}$$

Separating variable

$$d\left(Xe^{KT_c^\tau t}\right)=KT_c^\tau Se^{KT_c^\tau t}dt-DC^\tau y_0e^{-(DC^\tau-KT_c^\tau)t}dt$$

Integrating both side

$$\int\frac{d}{dt}\left(Xe^{KT_c^\tau t}\right)=KT_c^\tau S\int e^{KT_c^\tau t}\,dt-DC^\tau y_0\int e^{-(DC^\tau-KT_c^\tau)t}\,dt$$

$$Xe^{KT_c^\tau t}=\frac{KT_c^\tau S}{KT_c^\tau}e^{KT_c^\tau t}\frac{DC^\tau y_0}{(DC^\tau-KT_c^\tau)}e^{-(DC^\tau-KT_c^\tau)t}+h_1$$

Dividing through by $e^{KT_c^\tau t}$, we have

$$X(t)=\frac{KT_c^\tau S}{KT_c^\tau}+\frac{DC^\tau y_0}{(DC^\tau-KT_c^\tau)e^{KT_c^\tau t}}.e^{-(DC^\tau-KT_c^\tau)t}+h_1e^{-KT_c^\tau t}$$

$$X(t)=S+\frac{DC^\tau y_0}{(DC^\tau-KT_c^\tau)}.e^{-(DC^\tau)t}+he^{-KT_c^\tau t}$$

$$X(0)=X_0$$

$$X(0)=X_0=S+\frac{DC^\tau y_0}{(DC^\tau-KT_c^\tau)}+h$$

$$h=X_0-S-\frac{DC^\tau y_0}{(DC^\tau-KT_c^\tau)}$$

$$X(t)=S+\frac{DC^\tau y_0}{(DC^\tau-KT_c^\tau)}.e^{-(DC^\tau)t}+\left(X_0-S-\frac{DC^\tau y_0}{(DC^\tau-KT_c^\tau)}\right)e^{-KT_c^\tau t} \tag{10}$$

Thus, the predictive model for dissolved oxygen (DO) and Bio-oxygen Demand (BOD) for arbitrary parameter values are stated as follows

$$X(t) = S + \frac{DC^{\tau}y_0}{(DC^{\tau} - KT_c^{\tau})}.e^{-(DC^{\tau})t} + \left(X_0 - S - \frac{DC^{\tau}y_0}{(DC^{\tau} - KT_c^{\tau})}\right)e^{-KT_c^{\tau}t}$$

$$Y(t) = y_0 e^{-DC^{t}t}$$

METHOD OF SOLUTION

The numerical simulation aspects using the above solution trajectories of the BOD-DO model will be the core of this studies.

Following Akpodee (2019), when numerical solutions to initial value problems (IVPs) are required that cannot be obtained by analytical means, it is necessary to use numerical methods. From the numerical methods that exist in solving initial value problems, we have only considered the popular fourth-order Runge-Kutta method in this study as part of the mathematical preliminaries. The mathematical structure and the theoretical definitions of this method are presented as follows:

The fourth-order Runge-Kutta (R-K) method is an accurate and flexible method based on a Taylor series approximation to the function $f(x, y)$ in the initial value problem

$$\frac{dy}{dx} = f(x, y)$$

Subject to the initial condition $y\ (x_o) = y_o$

The increment *h* in *x* may be changed at each step, but is usually kept constant so that after *n* steps, we have

$$x_n = x_o + nh$$

The Runge-Kutta algorithm for the determination of the approximation y_{n+1} to $y(x_{n+1})$ is

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where,

$$k_1 = hf(x_{n,}y_{n,})$$
$$k_2 = hf(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_1)$$
$$k_3 = hf(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_2)$$
$$k_4 = hf(x_{n+1,}\ y_n + k_3)$$

The local error involved in the determination of y_{n+1} from y_n is $0(h^5)$

The above method can be extended to find solution to a system of differential equations such as

$$\frac{dy}{dx} = f(x, y, z)$$

$$\frac{dz}{dx} = g(x, y, z)$$

Subject to the initial condition $y\ (x_o) = y_o$ and $z\ (x_o) = z_o$
These are the types of equations considered by this study which consists of a system of two first order nonlinear differential equations.
At the *nth* integration step, using a step of length *h*, the Runge-Kutta Algorithm for the system takes the form

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$z_{n+1} = z_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

Where,

$$k_1 = hf(x_{n,}y_{n,}z_n)$$
$$k_2 = hf(x_n + \frac{1}{2}h\ ,\ \ y_n + \frac{1}{2}k_1,\ z_n + \frac{1}{2}K_1)$$
$$k_3 = hf(x_n + \frac{1}{2}h\ ,\ y_n + \frac{1}{2}k_{2,}\ \ z_n + \frac{1}{2}K_2)$$
$$k_4 = hf(x_n + h\ ,\ y_n + k_3,\ \ z_n + K_3)$$

and

$$K_1 = hg(x_{n,}y_{n,}z_n)$$
$$K_2 = hg(x_n + \frac{1}{2}h\ ,\ y_n + \frac{1}{2}k_{1,}\ z_n + \frac{1}{2}K_1)$$
$$K_3 = hg(x_n + \frac{1}{2}h\ \ ,\ y_n + \frac{1}{2}k_2,\ z_n + \frac{1}{2}K_2)$$
$$K_4 = hg(x_n + h\ ,\ y_n + k_3 z_n + K_3)$$

As with the Runge-Kutta method, the local error involved in the determination of y_{n+1} from y_n and z_{n+1} from z_n is $0(h^5)$

It is a good numerical analysis practice that in the event of a complex dynamical system that can not admit an analytic solution for sensitivity analysis of interacting variables, we have to adopt an alternative method to study the qualitative behavior of the unique positive co-existence steady-state solution of the DO and BOD as $t \to \infty$. This is a challenging environmental problem that will be tackle computationally using MATLAB ODE45 numerical scheme.

For the purpose of this study, we have used the following parameter values compressed from the field data stated as follows:

d1 = BOD decay rate/day

k = Oxygen Supply Proportional coefficient

C = BOD correction coefficient

$\tau = Oxygen\ Supply\ Temperature\ Difference = T - 20$

X = DO

Y = BOD

T= Temperature

T_c = temperature correction coefficient for Oxygen Supply

S = saturation concentration for DO

T_0=20; T=28.91; τ =T-To; Tc=1.0300; k=0.01; d1=0.1172, X (0) =4.13, Y(0)=2.08; S =4.50

Results and Discussion

Table 1.1: Scenario I of the impact of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) interactions between a time interval of 0(5)90 in days

Example	Time (Days)	DO (mg/l)	DO_m (mg/l)	BOD (mg/l)	BOD_m (mg/l)
1	**0**	**4.1300**	**4.1300**	**2.0800**	**2.0800**
2	**5.0000**	**3.2625**	**3.1972**	**1.1571**	**1.3714**
3	**10.0000**	**2.8458**	**2.6205**	**0.6445**	**0.9853**
4	**15.0000**	**2.6743**	**2.2596**	**0.3588**	**0.7387**
5	**20.0000**	**2.6357**	**2.0295**	**0.1997**	**0.5856**
6	**25.0000**	**2.6677**	**1.8670**	**0.1112**	**0.5247**
7	**30.0000**	**2.7356**	**1.7496**	**0.0619**	**0.4827**
8	**35.0000**	**2.8203**	**1.6611**	**0.0345**	**0.4462**
9	**40.0000**	**2.9114**	**1.5933**	**0.0192**	**0.4358**
10	**45.0000**	**3.0032**	**1.5338**	**0.0107**	**0.4294**
11	**50.0000**	**3.0930**	**1.4744**	**0.0059**	**0.4362**
12	**55.0000**	**3.1790**	**1.4146**	**0.0033**	**0.4470**
13	**60.0000**	**3.2608**	**1.3671**	**0.0018**	**0.4313**

14	**65.0000**	**3.3381**	**1.3194**	**0.0010**	**0.4287**
15	**70.0000**	**3.4109**	**1.2789**	**0.0006**	**0.4232**
16	**75.0000**	**3.4792**	**1.2401**	**0.0003**	**0.4445**
17	**80.0000**	**3.5434**	**1.1942**	**0.0002**	**0.4344**
18	**85.0000**	**3.6036**	**1.1564**	**0.0001**	**0.4373**
19	**90.0000**	**3.6600**	**1.1179**	**0.0001**	**0.4228**

Table 1.2: Scenario I of the impact of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) interactions between a time interval of 95(5)180 in days

Example	**Time (Days)**	**DO (mg/l)**	**DO_m (mg/l)**	**BOD (mg/l)**	**BOD_m (mg/l)**
1	**95.0000**	**3.7129**	**1.0967**	**0.0000**	**0.4094**
2	**100.0000**	**3.7625**	**1.0757**	**0.0000**	**0.4208**
3	**105.0000**	**3.8089**	**1.0474**	**0.0000**	**0.4297**
4	**110.0000**	**3.8524**	**1.0194**	**0.0000**	**0.4380**
5	**115.0000**	**3.8932**	**0.9912**	**0.0000**	**0.4423**
6	**120.0000**	**3.9315**	**0.9633**	**0.0000**	**0.4537**
7	**125.0000**	**3.9673**	**0.9360**	**0.0000**	**0.4457**
8	**130.0000**	**4.0008**	**0.9168**	**0.0000**	**0.4224**
9	**135.0000**	**4.0323**	**0.9002**	**0.0000**	**0.4222**
10	**140.0000**	**4.0617**	**0.8907**	**0.0000**	**0.4210**
11	**145.0000**	**4.0893**	**0.8711**	**0.0000**	**0.4549**
12	**150.0000**	**4.1152**	**0.8445**	**0.0000**	**0.4431**
13	**155.0000**	**4.1395**	**0.8222**	**-0.0000**	**0.4446**
14	**160.0000**	**4.1622**	**0.8046**	**-0.0000**	**0.4359**
15	**165.0000**	**4.1834**	**0.7904**	**-0.0000**	**0.4345**
16	**170.0000**	**4.2034**	**0.7799**	**0.0000**	**0.4232**
17	**175.0000**	**4.2221**	**0.7755**	**0.0000**	**0.4204**
18	**180.0000**	**4.2396**	**0.7659**	**-0.0000**	**0.4191**

Table 1.3: Scenario I of the impact of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for

DO(IC=4.13) and BOD(IC=2.08) interactions between a time interval of 185(5)270 in days

Example	Time (Days)	DO (mg/l)	DO_m (mg/l)	BOD (mg/l)	BOD_m (mg/l)
1	185.0000	4.2560	0.7559	0.0000	0.4448
2	190.0000	4.2714	0.7336	0.0000	0.4709
3	195.0000	4.2858	0.7148	0.0000	0.4326
4	200.0000	4.2993	0.7103	0.0000	0.4343
5	205.0000	4.3119	0.6990	0.0000	0.4414
6	210.0000	4.3237	0.6920	0.0000	0.4424
7	215.0000	4.3349	0.6895	0.0000	0.4095
8	220.0000	4.3453	0.6856	0.0000	0.4368
9	225.0000	4.3550	0.6767	0.0000	0.4452
10	230.0000	4.3641	0.6670	0.0000	0.4418
11	235.0000	4.3727	0.6564	0.0000	0.4495
12	240.0000	4.3807	0.6519	0.0000	0.4250
13	245.0000	4.3882	0.6512	0.0000	0.4207
14	250.0000	4.3953	0.6572	0.0000	0.4170
15	255.0000	4.4019	0.6622	0.0000	0.4262
16	260.0000	4.4080	0.6606	0.0000	0.4434
17	265.0000	4.4138	0.6633	0.0000	0.4155
18	270.0000	4.4193	0.6640	0.0000	0.4400

Table 1.4: Scenario I of the impact of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) interactions between a time interval of 275(5)360 in days

Example	Time (Days)	DO (mg/l)	DO_m (mg/l)	BOD (mg/l)	BOD_m (mg/l)
1	275.0000	4.4244	0.6562	0.0000	0.4337
2	280.0000	4.4291	0.6563	0.0000	0.4270
3	285.0000	4.4336	0.6487	0.0000	0.4393
4	290.0000	4.4378	0.6380	0.0000	0.4555
5	295.0000	4.4417	0.6315	0.0000	0.4348
6	300.0000	4.4454	0.6293	0.0000	0.4230

7	**305.0000**	**4.4488**	**0.6334**	**0.0000**	**0.4300**
8	**310.0000**	**4.4520**	**0.6324**	**0.0000**	**0.4392**
9	**315.0000**	**4.4551**	**0.6270**	**0.0000**	**0.4426**
10	**320.0000**	**4.4579**	**0.6251**	**0.0000**	**0.4352**
11	**325.0000**	**4.4605**	**0.6312**	**0.0000**	**0.4019**
12	**330.0000**	**4.4630**	**0.6388**	**0.0000**	**0.4229**
13	**335.0000**	**4.4654**	**0.6388**	**0.0000**	**0.4284**
14	**340.0000**	**4.4675**	**0.6406**	**0.0000**	**0.4175**
15	**345.0000**	**4.4696**	**0.6454**	**0.0000**	**0.4193**
16	**350.0000**	**4.4715**	**0.6478**	**0.0000**	**0.4132**
17	**355.0000**	**4.4733**	**0.6447**	**0.0000**	**0.4382**
18	**360.0000**	**4.4750**	**0.6377**	**0.0000**	**0.4248**

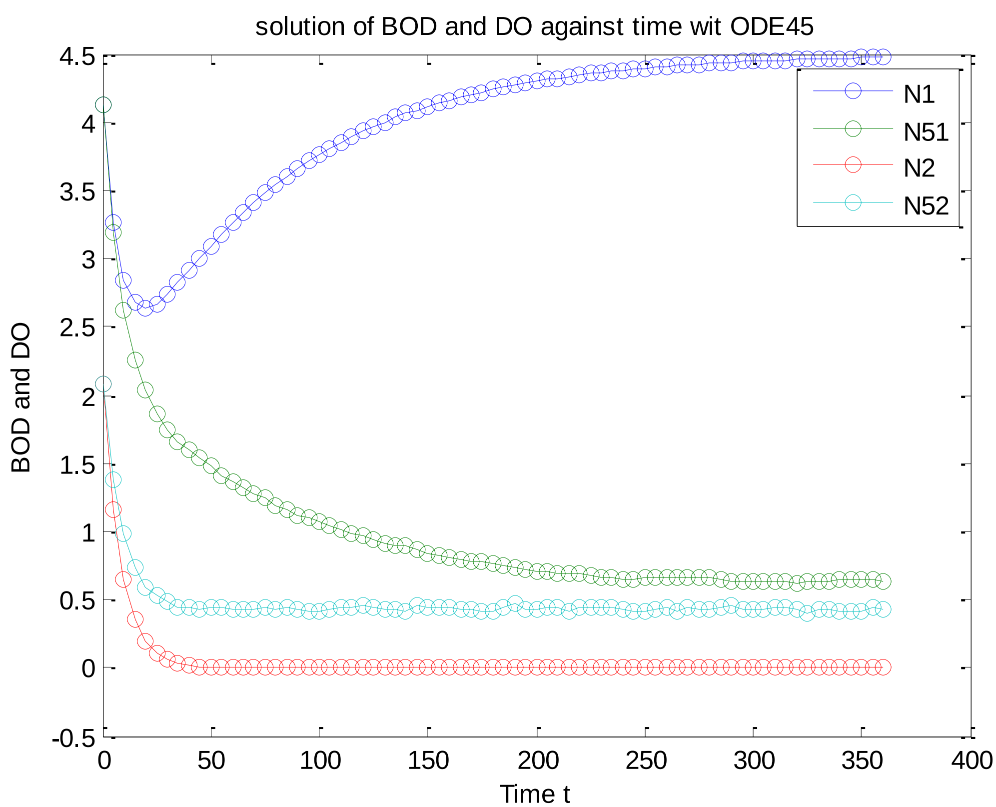

Figure 1.1: Solution trajectory of Scenario I of the impact of the inclusion of a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) interactions between a time interval of 0(5)360 in days

From the result obtained in Table 1.1 – Table 1.4 and Figure 4.12, we observed that when all model parameter values are fixed ranging from the time interval of 0:5:360 in days, the BOD and DO initial values here called the initial condition (IC) on the base day are recorded as BOD_0 = 2.080 and DO_0 = 4.130 due to pollution of the stream. Furthermore, as a result of the impact of the inclusion of a medium random environmental perturbation value RNI = 0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) in their interaction between a time interval at 0(50)90 in days, we observed grey area such as the time interval between (0 – 15) days called the interval of degradation with DO_0 = 4.13 on the base day up DO_m = 2.2960 on the sixteenth (16th) day against the actual the actual DO = 4.13 on the base day up to DO = 2.6743 on the sixteenth (16th) day before the control. The next grey area is the interval of active decomposition which is observed between (10 – 35) days with DO_m = 2.6205 on the eleventh (11th) day and DO_m = 1.6611 on the 36th day against the actual DO = 2.8458 on the 11th day and DO = 2.8203 on the 36th day before the control. The DO deficit of the modified coordinate is recorded as DO_{mc} = 2.0295 at the critical time (days) which fall on the twenty first (21st) day of our experimental time. Moreover, a further grey area captured is the interval of active recovery which ranges from (35 – 150) days has DO_m = 1.1661 on the 36th day and DO_m = 0.8445 on the 151st day been the point of inflexion against the actual DO = 2.6205 on the 36th day and DO = 1.6611 on the 151st day been the actual value of the point of inflexion before the control. Here, an important region or grey area called "the interval of complete saturation" has DO_m = 0.8445 at the point of inflexion on the 151st day and a saturated value of DO_m = 0.6377 on the 361st day of our experimental time against the actual DO = 4.1152 on the 151st day and DO = 4.4750 on the 361st day before control.

This observation due to a slight change in the initial condition of the inclusion of a a mild random environmental perturbation value of RNI=0.1 on the BOD coordinate for DO(IC=4.13) and BOD(IC=2.08) portrays a threat to the aquatic environment as the grey areas indicates a depletion in the coordinates of the DO down the trend, based on the monotonic decrease in the relative abundance of the DO coordinate with a critical DO_{mc} = 2.0295 against the actual DO_c = 2.6357 at the 21st day and recovery at DO_m = 1.8670 against actual recovery DO = 2.6677 on the 26th day of our experimental time before the control. In the same scenario, due to the inclusion of a mild environmental perturbation of random noise intensity value of 0.1 on the BOD coordinates, we observed a slight gain in the coordinate of the BOD with BOD_0 (IC = 2.08) on the base day which depleted to BOD_m = 1.3714 against the actual BOD = 1.5171 on the Sixth

(6^{th}) day. The BOD_m coordinates due to the modification, decreases drastically from $BOD_m = 1.3714$ on the sixth (6^{th}) day to a converging (saturating) value of $BOD_m = 0.4248$ on the 361^{st} day of our experimental time.

This observation is consistent and vital for environmental decision and policy making which will improve the scope of protecting the water bodies and aquatic environment as a result of pollution. The information discussed here captures grey area of vulnerability of the aquatic environment and will serve as a valuable tool that may be utilized by environmental protection agency and other related bodies concerned with pollution for monitoring, intervention and mitigation.

CONCLUSION

In studying the impact of environmental perturbation on polluted stream using analytical method and describing the qualitative characterization of the dynamical system will be quite tasking. Hence we have introduced the core method for this work which is the computational method in order to get an early insight of the expected results and also eliminate approximation errors. Through this method, we were able to predict the relative abundance of the BOD-DO in their interactions over a period of $360\ at\ an\ interval\ 0{:}\,5{:}\,360$ for a mild random environmental perturbation with random noise intensity value of 0.1 using MatLab ODE45 numerical scheme. It was observed that there was a decrease in the relative abundance of both coordinates and a point of recovery in the DO coordinates with its DO deficit up to its complete saturation and a randomized value of the BOD coordinates.

The differential equation model for effluent discharge used in this work is very effective in the determination and prediction of the relative abundance of the BOD-DO coordinates over a period of time.

RECOMMENDATION

To researchers who may want to do similar works as this, we recommend that:

The impact of a low and severe environmental perturbation should be studied.

The impact of the growth rate should be checked using computational method.

The impact varying initial conditions should be predicted using computational method.

Similar method should be adopted to study the qualitative behavior other physiochemical properties over time.

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