ANALYTICAL SOLUTION OF THE IMPACT OF VOCATIONAL EDUCATION ON THE UNEMPLOYMENT RATE IN NIGERIA: INCORPORATING CONTROL STRATEGY

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ABSTRACT

In this paper, we developed a mathematical modeling to investigate the impact of vocational education on the unemployment rate in Nigeria by incorporating control strategy. In developing our model, the population was compartmentalized into Vocational-Theoretical-Unemployed-Employed-Apprenticed (VTUEA). The model developed in a system of differential equations. Equilibrium points of the system and basic reproduction number were obtained, existence and uniqueness of the solution, invariant region, positivity of the solution was established. The dynamics of the rate of unemployment is determined by the Implementation Success Ratio of Vocational Education Program I_0, if $I_0 > 1$ the unemployment-free equilibrium is positive and if $I_0 < 1$ the unemployment will persist in the country. The results showed that the dynamic of the system is well-posedness. It's recommended that Parents should continue encouraging their children to study vocational education at all level of educational system and government should institute apprenticeship Centre across the country.

Keyword: Positivity, Existence and uniqueness, Invariant region, Unemployment free equilibrium, Endemic equilibrium, Implementation success ratio.

Introduction

Vocational education is educations that prepare people to work as a technician or to take up employment in skilled craft or trade as a tradesperson or artisan, it can take place at the post-secondary, further education or higher education level and can interact with the apprenticeship system [**15**]. Vocational excellence is a high quality of training and education which relevance to the world of work and to the attractiveness of the

educational offer to learners and to employers. It may also imply an enlarged, more comprehensive and inclusive conceptualization of skills provision that addressing innovation, pedagogy, social justice, lifelong learning, transversal skills, organizational and continuing professional learning and community needs [**5**]. The importance of vocational education in schools has long been recognized as a bridge between education and employment which provide a better match for the aptitudes and interests of some students, vocational education increases school engagement and reduces the risk of students leaving the education system with no qualifications [**16**]. Vocational education focuses on the actual attainment of proficiency in manual skills and it trains students for jobs that are connected with a specific occupation or trade [**12**].

The current system of education is theoretical in nature because of its emphasis on basic education at the expense of vocational education. Nigeria should prepare for doom as the current unemployment rate may reach the peak in the nearest future due to the neglect of technical and vocational education training [**1**].

Mathematics is the most comprehensive education area of the world that could be used in various ways in areas and topics that is not related to it [**2**]. In Nigeria, the National Bureau of Statistics describes an unemployed person as someone between age 15 – 64 years, capable and willing to work but unable to obtain any work or work for less than 40 hours within a period of 7 days [**10**]. The demand for vocational skills is complex to predict with precision, and is typically localized. The responsiveness of vocational education systems depends not on top-down planning, but on having a diversity of providers that are motivated to meet the demand for skills [**4**]. The youth unemployment figure has been rising year by year in Nigeria with 64 million youth unemployed and 1.6 million youth underemployed in 2017 [**9**]. Youth unemployment in Nigeria is attributed to a number of factors among which are: rural-urban drift, rapid population growth, corruption, neglect of agricultural sector, unfavorable government reform, low standard of education, erratic power supply, infrastructural decay, poor management practices, poor system of education and so on [**14, 3**].

Technical and vocational education and training (TVET) is the solution to the unemployment burden in developing countries [**17**]. The rates of unemployment in East Asian countries are low mainly because the population acquired employable technical and vocational skills [**6, 8**]. Technical and vocational education and training (TVET) is the education that highlights the application of skills, attitude and knowledge needed for employment in a given occupation or related occupations in any field of economic and social activities [**13**]. According to [**7**], the population of individuals within the age

bracket of 15 – 29 years in developing nations rose by 12.4% between 1993 and 2003, while the youth employment increased by just 0.6%. This is a precarious unemployment situation that can be remedied by encouraging job-seekers' opportunities in the informal labor markets through TVET **[12]**.

The population of Nigeria is more than 2.16 million which increase from previous year by 2.53% but according to international labor organization's standard of one-hour work per week, Nigeria record of unemployment rate is 11.7% out of 181countries with the rate published in 2018 and its rank as the 41st country with highest unemployment rate, Unemployment share of labor force that is without work but available for and seeking employment, Nigeria unemployment rate for 2020 was 9.01% which was increases averagely by 0.67 in the past four years **[11]**.

The remaining part of this paper is organized as follows; in the next section, Section 2 describes the structure of the model transmission dynamics, its basic properties and corresponding implementation success ratio of Vocational Education Program by using the method adopted from **[1]**, section 3 summarizes the entire work and finally, make conclusion and recommendation..

Materials and Methods

Model Formulation and Its Basic Properties

A model consider the impact of vocational education on the unemployment rate, a mathematical model is formulated. In the model, individuals are classified as Vocational class $V(t)$, Theoretical class $T(t)$, Employed class $E(t)$, Unemployed class $U(t)$ and Apprenticed class $A(t)$. Hence, the total population at is given by $N(t)=V(t)+T(t)+E(t)+A(t).$ rescaling, we can consider with $N(t)=1.$ We assumed that the rate at which individuals disengage from vocational education, theoretical education and present employment for looking a greener pastures is the same and is denoted by μ, It assumed that those that have been gainfully employed do not lose their jobs but remain employed unless they leave their present employment to look for greener pastures it also assumed that unemployed individuals either withdrawal or move to apprenticed class. The class of vocational $V(t)$ is made up of students who are into vocational education and are expected to be self-employed after graduation and the recruitment rate into the vocational class is $\Lambda\prod$, A proportion $0<\Lambda<1$ of enrollment individuals recruited into vocational class $V(t)$ at a rate $\prod$ while the rest $(1-\Lambda)$

recruited into theoretical class $T(t)$ at the same rate $\prod$. the gainfully employed either by setting up themselves with businesses after graduation or by getting recruited into the paid employment and those who gainfully recruited into employment through apprenticeship programmed individuals progress to employment class $E(t)$ at rate β and η which are the rate of employment creation in the economy where σ is the rate at which unemployment individuals in $U(t)$ class transient into $E(t)$ class and γ is the rate at which individuals in vocational class $V(t)$ transient into theoretical class $T(t)$.

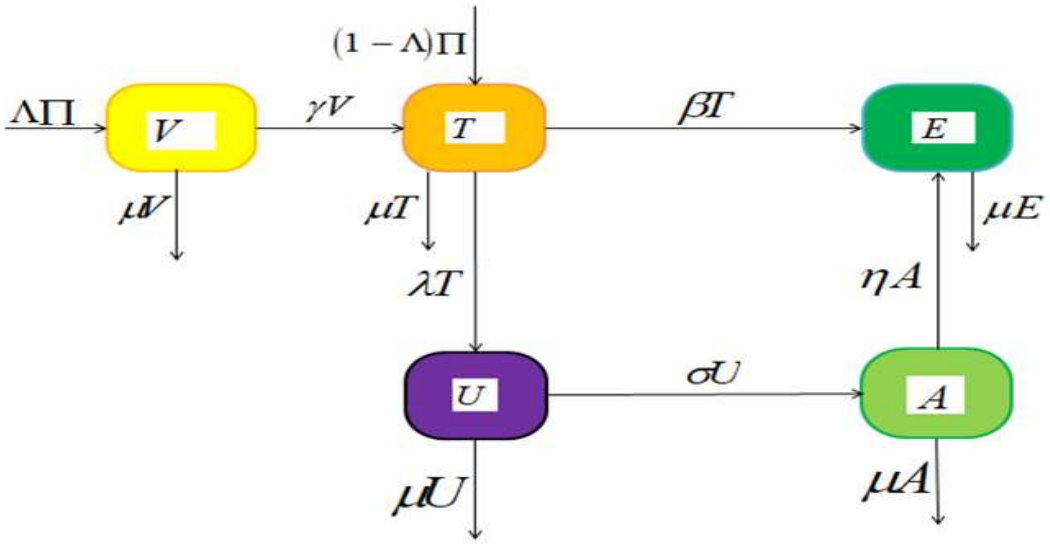

Fig1. The schematic diagram of the model

The model is given by the following differential equations

$$\left.\begin{aligned}
\frac{dV}{dt} &= \Lambda\prod - \gamma V - \mu V \\
\frac{dT}{dt} &= (1-\Lambda)\prod + \gamma V - \beta T - \lambda T - \mu T \\
\frac{dU}{dt} &= \lambda T - \sigma U - \mu U \\
\frac{dA}{dt} &= \sigma U - \eta A - \mu A \\
\frac{dE}{dt} &= \beta T + \eta A - \mu E
\end{aligned}\right\} \qquad 1$$

With the condition

$$V(t)+T(t)+U(t)+E(t)+A(t)=1$$

. Because of the absence of the employment class $E(t)$ in the model equations, we can study the following reduced system:

$$\left.\begin{aligned}\frac{dV}{dt} &= \Lambda\prod -\gamma V - \mu V\\ \frac{dT}{dt} &= (1-\Lambda)\prod +\gamma V - \beta T - \lambda T - \mu T\\ \frac{dU}{dt} &= \lambda T - \sigma U - \mu U\\ \frac{dA}{dt} &= \sigma U - \eta A - \mu A\end{aligned}\right\} \qquad 2$$

Table1: The model variables and their Description

Variables	Descriptions
$\prod$	Rate of recruitment
Λ	Proportion of individuals recruited into vocational education
μ	Rate of Withdrawal
γ	Rate transition of vocational education
β	Rate of transition of becoming employment
λ	Rate of transition of becoming unemployed
θ	Rate of transition of individuals to employment class
η	Rate of transition of individuals from apprenticed class to employment class
σ	Rate of transition of individuals to apprenticed class
$(1-\Lambda)$	Proportion of individuals recruited into theoretical education
$V(t)$	Number of vocational individuals at time t.
$T(t)$	Number of theoretical individuals at time t.
$U(t)$	Number of unemployment individuals at time t.
$A(t)$	Number of apprenticed individuals at time t.
$E(t)$	Number of employment individuals at time t.

The Existence and Uniqueness of Solutions of the Model

We use the theorem of Lipchitz continuity to establish the existence and uniqueness of solution to the system (2).

Consider the system of equation below

$$\left.\begin{aligned} x_1' &= f_1(t, x_1, x_2, \ldots x_n),\ x_1(t_0) = x_{10} \\ x_2' &= f_1(t, x_1, x_2, \ldots x_n),\ x_2(t_0) = x_{20} \\ &\cdot \\ &\cdot \\ &\cdot \\ x_n' &= f_n(t, x_1, x_2, \ldots x_n),\ x_n(t_0) = x_{n_0} \end{aligned}\right\}$$

(3)

We may write equation (3) in compact form as

$$x' = f_1(t, x),\ x_1(t_0) = x_0 \tag{4}$$

Where

$$X' = \begin{bmatrix} x_1' \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n' \end{bmatrix} \qquad \mathrm{F} = \begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ f_n \end{bmatrix} \qquad \text{and} \quad X(\mathrm{t}_0) = \begin{bmatrix} X_1(\mathrm{t}_0) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ X_n(\mathrm{t}_0) \end{bmatrix}$$

Theorem1: Let D denote the region

$$|t - t_o| \le a, \|x - x_o\| \le b,\ x = (x_1, x_2, \ldots, x_n),\ x_o = (x_{10}, x_{20}, \ldots, x_{n0}) \tag{4}$$

And suppose that $F(t, x)$ satisfies the Lipchitz condition

$$\|f(t, x_1) - f(t, x_2\| \le k\|x_1 - x_2\| \tag{5}$$

Whenever the pairs (t, x_1) and (t, x_2) belong to D, there is a constant $\delta > 0$ such that there exist a unique continues vector solution $\mathrm{X}(t)$ of the system (4) in the interval $|t - t_o| \le \delta.$

Proof

Let

$$\left.\begin{aligned} f_1 &= \Lambda \prod - \gamma V - \mu V \\ f_2 &= (1 - \Lambda) \prod + \gamma V - \beta T - \lambda T - \mu T \\ f_3 &= \lambda T - \sigma U - \mu U \\ f_4 &= \sigma U - \eta A - \mu A \end{aligned}\right\}$$

6

It is sufficient to show that $\frac{\partial f_i}{\partial x_j}, i, j = 1, 2, \ldots\ldots\ldots\ldots\ldots, 6$ are continuous.

Let $x = (V,T,U,A)$

Consider the partial derivatives for $f_1, f_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, f_6$ below

For f_1

$where\ f_1 = \Lambda\prod -(\gamma-\mu)V$

$$\frac{\partial f_1}{\partial V} = \left|-(\gamma+\mu)\right| < \infty, \quad \frac{\partial f_1}{\partial T} = \frac{\partial f_1}{\partial U} = \frac{\partial f_1}{\partial A} = 0 < \infty,$$

Clearly $\frac{\partial f_1}{\partial V}, \frac{\partial f_1}{\partial T}, \frac{\partial f_1}{\partial U}\ and\ \frac{\partial f_1}{\partial A}$ its continous and bounded

If we should continuous we proved for $f_2, f_3\ and\ f_4$.

So, it's clearly shows that all the partial derivatives of the model equations are continues and bounded, hence by theorem1, there exists a unique solution of equations (2) in the region D.

Positivity of Solution of the Model

For non-negative initial conditions of the model equations given by (4.1) the solution (V,T,U,A) of the model equations (1) are all non-negative for all time $t > 0$

Theorem 2: If $\{V(0) \geq T(0) \geq U(0) \geq A(0)\} \in \Omega$ the solutions $(V,\mathrm{T},\mathrm{U},\mathrm{A})$ of the model equations (2) are all non-negative for all time $t > 0$

Proof:

From equation (2) we have

$\frac{dV}{dt} = -(\gamma+\mu)V$ By comparison theorem, we deduced that

$$\frac{dV}{dt} \geq -(\gamma+\mu)V$$

Using method of separation of variables we get

$$\frac{dV}{V} \geq -(\gamma+\mu)\,\mathrm{dt}$$

Integrating both sides we have

$$\ln V \geq -\int(\gamma+\mu)dt$$

Taking exponent of both sides we get

$$V(t) \geq e^{-\int(\gamma+\mu)dt}$$
$$V(t) \geq 0$$

If we should continuous we obtained;

$$T(t) \geq e^{-\int(\beta+\lambda+\mu)\,dt} \quad U(t) \geq e^{-\int(\sigma+\mu)\,dt} \quad A(t) \geq e^{-\int(\eta+\mu)d}$$
$$T(t) \geq 0 \quad , \quad U(t) \geq 0 \quad A(t) \geq 0$$

Therefore, all the solution of system of the model equations with non-negative initial conditions remain non-negative for all time t>0.

Invariant Region

We consider the theorem on the region that the system of the model equations is restricted.

Theorem 3:

The *feasible region* $\boldsymbol{\Omega}$ *defined by*

$\boldsymbol{\Omega} = \left\{ V(t), T(t), U(t), A(t) \in R_4^+ \,\middle|\, 0 \leq N \leq \max\left\{ N(0), \frac{\Pi}{\mu} \right\} \right\}$ with initial conditions $V(0) \geq 0,\ T(0) \geq 0,\ U(0) \geq 0$ *and* $A(0) \geq 0$ is positively invariant and attracting with respect to system of equations for all $t > 0$.

Proof:

Let $N(t) = V(t) + T(t) + E(t) + A(t)$ Thus, summing up the model equations (2), we obtained the total population satisfies the differential equation equations (2)

That is $\frac{dN(t)}{dt} = \frac{dV(t)}{dt} + \frac{dT(t)}{dt} + \frac{dU(t)}{dt} + \frac{dA(t)}{dt}$

In the absence of unemployment it follows that

$$\Rightarrow \frac{dN(t)}{dt} + \mu N = \Pi$$

Using integrating factor (I.F) method to integrate, so that the solution of an equation in the form

$\frac{dN}{dt} + \mu N \leq \Pi$ (where μ and Λ are function of t) is given by

$$N(t) \leq \frac{\Pi}{\mu} + \left(N(0) - \frac{\Pi}{\mu} \right) e^{-ut}$$

$At \quad t \to \infty$

$$N(\mathrm{t}) \to \frac{\prod}{\mu}$$

We observe that as $t \to \infty$ the population size $N(\mathrm{t}) \to \frac{\prod}{\mu}$. So if $N(0) \le \frac{\prod}{\mu}$, then $\lim_{t\to\infty} N(t) = \frac{\prod}{\mu}$, so also if $N(0) > \frac{\prod}{\mu}$, then N will decrease to $\frac{\prod}{\mu}$ as $t \to \infty$. This means that $N(t) \le \max\left\{\mathrm{N}(0), \frac{\prod}{\mu}\right\}$

Therefore, $N(t)$ is bounded above. Subsequently $V(t), \mathrm{T}(t), \mathrm{U}(t)$ *and* $\mathrm{A}(t)$ are bounded above. Thus, in Ω system (2) is well posed. Hence, it is sufficient to study the dynamics of the system in Ω.

Equilibriums

To find equilibriums, we set the right-hand side of the system (2) equal to zero. There are two equilibriums in $(V, \mathrm{T}, \mathrm{U}, \mathrm{A})$ space:

(1) The unemployment-free equilibrium

$$U^0 = \left(\mathrm{V}^0, \mathrm{T}^0, \mathrm{U}^0, A^0\right) = \left(\frac{\Lambda\prod}{(\gamma+\mu)}, \frac{(\gamma+\mu+\mu\Lambda)\prod}{(\beta+\lambda+\mu)(\gamma+\mu)}, 0, 0\right)$$

(2) The endemic equilibrium $U^* = \left(V^*, T^*, \mathrm{U}^*, \mathrm{A}^*\right)$ where

$$\left.\begin{aligned} V^* &= \frac{(\beta+\lambda+\mu)(\sigma+\mu)U^* - (1-\Lambda)\lambda\prod}{\lambda} \\ T^* &= \frac{\prod(1-\Lambda)(\lambda-\Lambda\gamma) + \gamma\{(\beta+\lambda+\mu)(\sigma+\mu)\}U^*}{\lambda(\beta+\lambda+\mu)} \\ U^* &= \frac{\prod(1-\Lambda)(\lambda-\Lambda\gamma) + \gamma\{(\beta+\lambda+\mu)(\sigma+\mu)\}U^*}{(\sigma+\mu)(\beta+\lambda+\mu)} \\ A^* &= \frac{\sigma\left(\prod(1-\Lambda)(\lambda-\Lambda\gamma) + \gamma\{(\beta+\lambda+\mu)(\sigma+\mu)\}U^*\right)}{(\eta+\mu)(\sigma+\mu)(\beta+\lambda+\mu)} \end{aligned}\right\}$$

From the above expressions it is clear that U^* is positive only when $I_0 > 1$ hence the endemic equilibrium point U^* exists whenever $R < 1$

Implementation Success Ratio of Vocational Education Program I_0

The quantity I_0 is the quantity that measures the implementation success of the vocational education program in terms of reduction in the level of unemployment in the economy. Unlike in epidemic model, if $I_0>1$, the vocational education program is successfully implemented to the extent that more individuals are willing to enroll in the program and those that are already in the program are motivated to maintain the program. It would decrease the level of unemployment in the economy while if $I_0<1$, the vocational education program is poorly implemented to the extent that individuals are discouraged from enrolling in the program and those that are already in the program are withdrawing to the theoretical education compartment. The situation eventually leads to an upsurge in the level of unemployment in the economy. Since individuals in the theoretical education compartment are more liable to become unemployed after graduation due to the current prevailing economic situation, the compartment shall be used to derive I_0. Besides, for the endemic equilibrium U^* to exist in the feasible region $\Omega,\ 0<T^*<T^0$ or $\frac{1}{T^*}\left(T^0\right)>1$ is the necessary and sufficient condition. The inequality is true on the ground that at the unemployment endemic equilibrium, a good number of individuals in $T(t)$ compartment must have proceeded to $U(t)$ compartment after graduation. Therefore,

$$I_0=\frac{T^0}{T^*}=\frac{\lambda\prod(\gamma+\mu+\Lambda\prod)}{\left(\prod(1-\Lambda)(\lambda-\Lambda\gamma)+\gamma\right)U^*(\gamma+\mu)}$$

For I_0 to exist

$$\left[\prod(1-\Lambda)(\lambda-\Lambda\gamma)\right]<\gamma$$

For $I_0>1$

$$\left[\left(\prod(1-\Lambda)(\lambda-\Lambda\gamma)+\gamma\right)U^*(\gamma+\mu)\right]<\lambda\prod(\gamma+\mu+\Lambda\prod)$$

Thus, I_0 is clearly a threshold parameter that can be used as an indicator for unemployment control.

Conclusion

This research work presents modified model of the impact of vocational education on the unemployment rate in Nigeria by incorporating control strategy; we formulated a deterministic compartmental model and obtained the solutions of basic properties of the model such as positive, invariant region, existence and uniqueness and equilibriums point. There exist two equilibriums of the model space, namely the unemployment-free

equilibrium (UFE) and the endemic equilibrium point (EEP), the unemployment-free equilibrium is positive when I_0 is greater than 1 and the endemic equilibrium point exists whenever I_0 less than unity. The threshold for the Implementation Success Ratio I_0, of the vocational education program was also derived. From the result, it is sufficient to study the dynamics of the system since its well- \posedness.

Recommendation

The study, it is therefore recommended that;

i. Government should include vocational education into the primary and secondary school curriculum at all level.
ii. Government should try to convert majority of her conventional university to vocational and technical university in the country.
iii. Parents should continue to encouraging their children to study vocational education course.
iv. Government and non-governmental organization should build apprenticed Centre throughout across the country.

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