MODELLING AND NUMERICAL SIMULATION OF BIOCHEMICAL-OXYGEN DEMAND AND DISSOLVED-OXYGEN INTERACTIONS DUE TO POLLUTION IN RIVER GANGES

[1]R.E. AKPODEE; [2]I.E. NWAFOR
[1]Department of Mathematics, Faculty of Science, Rivers State University.
[2]Institutes of Geoscience and Environmental Management, Faculty of Science, Rivers State University.

ABSTRACT
In effluents discharge modelling, we have investigated the impact of temperature variation (Increasing Temperature, T=28unit) in the interaction between Biochemical-Oxygen Demand (BOD) and Dissolved Oxygen (DO) on a polluted river Ganges. Here, we have derived first an analytical solution that can predict the relative abundance of both coordinates in their interaction at any time, t. Furthermore, a numerical scheme of Runge-Kutta ODE45 computational approach was used to study the qualitative characteristics of the BOD-DO interaction for the impact of temperature variation (Increasing Temperature @ T=28unit) in the interaction between BOD (IC = 8.0) and DO (IC = 7.1) at an interval of 0(0.1)60 time in hrs, when all model parameter values are fixed ranging from the time interval of 0:0.1:60 in hrs. The BOD and DO initial values here called the IC's on the base time recorded as $BOD_0 = 8.0$ and $DO_0 = 7.1$ due to the pollution of the stream. Furthermore, from the base time, we observed a severe depletion of the DO downstream from a value of 7.1 units DO to a value of 4.0099 units DO for the uncontrol coordinate and 4.4353unit DO for the control (Modified) coordinate at 1.7hr of our experimental time which further recover with various DO deficit up to full saturation. The detail work and full results are presented and discussed in this study.

Keywords: Biochemical-Oxygen Demand, Dissolved Oxygen, Numerical Simulation, Biological Extinction, Model Parameters.

INTRODUCTION
The Oxford dictionary defines pollution as "the presence in or introduction into the environment of a substance which has harmful or poisonous effect. There are many types of pollution including air pollution, water pollution, land pollution, radioactive pollution and thermal pollution" (oxford dictionaries.com).

One of the major types of environmental pollution is water pollution of which the River Ganges is been highly polluted on a daily basis. Water the most important constituent for existence of life on earth is being polluted every day and its aquatic species existence and survival are vulnerable to biological extinction. This has become an increasing and challenging problem in environmental management and ecosystem functioning globally. The water bodies become polluted due to untreated waste, sewage, agricultural discharge and toxic metals. The effect of this is that the pollutants impart colour, taste, and odour, to receiving water thus making it unfit for use. The most challenging problem in the society is the change in the environment caused by the pollution affecting the long-term survival of the aquatic species, human lifestyle and bio-diversity of the habitat. A great quantity of toxicants and contaminants enters into the River Ganges continuously which threaten the survival of the exposed population including human beings. This challenging environmental problem will be solve computationally using numerical simulation approach (**Akpodee and Ekaka-a,2019**) on an interacting Dissolved-Oxygen and Bio-Oxygen Demand on the polluted river for water quality and aquatic species survival.

MATHEMATICAL FORMULATION

When water of river is polluted then the river has an ability to purify itself using some chemical and biological actions. This is known as self-purification. Self-purification can be proved as a good indicator for the status of a river, whenever pollutants is there in a river; 2 process takes place simultaneously.

1. De-oxygen
2. Re-aeration

Due to these processes, river has its self-cleansing property.

We will consider now the case of first-order decay equation of industrial wastes. Equation for two processes is as follows **(Kaushik, 2015)**:

$$\dot{X} = -DC^{\tau}Y + KT_c^{\tau}(S - X)$$
$$\dot{Y} = -DC^{\tau}Y$$

D = decay rate/day

K = proportional coefficient

C = correction coefficient

$\tau = T - 20$

X =Dissolved Oxygen (D.O); Y =Biochemical-Oxygen Demand (B.O.D).

T_c = temperature correction coefficient

S = saturation concentration

Thus, the predictive model for dissolved oxygen (DO) and Bio-oxygen Demand (BOD) for an arbitrary parameter values are stated as follows

$$\boldsymbol{X(t) = S + \frac{DC^{\tau}y_0}{(DC^{\tau} - KT_c^{\tau})} . e^{-(DC^{\tau})t} + \left(X_0 - S - \frac{DC^{\tau}y_0}{(DC^{\tau} - KT_c^{\tau})}\right) e^{-KT_c^{\tau}t}}$$
$$\boldsymbol{Y(t) = y_0 e^{-DC^t t}}$$

METHOD OF SOLUTION

Using the Data set: (Data source: Rajat Kaushik, 2015)

D = 0.3 Day^{-1}

K = 0.1 Day^{-1}

C = 1.048 Day^{-1}

T = 24^0C

$\tau = T - 20 = (24 - 20)^0 C$

S = 8.3374 mg/L

$X_0 = 7.1\ mg/L$ at t = 0

$y_0 = 8.0 mg/L$ at t = 0

$T_c = 1.0300$

Substituting the parameter into the solution map and evaluating. We have,

$$X(t) = 8.3374 + \left[\frac{(0.3)(1.048^4)(8.0)}{(0.3)(1.048^4) - (0.1)(1.03^4)}\right] e^{-(0.3)(1.048^4)t} + 7.1 - 8.3374 - \left[\frac{(0.3)(1.048^4)(8.0)}{(0.3)(1.048^4) - (0.1)(1.03^4)}\right] e^{-(0.1)(1.03^4)t}$$

$$X(t) = 8.3374 + \left[\frac{(2.895052023)}{(0.36188 - 0.112550881)}\right] e^{-(0.36188)t} + 7.1 - 8.3374 - \left[\frac{(2.895052023)}{(0.36188) - (0.112550881)}\right] e^{-(0.11255088)t}$$

$$X(t) = 8.3374 + \left[\frac{(2.895052023)}{(0.24932912)}\right] e^{-(0.36188)t} + 7.1 - 8.3374 - \left[\frac{(2.895052023)}{(0.24932912)}\right] e^{-(0.11255088)t}$$

$$X(t) = 8.3374 + 11.6113674 e^{-(0.36188)t} + (7.1 - 8.3374 - 11.61136743) e^{-(0.11255088)t}$$

$$X(t) = 8.3374 + 11.6113674 e^{-(0.36188)t} - 12.84876 e^{-(0.11255088)t}$$

The requirement solution trajectory of the DO and BOD on the polluted water Gange in thus states as:

$$\boldsymbol{X(t) = 8.3374 + 11.6113674 e^{-(0.36188)t} - 12.84876 e^{-(0.11255088)t}}$$

$$\boldsymbol{Y(t) = 8.0 e^{-0.36188t}}$$

The numerical simulation aspects using the above solution trajectories of the BOD-DO model will be the core of this studies.

Results and Discussion

Table1: Impact of Temperature Gradient on Self Purification of an Industrial Polluted Water Ganges

Time	DO	DO_m	BOD	BOD_m
0	7.1000	7.1000	8.0000	8.0000
0.1000	6.8311	6.7760	7.7157	7.6583
0.2000	6.5753	6.4707	7.4414	7.3312
0.3000	6.3320	6.1830	7.1770	7.0180
0.4000	6.1008	5.9123	6.9219	6.7183
0.5000	5.8813	5.6576	6.6759	6.4313
0.6000	5.6728	5.4184	6.4386	6.1566
0.7000	5.4750	5.1937	6.2097	5.8935
0.8000	5.2874	4.9829	5.9889	5.6416
0.9000	5.1095	4.7853	5.7758	5.4004
1.0000	4.9412	4.6004	5.5703	5.1694
1.1000	4.7820	4.4277	5.3720	4.9483
1.2000	4.6316	4.2665	5.1809	4.7367
1.3000	4.4897	4.1165	4.9966	4.5343
1.4000	4.3559	3.9771	4.8188	4.3405
1.5000	4.2301	3.8478	4.6475	4.1551
1.6000	4.1117	3.7281	4.4822	3.9777
1.7000	4.0007	3.6175	4.3230	3.8080
1.8000	3.8965	3.5156	4.1694	3.6456
1.9000	3.7991	3.4220	4.0214	3.4901
2.0000	3.7080	3.3361	3.8786	3.3413
2.1000	3.6231	3.2576	3.7410	3.1989
2.2000	3.5440	3.1860	3.6083	3.0624
2.3000	3.4704	3.1210	3.4803	2.9317
2.4000	3.4022	3.0622	3.3568	2.8064
2.5000	3.3391	3.0094	3.2376	2.6865
2.6000	3.2808	2.9623	3.1227	2.5716
2.7000	3.2270	2.9207	3.0117	2.4617
2.8000	3.1777	2.8842	2.9046	2.3564
2.9000	3.1327	2.8527	2.8013	2.2557
3.0000	3.0918	2.8260	2.7017	2.1593
3.1000	3.0549	2.8037	2.6056	2.0670
3.2000	3.0218	2.7856	2.5129	1.9788
3.3000	2.9924	2.7716	2.4235	1.8943
3.4000	2.9665	2.7613	2.3373	1.8134
3.5000	2.9439	2.7547	2.2542	1.7361
3.6000	2.9246	2.7514	2.1740	1.6620
3.7000	2.9084	2.7514	2.0968	1.5911
3.8000	2.8951	2.7543	2.0223	1.5233

3.9000	2.8846	2.7600	1.9505	1.4583
4.0000	2.8768	2.7683	1.8812	1.3960
4.1000	2.8715	2.7791	1.8144	1.3363
4.2000	2.8687	2.7923	1.7500	1.2792
4.3000	2.8682	2.8077	1.6879	1.2245
4.4000	2.8699	2.8252	1.6280	1.1721
4.5000	2.8736	2.8448	1.5702	1.1220
4.6000	2.8793	2.8663	1.5144	1.0740
4.7000	2.8869	2.8895	1.4606	1.0281
4.8000	2.8962	2.9144	1.4087	0.9842
4.9000	2.9073	2.9409	1.3586	0.9422
5.0000	2.9200	2.9689	1.3102	0.9019
5.1000	2.9342	2.9983	1.2636	0.8635
5.2000	2.9499	3.0289	1.2186	0.8266
5.3000	2.9671	3.0607	1.1753	0.7914
5.4000	2.9856	3.0937	1.1335	0.7577
5.5000	3.0054	3.1276	1.0932	0.7253
5.6000	3.0264	3.1624	1.0543	0.6944
5.7000	3.0486	3.1981	1.0168	0.6647
5.8000	3.0719	3.2346	0.9807	0.6363
5.9000	3.0962	3.2718	0.9459	0.6091
6.0000	3.1215	3.3096	0.9123	0.5831
6.1000	3.1477	3.3481	0.8799	0.5581
6.2000	3.1748	3.3872	0.8487	0.5342
6.3000	3.2026	3.4268	0.8186	0.5114
6.4000	3.2313	3.4668	0.7895	0.4895
6.5000	3.2606	3.5073	0.7615	0.4686
6.6000	3.2905	3.5483	0.7345	0.4486
6.7000	3.3211	3.5895	0.7084	0.4294
6.8000	3.3521	3.6311	0.6832	0.4111
6.9000	3.3838	3.6729	0.6589	0.3936
7.0000	3.4159	3.7150	0.6355	0.3768
7.1000	3.4484	3.7572	0.6128	0.3608
7.2000	3.4815	3.7996	0.5910	0.3454
7.3000	3.5149	3.8421	0.5700	0.3307
7.4000	3.5487	3.8847	0.5497	0.3166
7.5000	3.5828	3.9273	0.5301	0.3030
7.6000	3.6172	3.9700	0.5113	0.2901
7.7000	3.6520	4.0126	0.4931	0.2777
7.8000	3.6870	4.0553	0.4756	0.2658
7.9000	3.7223	4.0978	0.4587	0.2544
8.0000	3.7577	4.1404	0.4424	0.2436
8.1000	3.7934	4.1829	0.4267	0.2331
8.2000	3.8292	4.2253	0.4115	0.2232

8.3000	3.8651	4.2675	0.3970	0.2136
8.4000	3.9012	4.3097	0.3829	0.2045
8.5000	3.9374	4.3517	0.3693	0.1958
8.6000	3.9736	4.3936	0.3562	0.1874
8.7000	4.0099	4.4353	0.3436	0.1794
8.8000	4.0462	4.4768	0.3314	0.1718
8.9000	4.0825	4.5182	0.3196	0.1645
9.0000	4.1189	4.5593	0.3082	0.1575
9.1000	4.1552	4.6002	0.2973	0.1507
9.2000	4.1914	4.6408	0.2867	0.1443
9.3000	4.2277	4.6812	0.2765	0.1382
9.4000	4.2639	4.7214	0.2666	0.1322
9.5000	4.3000	4.7613	0.2572	0.1266
9.6000	4.3361	4.8009	0.2480	0.1212
9.7000	4.3721	4.8403	0.2392	0.1160
9.8000	4.4080	4.8794	0.2307	0.1110
9.9000	4.4439	4.9182	0.2225	0.1063
10.0000	4.4796	4.9567	0.2146	0.1017
10.1000	4.5152	4.9949	0.2069	0.0974
10.2000	4.5507	5.0329	0.1996	0.0932
10.3000	4.5860	5.0705	0.1925	0.0892
10.4000	4.6212	5.1079	0.1857	0.0854
10.5000	4.6562	5.1449	0.1791	0.0818
10.6000	4.6911	5.1816	0.1727	0.0783
10.7000	4.7259	5.2180	0.1666	0.0750
10.8000	4.7604	5.2541	0.1607	0.0718
10.9000	4.7948	5.2899	0.1550	0.0687
11.0000	4.8290	5.3254	0.1495	0.0658
11.1000	4.8629	5.3605	0.1442	0.0630
11.2000	4.8967	5.3953	0.1391	0.0603
11.3000	4.9303	5.4298	0.1341	0.0577
11.4000	4.9637	5.4639	0.1293	0.0552
11.5000	4.9969	5.4977	0.1247	0.0529
11.6000	5.0298	5.5312	0.1203	0.0506
11.7000	5.0626	5.5644	0.1160	0.0485
11.8000	5.0951	5.5972	0.1119	0.0464
11.9000	5.1275	5.6298	0.1079	0.0444
12.0000	5.1596	5.6620	0.1041	0.0425
12.1000	5.1915	5.6938	0.1004	0.0407
12.2000	5.2231	5.7254	0.0968	0.0390
12.3000	5.2546	5.7566	0.0934	0.0373
12.4000	5.2858	5.7875	0.0901	0.0357
12.5000	5.3168	5.8181	0.0869	0.0342
12.6000	5.3475	5.8484	0.0838	0.0327

12.7000	5.3780	5.8783	0.0808	0.0313
12.8000	5.4083	5.9079	0.0779	0.0300
12.9000	5.4383	5.9373	0.0752	0.0287
13.0000	5.4681	5.9663	0.0725	0.0275
13.1000	5.4977	5.9949	0.0699	0.0263
13.2000	5.5270	6.0233	0.0675	0.0252
13.3000	5.5561	6.0514	0.0651	0.0241
13.4000	5.5849	6.0791	0.0627	0.0231
13.5000	5.6135	6.1066	0.0605	0.0221
13.6000	5.6418	6.1337	0.0584	0.0211
13.7000	5.6699	6.1605	0.0563	0.0202
13.8000	5.6978	6.1871	0.0543	0.0194
13.9000	5.7254	6.2133	0.0523	0.0185
14.0000	5.7528	6.2393	0.0505	0.0178
14.1000	5.7799	6.2649	0.0487	0.0170
14.2000	5.8068	6.2903	0.0470	0.0163
14.3000	5.8335	6.3154	0.0453	0.0156
14.4000	5.8599	6.3402	0.0437	0.0149
14.5000	5.8861	6.3647	0.0421	0.0143
14.6000	5.9120	6.3889	0.0406	0.0137
14.7000	5.9378	6.4129	0.0392	0.0131
14.8000	5.9632	6.4365	0.0378	0.0125
14.9000	5.9885	6.4599	0.0365	0.0120
15.0000	6.0135	6.4831	0.0352	0.0115
15.1000	6.0383	6.5059	0.0339	0.0110
15.2000	6.0628	6.5285	0.0327	0.0105
15.3000	6.0871	6.5508	0.0316	0.0101
15.4000	6.1112	6.5729	0.0304	0.0096
15.5000	6.1350	6.5947	0.0294	0.0092
15.6000	6.1586	6.6162	0.0283	0.0088
15.7000	6.1820	6.6375	0.0273	0.0085
15.8000	6.2052	6.6586	0.0263	0.0081
15.9000	6.2281	6.6793	0.0254	0.0077
16.0000	6.2508	6.6999	0.0245	0.0074
16.1000	6.2733	6.7202	0.0236	0.0071
16.2000	6.2955	6.7402	0.0228	0.0068
16.3000	6.3176	6.7601	0.0220	0.0065
16.4000	6.3394	6.7796	0.0212	0.0062

16.5000	6.3610	6.7990	0.0204	0.0060
16.6000	6.3824	6.8181	0.0197	0.0057
16.7000	6.4036	6.8370	0.0190	0.0055
16.8000	6.4246	6.8556	0.0183	0.0052
16.9000	6.4453	6.8741	0.0177	0.0050
17.0000	6.4659	6.8923	0.0171	0.0048
17.1000	6.4862	6.9103	0.0164	0.0046
17.2000	6.5064	6.9280	0.0159	0.0044
17.3000	6.5263	6.9456	0.0153	0.0042
17.4000	6.5460	6.9629	0.0148	0.0040
17.5000	6.5656	6.9801	0.0142	0.0039
17.6000	6.5849	6.9970	0.0137	0.0037
17.7000	6.6040	7.0137	0.0132	0.0035
17.8000	6.6230	7.0302	0.0128	0.0034
17.9000	6.6417	7.0465	0.0123	0.0032
18.0000	6.6602	7.0626	0.0119	0.0031
18.1000	6.6786	7.0785	0.0115	0.0030
18.2000	6.6968	7.0943	0.0110	0.0028
18.3000	6.7147	7.1098	0.0107	0.0027
18.4000	6.7325	7.1251	0.0103	0.0026
18.5000	6.7501	7.1403	0.0099	0.0025
18.6000	6.7675	7.1552	0.0096	0.0024
18.7000	6.7848	7.1700	0.0092	0.0023
18.8000	6.8018	7.1846	0.0089	0.0022
18.9000	6.8187	7.1990	0.0086	0.0021
19.0000	6.8354	7.2133	0.0083	0.0020
19.1000	6.8519	7.2273	0.0080	0.0019
19.2000	6.8682	7.2412	0.0077	0.0018
19.3000	6.8844	7.2550	0.0074	0.0018
19.4000	6.9004	7.2685	0.0072	0.0017
19.5000	6.9162	7.2819	0.0069	0.0016
19.6000	6.9319	7.2951	0.0067	0.0015
19.7000	6.9474	7.3082	0.0064	0.0015
19.8000	6.9627	7.3211	0.0062	0.0014
19.9000	6.9779	7.3338	0.0060	0.0014
20.0000	6.9929	7.3464	0.0058	0.0013
20.1000	7.0077	7.3588	0.0056	0.0012
20.2000	7.0224	7.3711	0.0054	0.0012

20.3000	7.0370	7.3832	0.0052	0.0011
20.4000	7.0513	7.3951	0.0050	0.0011
20.5000	7.0655	7.4069	0.0048	0.0010
20.6000	7.0796	7.4186	0.0046	0.0010
20.7000	7.0935	7.4301	0.0045	0.0010
20.8000	7.1073	7.4415	0.0043	0.0009
20.9000	7.1209	7.4528	0.0042	0.0009
21.0000	7.1344	7.4639	0.0040	0.0008
21.1000	7.1477	7.4748	0.0039	0.0008
21.2000	7.1609	7.4856	0.0037	0.0008
21.3000	7.1739	7.4963	0.0036	0.0007
21.4000	7.1868	7.5069	0.0035	0.0007
21.5000	7.1996	7.5173	0.0033	0.0007
21.6000	7.2122	7.5276	0.0032	0.0006
21.7000	7.2247	7.5378	0.0031	0.0006
21.8000	7.2370	7.5478	0.0030	0.0006
21.9000	7.2492	7.5577	0.0029	0.0006
22.0000	7.2613	7.5675	0.0028	0.0005
22.1000	7.2732	7.5772	0.0027	0.0005
22.2000	7.2850	7.5867	0.0026	0.0005
22.3000	7.2967	7.5962	0.0025	0.0005
22.4000	7.3083	7.6055	0.0024	0.0005
22.5000	7.3197	7.6147	0.0023	0.0004
22.6000	7.3310	7.6237	0.0022	0.0004
22.7000	7.3422	7.6327	0.0022	0.0004
22.8000	7.3533	7.6416	0.0021	0.0004
22.9000	7.3642	7.6503	0.0020	0.0004
23.0000	7.3750	7.6589	0.0019	0.0003
23.1000	7.3857	7.6675	0.0019	0.0003
23.2000	7.3963	7.6759	0.0018	0.0003
23.3000	7.4068	7.6842	0.0017	0.0003
23.4000	7.4171	7.6924	0.0017	0.0003
23.5000	7.4274	7.7005	0.0016	0.0003
23.6000	7.4375	7.7085	0.0016	0.0003
23.7000	7.4475	7.7164	0.0015	0.0003
23.8000	7.4574	7.7242	0.0015	0.0002
23.9000	7.4672	7.7319	0.0014	0.0002
24.0000	7.4769	7.7395	0.0014	0.0002

24.1000	7.4865	7.7471	0.0013	0.0002
24.2000	7.4960	7.7545	0.0013	0.0002
24.3000	7.5053	7.7618	0.0012	0.0002
24.4000	7.5146	7.7690	0.0012	0.0002
24.5000	7.5238	7.7762	0.0011	0.0002
24.6000	7.5329	7.7833	0.0011	0.0002
24.7000	7.5418	7.7902	0.0011	0.0002
24.8000	7.5507	7.7971	0.0010	0.0002
24.9000	7.5595	7.8039	0.0010	0.0002
25.0000	7.5681	7.8106	0.0009	0.0001
25.1000	7.5767	7.8172	0.0009	0.0001
25.2000	7.5852	7.8238	0.0009	0.0001
25.3000	7.5936	7.8302	0.0008	0.0001
25.4000	7.6019	7.8366	0.0008	0.0001
25.5000	7.6101	7.8429	0.0008	0.0001
25.6000	7.6182	7.8491	0.0008	0.0001
25.7000	7.6262	7.8553	0.0007	0.0001
25.8000	7.6341	7.8613	0.0007	0.0001
25.9000	7.6420	7.8673	0.0007	0.0001
26.0000	7.6497	7.8732	0.0007	0.0001
26.1000	7.6574	7.8791	0.0006	0.0001
26.2000	7.6650	7.8848	0.0006	0.0001
26.3000	7.6725	7.8905	0.0006	0.0001
26.4000	7.6799	7.8962	0.0006	0.0001
26.5000	7.6873	7.9017	0.0005	0.0001
26.6000	7.6945	7.9072	0.0005	0.0001
26.7000	7.7017	7.9126	0.0005	0.0001
26.8000	7.7088	7.9179	0.0005	0.0001
26.9000	7.7158	7.9232	0.0005	0.0001
27.0000	7.7228	7.9284	0.0005	0.0001
27.1000	7.7296	7.9336	0.0004	0.0001
27.2000	7.7364	7.9387	0.0004	0.0001
27.3000	7.7431	7.9437	0.0004	0.0001
27.4000	7.7498	7.9486	0.0004	0.0001
27.5000	7.7563	7.9535	0.0004	0.0000
27.6000	7.7628	7.9583	0.0004	0.0000
27.7000	7.7692	7.9631	0.0004	0.0000
27.8000	7.7756	7.9678	0.0003	0.0000

27.9000	7.7818	7.9725	0.0003	0.0000
28.0000	7.7881	7.9771	0.0003	0.0000
28.1000	7.7942	7.9816	0.0003	0.0000
28.2000	7.8003	7.9861	0.0003	0.0000
28.3000	7.8063	7.9905	0.0003	0.0000
28.4000	7.8122	7.9949	0.0003	0.0000
28.5000	7.8181	7.9992	0.0003	0.0000
28.6000	7.8239	8.0034	0.0003	0.0000
28.7000	7.8296	8.0076	0.0002	0.0000
28.8000	7.8353	8.0118	0.0002	0.0000
28.9000	7.8409	8.0159	0.0002	0.0000
29.0000	7.8464	8.0199	0.0002	0.0000
29.1000	7.8519	8.0239	0.0002	0.0000
29.2000	7.8574	8.0279	0.0002	0.0000
29.3000	7.8627	8.0318	0.0002	0.0000
29.4000	7.8680	8.0356	0.0002	0.0000
29.5000	7.8733	8.0394	0.0002	0.0000
29.6000	7.8785	8.0432	0.0002	0.0000
29.7000	7.8836	8.0469	0.0002	0.0000
29.8000	7.8887	8.0505	0.0002	0.0000
29.9000	7.8937	8.0541	0.0002	0.0000
30.0000	7.8986	8.0577	0.0002	0.0000
30.1000	7.9035	8.0612	0.0001	0.0000
30.2000	7.9084	8.0647	0.0001	0.0000
30.3000	7.9132	8.0681	0.0001	0.0000
30.4000	7.9179	8.0715	0.0001	0.0000
30.5000	7.9226	8.0749	0.0001	0.0000
30.6000	7.9273	8.0782	0.0001	0.0000
30.7000	7.9318	8.0814	0.0001	0.0000
30.8000	7.9364	8.0846	0.0001	0.0000
30.9000	7.9409	8.0878	0.0001	0.0000
31.0000	7.9453	8.0910	0.0001	0.0000
31.1000	7.9497	8.0941	0.0001	0.0000
31.2000	7.9540	8.0971	0.0001	0.0000
31.3000	7.9583	8.1002	0.0001	0.0000
31.4000	7.9625	8.1031	0.0001	0.0000
31.5000	7.9667	8.1061	0.0001	0.0000
31.6000	7.9709	8.1090	0.0001	0.0000

31.7000	7.9750	8.1119	0.0001	0.0000
31.8000	7.9790	8.1147	0.0001	0.0000
31.9000	7.9830	8.1175	0.0001	0.0000
32.0000	7.9870	8.1203	0.0001	0.0000
32.1000	7.9909	8.1230	0.0001	0.0000
32.2000	7.9948	8.1257	0.0001	0.0000
32.3000	7.9986	8.1284	0.0001	0.0000
32.4000	8.0024	8.1310	0.0001	0.0000
32.5000	8.0062	8.1336	0.0001	0.0000
32.6000	8.0099	8.1362	0.0001	0.0000
32.7000	8.0135	8.1387	0.0001	0.0000
32.8000	8.0172	8.1412	0.0001	0.0000
32.9000	8.0207	8.1437	0.0001	0.0000
33.0000	8.0243	8.1461	0.0001	0.0000
33.1000	8.0278	8.1485	0.0001	0.0000
33.2000	8.0312	8.1509	0.0000	0.0000
33.3000	8.0347	8.1532	0.0000	0.0000
33.4000	8.0381	8.1556	0.0000	0.0000
33.5000	8.0414	8.1579	0.0000	0.0000
33.6000	8.0447	8.1601	0.0000	0.0000
33.7000	8.0480	8.1623	0.0000	0.0000
33.8000	8.0512	8.1646	0.0000	0.0000
33.9000	8.0544	8.1667	0.0000	0.0000
34.0000	8.0576	8.1689	0.0000	0.0000
34.1000	8.0607	8.1710	0.0000	0.0000
34.2000	8.0638	8.1731	0.0000	0.0000
34.3000	8.0669	8.1752	0.0000	0.0000
34.4000	8.0699	8.1772	0.0000	0.0000
34.5000	8.0729	8.1792	0.0000	0.0000
34.6000	8.0759	8.1812	0.0000	0.0000
34.7000	8.0788	8.1832	0.0000	0.0000
34.8000	8.0817	8.1851	0.0000	0.0000
34.9000	8.0845	8.1870	0.0000	0.0000
35.0000	8.0874	8.1889	0.0000	0.0000
35.1000	8.0902	8.1908	0.0000	0.0000
35.2000	8.0929	8.1926	0.0000	0.0000
35.3000	8.0957	8.1945	0.0000	0.0000
35.4000	8.0984	8.1963	0.0000	0.0000

35.5000	8.1011	8.1980	0.0000	0.0000
35.6000	8.1037	8.1998	0.0000	0.0000
35.7000	8.1063	8.2015	0.0000	0.0000
35.8000	8.1089	8.2032	0.0000	0.0000
35.9000	8.1115	8.2049	0.0000	0.0000
36.0000	8.1140	8.2066	0.0000	0.0000
36.1000	8.1165	8.2082	0.0000	0.0000
36.2000	8.1190	8.2099	0.0000	0.0000
36.3000	8.1214	8.2115	0.0000	0.0000
36.4000	8.1238	8.2131	0.0000	0.0000
36.5000	8.1262	8.2146	0.0000	0.0000
36.6000	8.1286	8.2162	0.0000	0.0000
36.7000	8.1309	8.2177	0.0000	0.0000
36.8000	8.1332	8.2192	0.0000	0.0000
36.9000	8.1355	8.2207	0.0000	0.0000
37.0000	8.1378	8.2222	0.0000	0.0000
37.1000	8.1400	8.2236	0.0000	0.0000
37.2000	8.1422	8.2250	0.0000	0.0000
37.3000	8.1444	8.2265	0.0000	0.0000
37.4000	8.1465	8.2278	0.0000	0.0000
37.5000	8.1487	8.2292	0.0000	0.0000
37.6000	8.1508	8.2306	0.0000	0.0000
37.7000	8.1529	8.2319	0.0000	0.0000
37.8000	8.1549	8.2333	0.0000	0.0000
37.9000	8.1570	8.2346	0.0000	0.0000
38.0000	8.1590	8.2359	0.0000	0.0000
38.1000	8.1610	8.2371	0.0000	0.0000
38.2000	8.1630	8.2384	0.0000	0.0000
38.3000	8.1649	8.2397	0.0000	0.0000
38.4000	8.1669	8.2409	0.0000	0.0000
38.5000	8.1688	8.2421	0.0000	0.0000
38.6000	8.1707	8.2433	0.0000	0.0000
38.7000	8.1725	8.2445	0.0000	0.0000
38.8000	8.1744	8.2457	0.0000	0.0000
38.9000	8.1762	8.2468	0.0000	0.0000
39.0000	8.1780	8.2479	0.0000	0.0000
39.1000	8.1798	8.2491	0.0000	0.0000
39.2000	8.1815	8.2502	0.0000	0.0000

39.3000	8.1833	8.2513	0.0000	0.0000
39.4000	8.1850	8.2524	0.0000	0.0000
39.5000	8.1867	8.2534	0.0000	0.0000
39.6000	8.1884	8.2545	0.0000	0.0000
39.7000	8.1901	8.2555	0.0000	0.0000
39.8000	8.1917	8.2566	0.0000	0.0000
39.9000	8.1933	8.2576	0.0000	0.0000
40.0000	8.1950	8.2586	0.0000	0.0000
40.1000	8.1966	8.2596	0.0000	0.0000
40.2000	8.1981	8.2606	0.0000	0.0000
40.3000	8.1997	8.2615	0.0000	0.0000
40.4000	8.2012	8.2625	0.0000	0.0000
40.5000	8.2028	8.2634	0.0000	0.0000
40.6000	8.2043	8.2644	0.0000	0.0000
40.7000	8.2057	8.2653	0.0000	0.0000
40.8000	8.2072	8.2662	0.0000	0.0000
40.9000	8.2087	8.2671	0.0000	0.0000
41.0000	8.2101	8.2680	0.0000	0.0000
41.1000	8.2115	8.2689	0.0000	0.0000
41.2000	8.2130	8.2697	0.0000	0.0000
41.3000	8.2143	8.2706	0.0000	0.0000
41.4000	8.2157	8.2714	0.0000	0.0000
41.5000	8.2171	8.2722	0.0000	0.0000
41.6000	8.2184	8.2731	0.0000	0.0000
41.7000	8.2198	8.2739	0.0000	0.0000
41.8000	8.2211	8.2747	0.0000	0.0000
41.9000	8.2224	8.2755	0.0000	0.0000
42.0000	8.2237	8.2762	0.0000	0.0000
42.1000	8.2249	8.2770	0.0000	0.0000
42.2000	8.2262	8.2778	0.0000	0.0000
42.3000	8.2274	8.2785	0.0000	0.0000
42.4000	8.2287	8.2793	0.0000	0.0000
42.5000	8.2299	8.2800	0.0000	0.0000
42.6000	8.2311	8.2807	0.0000	0.0000
42.7000	8.2323	8.2814	0.0000	0.0000
42.8000	8.2335	8.2821	0.0000	0.0000
42.9000	8.2346	8.2828	0.0000	0.0000
43.0000	8.2358	8.2835	0.0000	0.0000

43.1000	8.2369	8.2842	0.0000	0.0000
43.2000	8.2380	8.2849	0.0000	0.0000
43.3000	8.2392	8.2855	0.0000	0.0000
43.4000	8.2403	8.2862	0.0000	0.0000
43.5000	8.2413	8.2868	0.0000	0.0000
43.6000	8.2424	8.2874	0.0000	0.0000
43.7000	8.2435	8.2881	0.0000	0.0000
43.8000	8.2445	8.2887	0.0000	0.0000
43.9000	8.2456	8.2893	0.0000	0.0000
44.0000	8.2466	8.2899	0.0000	0.0000
44.1000	8.2476	8.2905	0.0000	0.0000
44.2000	8.2486	8.2911	0.0000	0.0000
44.3000	8.2496	8.2917	0.0000	0.0000
44.4000	8.2506	8.2923	0.0000	0.0000
44.5000	8.2516	8.2928	0.0000	0.0000
44.6000	8.2525	8.2934	0.0000	0.0000
44.7000	8.2535	8.2939	0.0000	0.0000
44.8000	8.2544	8.2945	0.0000	0.0000
44.9000	8.2553	8.2950	0.0000	0.0000
45.0000	8.2563	8.2956	0.0000	0.0000
45.1000	8.2572	8.2961	0.0000	0.0000
45.2000	8.2581	8.2966	0.0000	0.0000
45.3000	8.2590	8.2971	0.0000	0.0000
45.4000	8.2598	8.2976	0.0000	0.0000
45.5000	8.2607	8.2981	0.0000	0.0000
45.6000	8.2616	8.2986	0.0000	0.0000
45.7000	8.2624	8.2991	0.0000	0.0000
45.8000	8.2632	8.2996	0.0000	0.0000
45.9000	8.2641	8.3001	0.0000	0.0000
46.0000	8.2649	8.3006	0.0000	0.0000
46.1000	8.2657	8.3010	0.0000	0.0000
46.2000	8.2665	8.3015	0.0000	0.0000
46.3000	8.2673	8.3019	0.0000	0.0000
46.4000	8.2681	8.3024	0.0000	0.0000
46.5000	8.2689	8.3028	0.0000	0.0000
46.6000	8.2696	8.3033	0.0000	0.0000
46.7000	8.2704	8.3037	0.0000	0.0000
46.8000	8.2711	8.3041	0.0000	0.0000

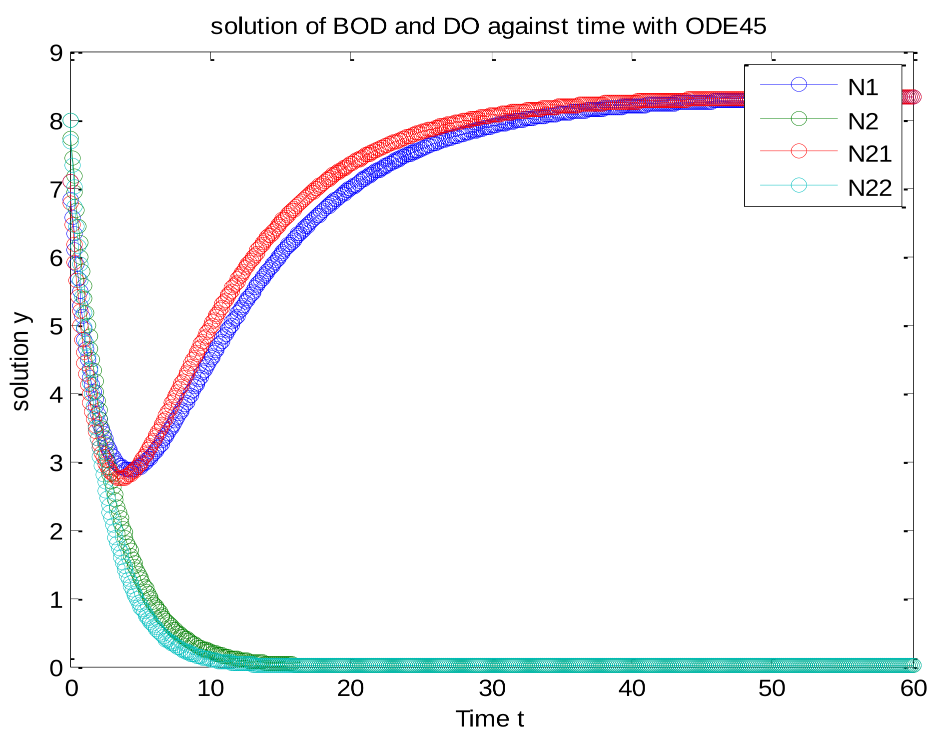
46.9000	8.2719	8.3045	0.0000	0.0000
47.0000	8.2726	8.3049	0.0000	0.0000
47.1000	8.2733	8.3053	0.0000	0.0000
47.2000	8.2741	8.3057	0.0000	0.0000
47.3000	8.2748	8.3061	0.0000	0.0000
47.4000	8.2755	8.3065	0.0000	0.0000
47.5000	8.2762	8.3069	0.0000	0.0000
47.6000	8.2768	8.3073	0.0000	0.0000
47.7000	8.2775	8.3077	0.0000	0.0000
47.8000	8.2782	8.3081	0.0000	0.0000
47.9000	8.2789	8.3084	0.0000	0.0000
48.0000	8.2795	8.3088	0.0000	0.0000
48.1000	8.2802	8.3092	0.0000	0.0000
48.2000	8.2808	8.3095	0.0000	0.0000
48.3000	8.2814	8.3099	0.0000	0.0000
48.4000	8.2821	8.3102	0.0000	0.0000
48.5000	8.2827	8.3105	0.0000	0.0000
48.6000	8.2833	8.3109	0.0000	0.0000
48.7000	8.2839	8.3112	0.0000	0.0000
48.8000	8.2845	8.3115	0.0000	0.0000
48.9000	8.2851	8.3119	0.0000	0.0000
49.0000	8.2857	8.3122	0.0000	0.0000
49.1000	8.2863	8.3125	0.0000	0.0000
49.2000	8.2868	8.3128	0.0000	0.0000
49.3000	8.2874	8.3131	0.0000	0.0000
49.4000	8.2880	8.3134	0.0000	0.0000
49.5000	8.2885	8.3137	0.0000	0.0000
49.6000	8.2891	8.3140	0.0000	0.0000
49.7000	8.2896	8.3143	0.0000	0.0000
49.8000	8.2901	8.3146	0.0000	0.0000
49.9000	8.2907	8.3149	0.0000	0.0000
50.0000	8.2912	8.3152	0.0000	0.0000
50.1000	8.2917	8.3155	0.0000	0.0000
50.2000	8.2922	8.3157	0.0000	0.0000
50.3000	8.2927	8.3160	0.0000	0.0000
50.4000	8.2932	8.3163	0.0000	0.0000
50.5000	8.2937	8.3166	0.0000	0.0000
50.6000	8.2942	8.3168	0.0000	0.0000

50.7000	8.2947	8.3171	0.0000	0.0000
50.8000	8.2952	8.3173	0.0000	0.0000
50.9000	8.2956	8.3176	0.0000	0.0000
51.0000	8.2961	8.3178	0.0000	0.0000
51.1000	8.2966	8.3181	0.0000	0.0000
51.2000	8.2970	8.3183	0.0000	0.0000
51.3000	8.2975	8.3186	0.0000	0.0000
51.4000	8.2979	8.3188	0.0000	0.0000
51.5000	8.2984	8.3190	0.0000	0.0000
51.6000	8.2988	8.3193	0.0000	0.0000
51.7000	8.2992	8.3195	0.0000	0.0000
51.8000	8.2997	8.3197	0.0000	0.0000
51.9000	8.3001	8.3199	0.0000	0.0000
52.0000	8.3005	8.3202	0.0000	0.0000
52.1000	8.3009	8.3204	0.0000	0.0000
52.2000	8.3013	8.3206	0.0000	0.0000
52.3000	8.3017	8.3208	0.0000	0.0000
52.4000	8.3021	8.3210	0.0000	0.0000
52.5000	8.3025	8.3212	0.0000	0.0000
52.6000	8.3029	8.3214	0.0000	0.0000
52.7000	8.3033	8.3216	0.0000	0.0000
52.8000	8.3037	8.3218	0.0000	0.0000
52.9000	8.3040	8.3220	0.0000	0.0000
53.0000	8.3044	8.3222	0.0000	0.0000
53.1000	8.3048	8.3224	0.0000	0.0000
53.2000	8.3052	8.3226	0.0000	0.0000
53.3000	8.3055	8.3228	0.0000	0.0000
53.4000	8.3059	8.3230	0.0000	0.0000
53.5000	8.3062	8.3231	0.0000	0.0000
53.6000	8.3066	8.3233	0.0000	0.0000
53.7000	8.3069	8.3235	0.0000	0.0000
53.8000	8.3073	8.3237	0.0000	0.0000
53.9000	8.3076	8.3239	0.0000	0.0000
54.0000	8.3079	8.3240	0.0000	0.0000
54.1000	8.3083	8.3242	0.0000	0.0000
54.2000	8.3086	8.3244	0.0000	0.0000
54.3000	8.3089	8.3245	0.0000	0.0000
54.4000	8.3092	8.3247	0.0000	0.0000

54.5000	8.3095	8.3248	0.0000	0.0000
54.6000	8.3099	8.3250	0.0000	0.0000
54.7000	8.3102	8.3252	0.0000	0.0000
54.8000	8.3105	8.3253	0.0000	0.0000
54.9000	8.3108	8.3255	0.0000	0.0000
55.0000	8.3111	8.3256	0.0000	0.0000
55.1000	8.3114	8.3258	0.0000	0.0000
55.2000	8.3117	8.3259	0.0000	0.0000
55.3000	8.3119	8.3260	0.0000	0.0000
55.4000	8.3122	8.3262	0.0000	0.0000
55.5000	8.3125	8.3263	0.0000	0.0000
55.6000	8.3128	8.3265	0.0000	0.0000
55.7000	8.3131	8.3266	0.0000	0.0000
55.8000	8.3133	8.3267	0.0000	0.0000
55.9000	8.3136	8.3269	0.0000	0.0000
56.0000	8.3139	8.3270	0.0000	0.0000
56.1000	8.3141	8.3271	0.0000	0.0000
56.2000	8.3144	8.3273	0.0000	0.0000
56.3000	8.3147	8.3274	0.0000	0.0000
56.4000	8.3149	8.3275	0.0000	0.0000
56.5000	8.3152	8.3276	0.0000	0.0000
56.6000	8.3154	8.3278	0.0000	0.0000
56.7000	8.3157	8.3279	0.0000	0.0000
56.8000	8.3159	8.3280	0.0000	0.0000
56.9000	8.3161	8.3281	0.0000	0.0000
57.0000	8.3164	8.3282	0.0000	0.0000
57.1000	8.3166	8.3284	0.0000	0.0000
57.2000	8.3168	8.3285	0.0000	0.0000
57.3000	8.3171	8.3286	0.0000	0.0000
57.4000	8.3173	8.3287	0.0000	0.0000
57.5000	8.3175	8.3288	0.0000	0.0000
57.6000	8.3177	8.3289	0.0000	0.0000
57.7000	8.3180	8.3290	0.0000	0.0000
57.8000	8.3182	8.3291	0.0000	0.0000
57.9000	8.3184	8.3292	0.0000	0.0000
58.0000	8.3186	8.3293	0.0000	0.0000
58.1000	8.3188	8.3294	0.0000	0.0000
58.2000	8.3190	8.3295	0.0000	0.0000

58.3000	8.3192	8.3296	0.0000	0.0000
58.4000	8.3194	8.3297	0.0000	0.0000
58.5000	8.3196	8.3298	0.0000	0.0000
58.6000	8.3198	8.3299	0.0000	0.0000
58.7000	8.3200	8.3300	0.0000	0.0000
58.8000	8.3202	8.3301	0.0000	0.0000
58.9000	8.3204	8.3302	0.0000	0.0000
59.0000	8.3206	8.3303	0.0000	0.0000
59.1000	8.3208	8.3304	0.0000	0.0000
59.2000	8.3210	8.3305	0.0000	0.0000
59.3000	8.3212	8.3306	0.0000	0.0000
59.4000	8.3214	8.3306	0.0000	0.0000
59.5000	8.3215	8.3307	0.0000	0.0000
59.6000	8.3217	8.3308	0.0000	0.0000
59.7000	8.3219	8.3309	0.0000	0.0000
59.8000	8.3221	8.3310	0.0000	0.0000
59.9000	8.3222	8.3311	0.0000	0.0000
60.0000	8.3224	8.3311	0.0000	0.0000

Trend of the solution trajectory



From the result obtained in the Table displayed on the **impact of temperature variation (Increasing Temperature @ T=28unit)** in the interaction between BOD (IC = 8.0) and DO (IC = 7.1) at an interval of 0(0.1)60 time in hrs, when all model parameter values are fixed ranging from the time interval of 0:0.1:60 in hrs. The BOD and DO initial values here called the IC's on the base time recorded as $BOD_0 = 8.0$ and $DO_0 = 7.1$ due to the pollution of the stream. Furthermore, from the base time, we observed a severe depletion of the DO downstream from a value of 7.1 units DO to a value of 4.0099 units DO for the uncontrol coordinate and 4.4353unit DO for the control (Modified) coordinate at 1.7hr of our experimental time.

In this scenario, above the point of waste water entry (above the DO IC on base day) is classified as the upstream which is called "clean water zone" and is characterized by clear and fresh water containing a stable aquatic environment which is conducive for natural fish, micro-invertebrates and plankton population as DO levels within this zone are near saturation.

However, as the pollution enters the stream, a short zone which is observed from the base day of our experimental time up to 1.7hrs is identified as the "zone of degradation" because of its severe depletion in the DO coordinates downstream. This zone is considered to be turbid as a result of the severe depletion of the DO and as such sunlight is usually reduced with depth in the stream. Moreso, this zone degradation will record a drastic change in chemical characteristics of the stream which is captured in the data been analysed, which include scenarios such as up to a 40% reduction of the DO from the initial condition as observed from the base time up to 1.7hrs with an increase in CO_2 concentration, Nitrogen present in organic form as well as bacterial activities increases. Green and blue algae will be present, fungi will be present, and protozoa (ciliates) will be in relative abundance as well as blood worm.

Again, from the interval of (1.7-8.7) hrs is regarded as a "zone of active decomposition" downstream due to pollution with a critical DO deficit of 2.8682 unit at a critical time at 4.3hrs of our experimental time on the uncontrol coordinates and a critical DO_m deficit of 2.7514unit at a critical time between 3.6-3.7hrs of our experimental time for the control coordinates. The physical characteristics of this period is featured with water that is grey or black in colour, the presence of offensive odours and no light penetration through the stream. As the stream travels within this interval in days, DO and DO_m concentration starts at 40% of the initial condition as seen at the 1.7hrs of our experimental time which drops to the critical DO deficit base on the relative minimum curve as seen at 4.3hrs for the uncontrol coordinates and 3.6-3.7hrs for the control coordinates(DO_m) of our experimental time and as such this interval is a local minimum interval or relative minimum interval with a turning point and its associated critical DO deficit and critical characteristics of gases such as H_2S, C_H4 and NH_3, which are normally produced as DO is depleted and contributes to the offensive odour problem in the stream.

Most importantly, as DO level drops, bacteria and algae may be the only life form present in the stream column, while other aquatic species both flora and fauna are vulnerable or may tend to biological extinction or possible migration. Though a slight recovery in the coordinate of the DO at 4.4hrs with DO= 2.8699unit for the uncontrol coordinate and DO_m at 3.8hrs with DO_m =2.7543 for the control coordinate of our experimental time. We observed that after the grey area of active decomposition, there is a relative long interval which starts from 8.7hrs up to 20.00hrs which is the inflexion point (hrs). This interval which occurs between the 8.7-20.00hrs is called "interval of active recovery" up to the point of inflexion. The feature of this interval has chemical characteristics which includes: steady rise in DO concentration from 40% of the initial value of the DO concentration as observed at 4.4hrs and the DO_m concentration as observed at 3.8hrs increased monotonically up the saturation level of the DO. Due to this steady increase in the DO, the CO_2 level and Nitrogen present as NH3, and organic form decreases as well. Moreso, biological characteristics within this interval has features such as drastic decrease in the relative abundance of the bacteria present due to external temperature influences and a decrease of the presence of protozoa, blue and green algae as well as decrease of the presence of bloodworm. Most importantly, after this zone or interval of active recovery, we observed another interval starting from the point of inflexion which occur on the 20.0hrs up to 60.0hrs of our experimental time. This interval is called a zone of cleaner water and conducive aquatic environment, the characteristics of this interval is due to intervention of external oxygen, temperature and other meteorological factors. The physical, chemical, and biological characteristics of the stream can be said to have returned to normalcy, and to the conditions present upstream of the pollution source with complete saturation on the 60.0hrs of our experimental time with DO value of 8.3224unit and DO_m value of 8.3311.

However, in the coordinates of the BOD, we observed a monotonic decreasing pattern in the concentration from the base time which depleted drastically down the trend to a saturated BOD at 60.0hrs for both the control and uncontrol coordinates.

This observation is consistent and vital for environmental policy making and decision which will improve the scope of protecting the river banks and aquatic environment as a result of pollution. The information discussed here captures grey area of vulnerability of the aquatic environment and will serve as a data base monitoring system for intervention and mitigation due to the pollution of the stream which will be very useful to the environmental protection agency and other related bodies concerned with pollution.

CONCLUSION

Using the analytical method to study the qualitative characterization of the dynamical system is tasking and did not give an early insight of the expected results and also involved some errors due to approximation, we introduced the core method for this work which is the computational method. Through this method, we were able to predict the relative abundance of the BOD-DO in their interactions over a period of $60hrs\ at\ an\ interval\ 0:0.1:60\ hrs$ using MatLab ODE45 numerical scheme. It was observed that there was a decrease in the in the relative abundance of both coordinates and a point of recovery in the DO coordinates with its DO deficit up to it complete saturation.

The differential equation model for effluents discharged used in this work is very effective in the determination and prediction of the relative abundance of the BOD-DO coordinates over a period of time.

RECOMMENDATION

To researchers who may want to do similar works as this, we recommend that:

The impact of the growth rate should be check using computational method.

The impact varying initial conditions should be predicted using computational method.

Similar method should be adopted to study the qualitative behavior other physiochemical properties over time.

REFERENCES

Ahamad, S., & Rao M.R.M. (1999). Theory of Ordinary Differential Equations with Applications in Biology and Engineering, East West Press Privet Ltd. New Delhi.

Akpodee, R.E. (2019). Semi-Stochastic Analysis of Environmental Variables Using Numerical Method on a System of Second Order Ordinary Differential Equations. A PhD. Thesis Submitted to the Department of Mathematics, Rivers State University, Port Harcourt, Nigeria.

Akpodee, R.E., & E.N. Ekaka-a. (2019). The Effect of Random Environmental Perturbation on the Bifurcation Interval and the Type of Stability of Two Interacting Environmental Variables Using a Second Order Ordinary Differential Equation. MPRI-JERT, International journal of Engineering and Research Technology Vol10 (5) Dec-2019 ISSN:1074-4741.

Ansari T. M., Marr, I. L., & Tariq N. (2004). Heavy Metals in Marine Pollution Perspective: A mini Review. Journal of Applied Science 4(1), 1-20, 2004.

Kapur, J. N. (2015). Mathematical Modelling, Second Edition, New age international publishers.

Kaushik, R. (2015). Mathematical modelling on water pollution and self-purification of river Ganges. Pelagia Res Libr, 6(7), 57-64.

Korotenko, K. A. (2016). High-resolution numerical model for predicting the transport and dispersal of oil spill in result of accidental deepwater blowout in the Black Sea. In The 26th International Ocean and Polar Engineering Conference.

Shukla, J. B., Misra, A. K., & Chandra, P. (2007). Mathematical modelling of the survival of a biological species in polluted water bodies. Differential Equations and Dynamical Systems, 15(3/4), 209-230.Jeffrey, A and Dai, H. H (2008). Handbook of mathematical formulars and integrals 4th ed. London, United Kingdom; Academic Press, Elsevier

Siddiqua, S., Chaturvedi, A., & Gupta, R. (2020). A Review-Mathematical Modelling on Water Pollution and Its Effects on Aquatic Species. In Advances in Applied Mathematics Conference (pp. 835-842).

Streeter, H. W. (1925). Studies of the pollution and natural purification of the Ohio River, Part III, Factors concerned in the phenomena of oxidation and reareation. Public health bulletin, (146).

Sunita, N., (2014) Ganga- the River, its pollution and what can we do to clean it, Centre for Science and Environment.

Tan, A. K. J. (2005). Vessel-source marine pollution: the law and politics of international regulation (Vol. 45). Cambridge University Press. TOPF. (2002). Fate of marine oil spill. Retrieved June 2013, from http://www.itopf.com/marinespills/fate/weatheringprocess/documents/tip2.pdf

Tchobanoglous, G., Ruppe, L., Leverenz, H., & Darby, J. (2004). Decentralized wastewater management: challenges and opportunities for the twenty-first century. Water Science and Technology: Water Supply, 4(1), 95-102.

Tchobanoglus, G., Burton, F., & Stensel, H. D. (2003). Wastewater engineering: treatment and reuse. American Water Works Association. Journal, 95(5), 201.

Terzhevik, A., Golosov, S., Palshin, N., Mitrokhov, A., Zdorovennov, R., Zdorovennova, G., ... & Zverev, I. (2009). Some features of the thermal and dissolved oxygen structure in boreal, shallow ice-covered Lake Vendyurskoe, Russia. Aquatic Ecology, 43(3), 617-627.