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APPLICATION OF MODIFIED LOKTA VOLTERRA MODEL FOR THE ESTABLISHMENT OF REGION OF STABILITY IN A PREDOMINANTLY INFECTED FINANCIAL MARKET.

[1]OZUITEM ISAAC DICKSON, [2]IBE GODSWILL ANAYO, [3]ISIBOR LUCKY; & [4]UDOH NDIPMONG

[1]Dept. of Cooperative Economy and Management, Federal Polytechnic, Nekede, Owerri. [2]Mathematics and Statistics Department, Federal Polytechnic, Nekede, Owerri. [3]Administrative Department, Emerald College, Port Harcourt. [4]Department of Mathematics, Niger Delta University, Bayelsa State.

Abstract:

Prey-Predator models applied to eco-system is a well known area of scientific research which is gaining prominence in other areas of application particularly in financial analysis. This research work is an effort to apply a modified version of the Lokta Volterra Prey-Predator model in financial market-Nigerian stock market. We considered a market predominantly infested with fraudulent operators, so as to ascertain region of stability where it will be safe for an investor to play in the market with less risk. The study employed a symbiosis or mutualism model whose analytic results were compared with the simulated result. Our findings which do not deviate much from those established in literature Lettitia et al (2017) shows that by carefully monitoring the changes in relevant parameter values of the model, an investor will be able to ascertain regions of stability where it is safe to invest. These stability parameter values would allow better decision-making especially when a large company is interested in investing in smaller companies.

Keywords: *Modified Prey-Predator Model, Stability region, Fraudulent Financial Market, Simulation.*

Introduction:

Corruption and Fraud in Financial Markets identifies potential issues surrounding all types of fraud, misconduct, price/volume manipulation and other forms of malpractices. It embodies the practices of informed but corrupt traders manipulating prices in dark pools which are run by investment banks. By implementing anonymous deals, they move prices in their own favour, extracting value from ordinary investors time and time again. More generally, financial market misconduct and fraud affects about 15 percent of publicly listed companies each year and the resulting fines can devastate an organisation's budget and initiate a tailspin from which it may never recover. Stephen *et al* (2015). The possibility of detection, prevention and regulation of corruption and fraud within different financial markets becomes a vital component in such market domain and academic research endeavors.

The Lotka-Volterra equations are nonlinear differential equations of the first order, also known as the predator-prey equations. This has been used to explain the dynamics of biological environments in which two or more animals interact, one as a predator and the other as prey. Lotka–Volterra models can be applied in studies of many natural phenomena, including but not limited to finance business, epidemiology, engineering systems and many other areas of studies. Padma *et al* (2020).

The dynamical relationship between predators, in this case financial experts and their prey, potential Investors has been in existence and will continue to be one of the dominant areas of study in financial ecology in relation to biological ecology. This is due to the universal applicability and importance in many field of studies, since species interaction in natural and wildlife environments is unavoidable. These leads to the question, how does this primal concern on survival impact the way in which species interact? Lokta, a chemist and statistician and Volterra, a mathematician studied the ecological problem of interaction between biological species and came up with a model often called Lokta Volterra Prey-Predator model. Different species occupying similar ecosystems and living amongst each other will regularly interact.

The interaction between two or more species could be positive (+), negative (-) or Neutral (0) and the combination of these effects results in one or a combination of the following types of interactions: Mutualism or symbiolysis interaction (+,+), Exploitative interaction (+,-), Commensalism (+, 0), Inter-Specific Competition (-, -), Amensalism (-, 0), Neutrality (0, 0). Murray (2002) Ibe *et al* (2021).

The interaction between the operators of financial market is aimed at making returns on investment to both the prey and the predator populations. This desire to make return has lead to application of different techniques some of which maybe fraudulent by both parties in other to maximize profit. On the other hand, every investor, honest or dishonest seeks to protect their investment. Lettitia *et al*(2017) considered a case of inter specific competition between two predators interacting with a prey species in an exploitative interaction. In this current study, we will dwell on a case where the predators interact in a symbiolysis or mutualism relationship with the prey so that we can establish a region of safety for profitability and security for both investors and proffer mechanism to control or detect fraud in financial market Ghezzi (1992).

We considered a financial market which is predominantly infected as it were, being fraud laden using Modified Lokta Volterra to investigate the effect of infection (fraud) on the coexistence of operators and to demonstrate the stability of the model with respect to the number of potential financial expert and investors and find the optimal parameters for which the financial operators can coexist to the mutual interest of both.

Literature

Lokta Volterra model is a system of coupled non linear ordinary differential equations which could be applied in a variety of studies involving interaction between species. Tingchun and Wei (2012) refers to such interaction as a struggle for existence of two or more species in an environment; one such specie is termed prey and the other, a predator. Letitia *et al* (2017) applied this idea in the study of financial market. Such prey-predator mathematical population models have been used to study the dynamics of prey predator systems since Lotka (1925) and Volterra (1927) proposed the simple model of prey-predator interactions and so many mathematical models has been constructed based on this simple model.Ahmed Buseri & Dawit Melese Gebru (2017) worked on the theme; 'Mathematical Modeling of a Predator-Prey Model with Modified Leslie-Gower and Holling-type II Schemes'. They used, a two-dimensional continuous predator-prey system with Holling type II functional response and a modification of Leslie–Gower to compare two schemes, one with prey refuge and the other without a prey refuge. In both cases, by non-dimensionalize the system, the fixed points were computed and condition for local and global asymptotic stability of the system are obtained. Moreover, the global asymptotic stability of the system is proved by defining appropriate Dulac function. Numerical simulations are also carried out to verify the analytical results.

Abraham and Masuda (1993), noted that the cyclic behavior of economy is explained by the financial structure of investment, while Wilcox (1999) maintained that the dynamics in investment markets are analogous to organisms competing for information on prices. Hence, ecological ideas can assist in the partial understanding of the complex dynamics involved in the financial eco-system. The Great Depression of 1929 and the financial recession crisis in 2008 have both inspired financial experts and mathematicians alike, to study models which can assist in prediction of such anomaly events and is the motivation for the current study.

Rachel (2013) Predator Prey Models in Competitive Corporations aimed at determining whether the relationship between competitive corporations can be significantly modeled by a predator prey model. Examining the application of Predator prey models as mathematical models used by bio-mathematicians to describe relative population sizes of a predator and its prey in a competitive environment over time, the study thus decided to apply same in competitive Corporations. He followed the pattern found in literature of previous studies used to analyze any number of economic situations, including but not limited to competition in the Korean stock market Lee, Lee & Oh, (2005) and the competition between ballpoint and fountain pens Modis, (2003). The simplest predator prey model used for this work is based on the Lotka-Volterra model, which is the most common of predator-prey models and relates one type of predator to one type of prey.

Padma *et al* (2020) examined first, the existence of non negative solutions for the several species Lotka-Volterra systems (cross-diffusion). This was done by employing Sobolev embedding theorems in other to probe the coexistence region in the plane of argument. Secondly, they applied the persistent deforming analysis method (PDAM) to investigate the change of state brought about by the passage of time on the several species Lotka-Volterra models. Based on their investigations, they were able to provide the right and most appropriate persistent solution for the several nonlinear species Lotka-Volterra system. The accuracy of their solution is equivalent to the accuracy of the solution which was obtained by a purely numerical fourth-order method in confirmation of the method.

The Lotka-Volterra model has since been expanded and modified in numerous ways to better model certain situations. In this paper we will also utilize a two-predator, one-prey model and a ratio-dependent model.

Letetia *et al* (2017) proposed a modified predator-prey model with logistic growth in both Zrey and predator populations with an infection in the predator population. They used the original biological LotkaVolterra model to imitate a financial market in which investors interact with financial experts in a prey-predator mode. predation. In their work, the co-existence of the potential investors and the financial experts are treated as a system that needs to be analysed using Hopf bifurcation and stability analysis to determine the region of stability. By using different datasets, they verified the results were using numerical simulations in MATLAB and showed that variation of different parameters can affect the stability of the system and the co-existence of potential investors and financial experts over time.

Methodology:

The Lokta Volterra model is a nonlinear first order ordinary differential equation. The model was modified by subjecting the interacting variables to positive interaction Addicot (2000). Linear stability analysis was carried out to see the dynamical behavior of the new system. We further carried out numerical simulation using excel based Matlab principles so as to compare results with analytic solution.

The Lokta Volterra Model of interaction between three species is given as

$$\frac{dx_1}{dt} = a_1 x_1\left(1-\frac{x_1}{k_1}\right) - m_{12}x_1x_2 - v_1x_1yf_1(x_1,x_2,y) \qquad (3.1)$$

$$\frac{dx_2}{dt} = a_2 x_2\left(1-\frac{x_2}{k_2}\right) - m_{21}x_1x_2 - v_2x_2yf_2(x_1,x_2,y) \qquad (3.2)$$

$$\frac{dy}{dt} = -\gamma y + c_1v_1x_1yf_1(x_1,x_2,y) + c_2v_2x_2yf_2(x_1,x_2,y) \qquad (3.3)$$

Where

$$f_1(x_1,x_2,y) = \frac{1}{1+\left(\frac{d_2x_2+e_2y}{x_1}\right)^n} \text{ and } f_2(x_1,x_2,y) = \frac{1}{1+\left(\frac{d_1x_1+e_1y}{x_2}\right)^n} \qquad (3.4)$$

Equation (3.4) is a predatory function which is assumed to be smooth and positive with Taylors's expansion about (x_1 and x_2) with an equilibrium point given as (x_i, x_2, y). *n:* multiplicative effect due to predatory functional response.

If the multiplicative effect due to the predatory functional response n is unity, the equation (3.4) becomes

$$f_1(x_1, x_2, y) = \frac{x_1}{x_1 + a_2 x_2 + b_2 y} \text{ and } f_2(x_1, x_2, y) = \frac{x_2}{x_2 + a_1 x_1 + b_1 y} \qquad (3.5)$$

On substituting equation (3.5) into equestion (3.1 – 3.3), we have

$$\frac{dx_1}{dt} = a_1 x_1 \left(1 - \frac{x_1}{k_1}\right) - m_{12} x_1 x_2 - \frac{v_1 (x_2)^2 y}{x_1 + a_2 x_2 + b_2 y} \qquad (3.6)$$

$$\frac{dx_2}{dt} = a_2 x_2 \left(1 - \frac{x_2}{k_2}\right) - m_{21} x_1 x_2 - \frac{v_2 (x_1)^2 y}{x_2 + a_1 x_1 + b_1 y} \qquad (3.7)$$

$$\frac{dy}{dt} = -by + \frac{c_1 v_1 (x_1)^2 y}{x_1 + a_2 x_2 + b_2 y} + c_2 v_2 x_2 y \frac{c_2 v_2 (x_2) y}{x_2 + a_1 x_1 + b_1 y} \qquad (3.8)$$

In the above model, (3.6 – 3.8), we define the relative share price as the price of the share divided by average stock exchange stock price. We represent this by the variables x_1 and x_2 and declare them the competing prey financial companies on which are to be feasted the predator company y. y is the relative price per share of the predator company. The key players are potential investors – the Prey and the financial experts made up of financial experts such as bankers brokers, agents etc. in two groups of either honest or dishonest operators who compete for the shares of the predator either by honest means or otherwise.
Thus suppose, the relative share prices x_i for $i = 1, 2$ S_r is taken to be the ratio of the price of share S_p and the volume of share S_v. the relative share price

$$x_i = \text{share price/share volume} = S_r = S_p/S_v \qquad (3.5)$$

The model parameters are said to be all non-negatives and defined as follows:
a_i: the intrinsic growth factor which determines the rate at which the prey's share price grows
γ: the rate at which the share price of the predator declines
m_{ij}: the inter-specific competition between the prey's market shares for $i, j = 1, 2$
c_i: rate at which the prey company's share is converted to the predator company's share

v_i: the likely hood that predator company y will invest in prey company x_i (i.e capture rate)
d_i : rate at which the prey shares are harvested by the predator
e_i: Predatory resistance action of prey share prices
k_i: is the maximum carrying capacity of the prey share.
The above model was analysed by Lettitia et al (2017), we thus modify them to reflect a symbiosis interaction.

$$\frac{dx_1}{dt} = a_1 x_1\left(1-\frac{x_1}{k_1}\right) + m_{12}x_1x_2 - \frac{v_1(x_2)^2 y}{x_1 + a_2x_2 + b_2 y} \quad (3.9)$$

$$\frac{dx_2}{dt} = a_2 x_2\left(1-\frac{x_2}{k_2}\right) + m_{21}x_1x_2 + \frac{v_2(x_1)^2 y}{x_2 + a_1x_1 + b_1 y} \quad (3.10)$$

$$\frac{dy}{dt} = -by + \frac{c_1v_1(x_1)^2 y}{x_1 + a_2x_2 + b_2 y} + c_2v_2x_2y\,\frac{c_2v_2(x_2)y}{x_2 + a_1x_1 + b_1 y} \quad (3.11)$$

Equation (3.9 – 3.11) is a prey-predator mutualism model.

Analytical Solution

Stationery State Solutions (Equilibrium Point)

To obtain the point of equilibrium we set

$$\frac{dx_1}{dt} = \frac{dx_2}{dt} = \frac{dy}{dt} = 0$$

equation (3.9 – 3.11) becomes

$$a_1 x_1\left(1-\frac{x_1}{k_1}\right) + m_{12}x_1x_2 - \frac{v_1(x_2)^2 y}{x_1 + a_2x_2 + b_2 y} = 0 \quad (3.12)$$

$$a_2 x_2\left(1-\frac{x_2}{k_2}\right) + m_{21}x_1x_2 - \frac{v_2(x_1)^2 y}{x_2 + a_1x_1 + b_1 y} = 0 \quad (3.13)$$

$$-\gamma y + \frac{c_1v_1(x_1)^2 y}{x_1 + a_2x_2 + b_2 y} + \frac{c_2v_2(x_2)^2 y}{x_2 + a_1x_1 + b_1 y} = 0 \quad (3.14)$$

Equation (3.12) and (3.13) gives

$$a_1 x_1\left(1-\frac{x_1}{k_1}\right) + m_{12}x_1x_2 = \frac{v_1(x_2)^2 y}{x_1 + a_2x_2 + b_2 y} \quad (3.15)$$

$$a_2x_2\left(1-\frac{x_2}{k_2}\right)+m_{21}x_1x_2=-\frac{v_2(x_1)^2y}{x_2+a_1x_1+b_1y} \quad (3.16)$$

Using (3.15) and (3.16) we obtain three relations for y as follows

$$y=\frac{1}{\gamma}\left\{c_1(a_1x_1\left(1-\frac{x_1}{k_1}\right)+m_{12}x_1x_2)-c_2(a_2x_2\left(1-\frac{x_2}{k_2}\right)+m_{21}x_1x_2)\right\} \quad (3.17)$$

$$y=\frac{\left(c_1\left(1-\frac{x_1}{k_1}\right)+m_{12}x_2)\right)(x_1+a_2x_2)}{v_1x_1+b_2\left((m_{12}x_2-a_1\left(1-\frac{x_1}{k_2}\right)\right)} \quad (3.18)$$

$$y=\frac{\left(c_2\left(1-\frac{x_2}{k_2}\right)+m_{21}x_1)\right)(x_2+a_1x_1)}{v_2x_2+b_1\left((m_{21}x_1-a_2\left(1-\frac{x_2}{k_1}\right)\right)} \quad (3.19)$$

Equation (3.17 – 3.19) leads to

$$f(x_1,x_2)=\frac{\left(a_1\left(1-\frac{x_1}{k_1}\right)+m_{12}x_2\right)(x_1+a_2x_2)}{v_1x_1+b_2\left(m_{12}x_2-a_1\left(1-\frac{x_1}{k_1}\right)\right)}-\frac{1}{\gamma}\left\{c_1\left(a_1x_1\left(1-\frac{x_1}{k_1}\right)+m_{12}x_1x_2\right)+c_2\left(a_2x_2\left(1-\frac{x_2}{k_2}\right)+m_{21}x_1x_2\right)\right\}=0 \quad (3.20)$$

$$g(x_1,x_2)=\frac{\left(a_2\left(1-\frac{x_2}{k_2}\right)+m_{21}x_1\right)(x_2+a_1x_1)}{v_2x_2+b_1\left(m_{21}x_1-a_2\left(1-\frac{x_2}{k_2}\right)\right)}-\frac{1}{\gamma}\left\{c_1\left(a_1x_1\left(1-\frac{x_1}{k_1}\right)+m_{12}x_1x_2\right)+c_2\left(a_2x_2\left(1-\frac{x_2}{k_2}\right)+m_{21}x_1x_2\right)\right\}=0 \quad (3.21)$$

Equation (3.20) and (3.21) provides a solution for x_1 and x_2.

Given that x_1 and x_2 are all positive, the solution should satisfy the conditions

$$\Omega_1 > 0 \text{ and } \Omega_2 > 0 \quad (3.22)$$

where $\Omega_1 = v_1x_1 + b_2(m_{12} - \alpha_1(1 - x_1/k_1))$

and

$\Omega_2 = v_2x_2 + b_1(m_{21} - \alpha_2(1 - x_2/k_2))$

This follows from the fact that the denominators are always positive, therefore the parameters are positive.

Considering equation (3.20) and letting $x_1 \to 0$, and $x_2 \to 0$, we have

$$f(0,x_2)=\frac{(a_1+m_{12}x_2)(a_2x_2)}{b_2(m_{12}x_2-a_1)}-\left\{+c_2\left(a_2x_2\left(1-\frac{x_2}{k_2}\right)\right)\right\}=0$$

(3.23)

and

$$g(x_1,0)=\frac{(a_2+m_{21}x_1)(a_1x_1)}{vb_1(m_{21}x_1-a_2)}-\frac{1}{\gamma}\left\{c_1\left(a_1x_1\left(1-\frac{x_1}{k_1}\right)\right)\right\}=0$$

(3.24)

So as $x_1 \rightarrow 0, x_2 \rightarrow x_{2a}$ and $x_2 \rightarrow 0, x_1 \rightarrow x_{2b}$ where x_{2a} and x_{2b} are respectively

$$x_{2a} = \frac{(c_2\alpha_2 b_1+\gamma)k_2}{c_2\alpha_2 b_1+c_2 v_2} \quad (3.25)$$

$$x_{2b} = \frac{(c_1\alpha_1 b_1+a_1\gamma)k_1}{c_1\alpha_1 b_1} \quad (3.26)$$

Considering equation (3.21) and letting $x_1 \rightarrow 0$, and $x_2 \rightarrow 0$we have

$$f(0,x_2)=\frac{(a_1+m_{12}x_2)(a_2x_2)}{b_2(m_{12}x_2-a_1)}-\left\{+c_2\left(a_2x_2\left(1-\frac{x_2}{k_2}\right)\right)\right\}=0$$

$$g(x_1,0)=\frac{(a_2+m_{21}x_1)(a_1x_1)}{vb_1(m_{21}x_1-a_2)}-\frac{1}{\gamma}\left\{c_1\left(a_1x_1\left(1-\frac{x_1}{k_1}\right)\right)\right\}=0$$

So as $x_2 \rightarrow 0, x_1 \rightarrow x_{1a}$ and $x_1 \rightarrow 0, x_2 \rightarrow x_{1b}$ where x_{1a} and x_{1b} are respectively

$$x_{1a} = \frac{(c_1\alpha_1 b_2+\gamma)k_1}{c_1\alpha_1 b_2+k_1 v_1} \quad (3.27)$$

$$x_{1b} = \frac{(c_2\alpha_2 b_2+\alpha_2\gamma)k_2}{c_2\alpha_2 b_2} \quad (3.28)$$

The stationary state solutions of equations (3.23) and (3.24) are given by equations (3.25 – 3.26) so far as all the parameters are positive .

.It can be shown that $x_{2a} < x_{2b}$ and $x_{1a} \leq x_{1b}$ from which we state that the positive equilibrium points (x_1, x_2, $f(x_1)$ and $f(x_2)$) exists *iff* $x_1 > 0$ and $x_2 > 0$ satisfying the condition stated in equation (3.22).

Stability Analysis

Let us consider linearising the original system by introducing a little disturbance (i.e perturbation)

$$X_1 = X_1+\overline{X_1}; \quad X_2 = X_2+\overline{X_2} \text{ and } Y=Y+\overline{Y} \quad (3.29)$$

Where $\overline{X_1}$, $\overline{X_2}$ and $\overline{Y}$ are small perturbations about the stationary state solution of the system.

Expanding the terms in equation (3.29) about the equilibrium points and neglecting higher powers of $\overline{X_1}$, $\overline{X_2}$ and $\overline{Y}$. The characteristic polynomial is given as

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0 \qquad (3.30)$$

Where *P*, *Q* and *R* are constant coefficients given as

$P = -J11 - J22 - J33;$ $Q = J_{22}J_{11} - J_{12}J_{21} + J_{33}(J_{11} + J_{22}) - J_{31}J_{13} - J_{32}J_{23}$ and

$R = -J_{11}J_{22}J_{33} + J_{21}J_{12}J_{33} - J_{12}J_{31}J_{23} + J_{11}J_{32}J_{23} - J_{12}J_{21}J_{32} + J_{13}J_{31}J_{22}$

J_{ij} for $i = 1, 2, 3$ and $j = 1, 2, 3$ are Jacobian Matrix at the equilibrium points given as

$$J_{11} = \alpha_1\left(1 - \frac{2x_1}{k_1}\right) + m_{12}x_2 - v_1x_1y\frac{2(a_2x_2 + b_2y) + x_1}{(x_1 + \alpha_2x_2 + b_2y)^2}$$

$$J_{12} = +m_{12}x_1 + v_1x_1^2y\frac{a_2}{(x_1 + \alpha_2x_2 + b_2y)^2}$$

$$J_{13} = -v_1(x_1^2)\frac{x_1 + a_2x_2}{(x_1 + \alpha_2x_2 + b_2y)^2}$$

$$J_{21} = +m_{21}x_2 + v_2x_1^2y\frac{a_1}{(x_2 + \alpha_1x_1 + b_{2=1}y)^2}$$

$$J_{22} = \alpha_2\left(1 - \frac{2x_2}{k_1}\right) + m_{21}x_1 - v_2x_2y\frac{2(a_1x_1 + b_1y) + x_2}{(x_2 + \alpha_1x_1 + b_1y)^2} \qquad J_{23} = -v_2x_1^2\frac{x_2 + a_1x_1}{(x_2 + \alpha_1x_1 + b_1y)^2}$$

$$J_{31} = c_1v_1x_1y\frac{2(a_2x_2 + b_2y) + x_1}{(x_1 + \alpha_2x_2 + b_2y)^2} - c_2v_2x_1^2y\frac{a_1}{x_2 + a_2x_2 + b_1y}$$

$$J_{32} = c_2v_2x_2y\frac{2(a_1x_2 + b_2y) + x_2}{(x_2 + \alpha_1x_1 + b_2y)^2} - c_1v_1x_1^2y\frac{a_2}{(x_2 + a_2x_2 + b_2y)^2}$$

$$J_{33} = -\gamma + c_1v_1x_1^2\frac{x_1 + a_2x_2}{(x_1 + a_2x_2 + b_2y)^2} + c_2v_2x_2^2\frac{x_2 + a_1x_1}{(x_2 + \alpha_1x_1 + b_1y)^2}$$

The condition for stability is $P > 0$; $R > 0$ and $PQ - R > 0$. Lettitia *et al* (2016).

Numerical Result

Recall equations (3.25 – 3.28) given as

$x_{2a} = \frac{(c_2\alpha_2 b_1 + \gamma)k_2}{c_2\alpha_2 b_1 + c_2 v_2}$; $x_{2b} = \frac{(c_1\alpha_1 b_2 + \gamma)k_1}{c_1\alpha_1 b_2 + c_1 v_1}$; $x_{1a} = \frac{(c_1\alpha_1 b_2 + \gamma)k_1}{c_1\alpha_1 b_2 + k_1 v_1}$; and $x_{1b} = \frac{(c_2\alpha_2 b_2 + \alpha_2\gamma)k_2}{c_2\alpha_2 b_2}$

Using the parameters values adopted from Letitia *et al* (2007).

Table 4.1: Data set

Alpha1	*Alpha2*	k_1	k_2	m_{12}	m_{21}	a_1	a_2
0.04	0.05	5	10	0.01	0.02	0.02	1.5
b_1	b_2	c_1	c_2	v_1	v_2	*gema*	
0.01	0.03	0.1	0.2	0.03	0.03	0.02	

By using the stationary state solutions of equations (3.25 – 3.28), we simulated the interaction by varying the share's conversion rate between 10% and 200% while other parameters are kept constant. The result is displayed in the table below.

Table 4.2a: Simulation values before Variation

Period	x1a	x2a	x1b	x2b
1	0.67013	0.3295	0.0677	1.5758
2	0.50270	0.2481	0.0677	1.2093
3	0.40221	0.1990	0.0677	0.9811
4	0.33520	0.1661	0.0677	0.8254
5	0.28733	0.1426	0.0677	0.7123
6	0.25142	0.1248	0.0677	0.6265
7	0.22350	0.1110	0.0677	0.5591
8	0.20115	0.1000	0.0677	0.5049
9	0.18287	0.0910	0.0677	0.4602
10	0.16763	0.0834	0.0677	0.4228
11	0.15474	0.0770	0.0677	0.3910
12	0.14369	0.0715	0.0677	0.3636
13	0.13411	0.0668	0.0677	0.3399
14	0.12573	0.0626	0.0677	0.3190
15	0.11834	0.0589	0.0677	0.3006
16	0.11834	0.0589	0.0677	0.3006

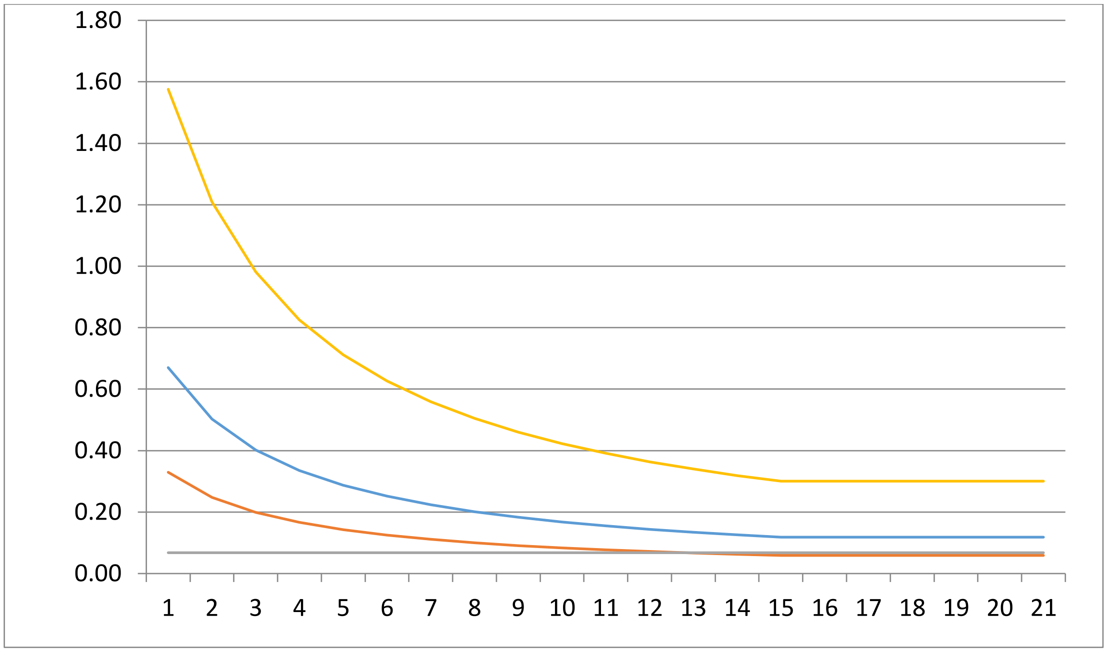
17	0.11834	0.0589	0.0677	0.3006
18	0.11834	0.0589	0.0677	0.3006
19	0.11834	0.0589	0.0677	0.3006
20	0.11834	0.0589	0.0677	0.3006
21	0.11834	0.0589	0.0677	0.3006
Total	**4.61077**	**2.28613**	**1.42223**	**11.39491**
Average	**0.21956**	**0.10886**	**0.06773**	**0.54261**

Table 4.2b: Simulation table after variation

Period	x1a	x2a	x1b	x2b
1	0.67013	0.3295	0.0677	1.5758
2	0.50027	2.4704	0.6752	9.5814
3	0.40044	0.9911	0.3377	4.0000
4	0.33389	0.5518	0.2252	2.3069
5	0.28636	0.3553	0.1690	1.5342
6	0.25071	0.2491	0.1352	1.1084
7	0.22299	0.1847	0.1127	0.8459
8	0.20081	0.1426	0.0967	0.6713
9	0.18266	0.1136	0.0846	0.5487
10	0.16754	0.0926	0.0752	0.4589
11	0.15474	0.0770	0.0677	0.3910
12	0.14377	0.0651	0.0616	0.3382
13	0.13427	0.0557	0.0565	0.2963
14	0.12595	0.0482	0.0521	0.2624
15	0.11861	0.0422	0.0484	0.2345
16	0.11868	0.1176	0.1352	0.2261
17	0.11875	0.0980	0.1127	0.2162
18	0.11882	0.0841	0.0967	0.2075
19	0.11889	0.0736	0.0846	0.1997
20	0.11896	0.0655	0.0752	0.1928
21	0.11903	0.0296	0.0340	0.1850
Total	**4.60626**	**6.23731**	**2.80408**	**25.38119**
Average	**0.21935**	**0.29701**	**0.13353**	**1.20863**

Table 4.2a and b are represented in the above figures which shows that the interaction lead to a rise in share values of the entire operator as against the gradual decline witnessed before the interaction.

Fig 4.1: Comparing Predator's share values before and after interaction with the prey's shares values



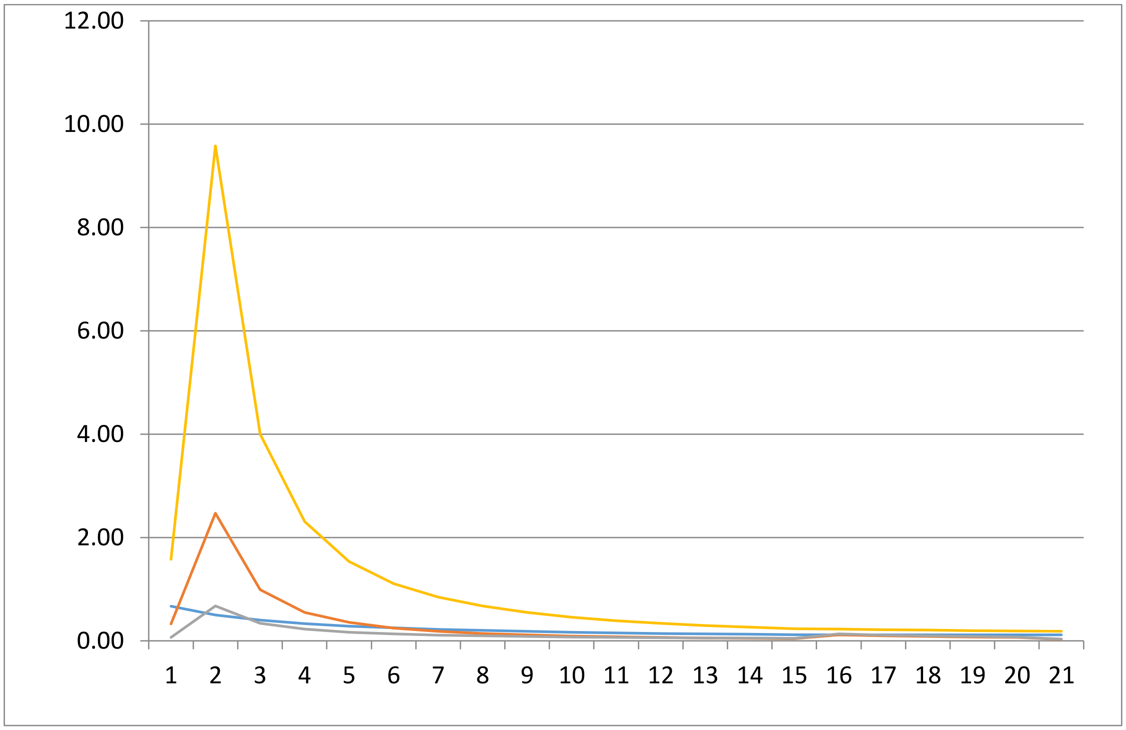
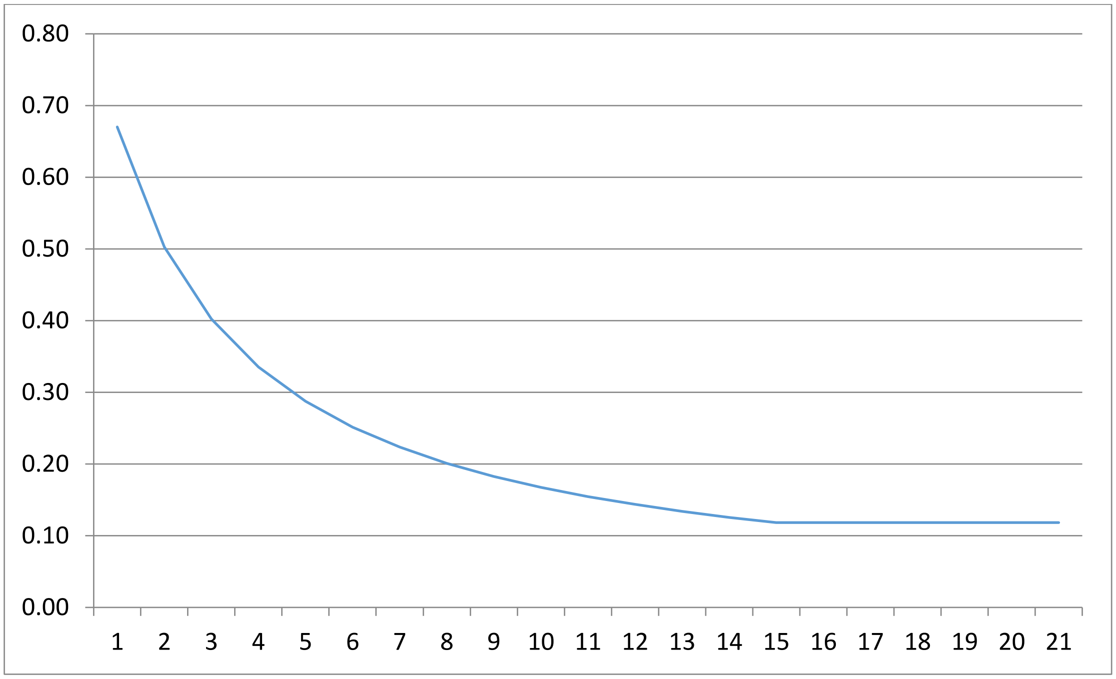
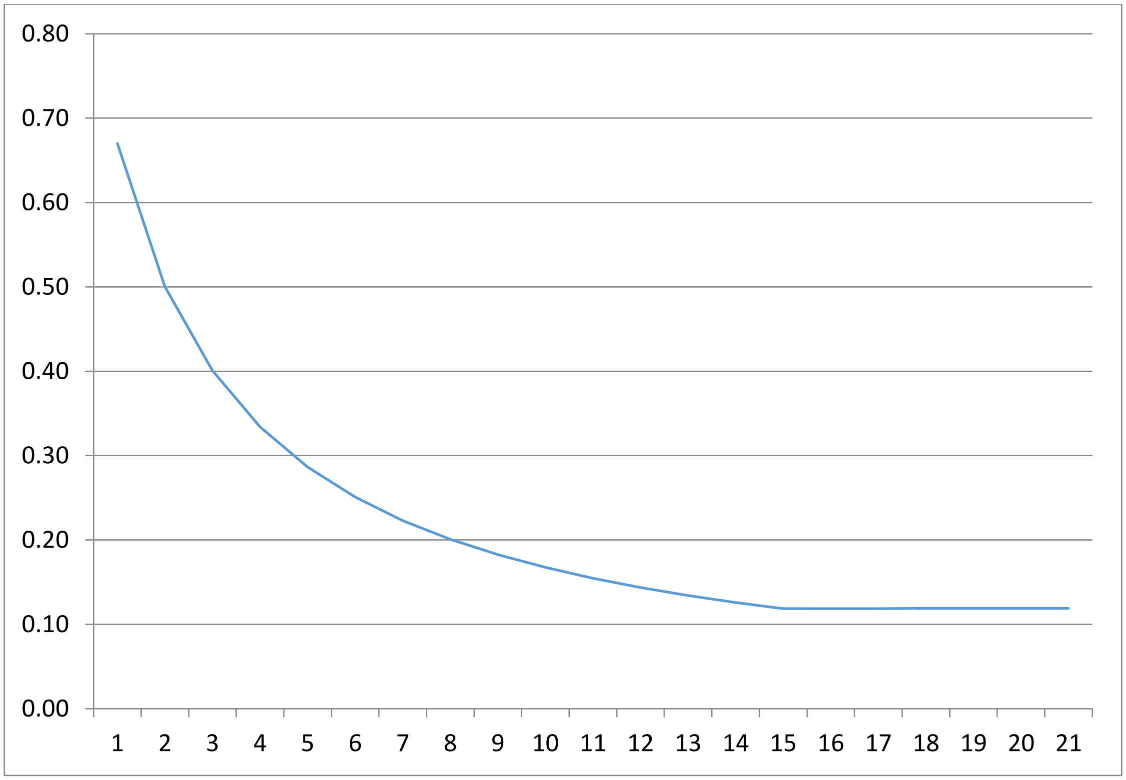


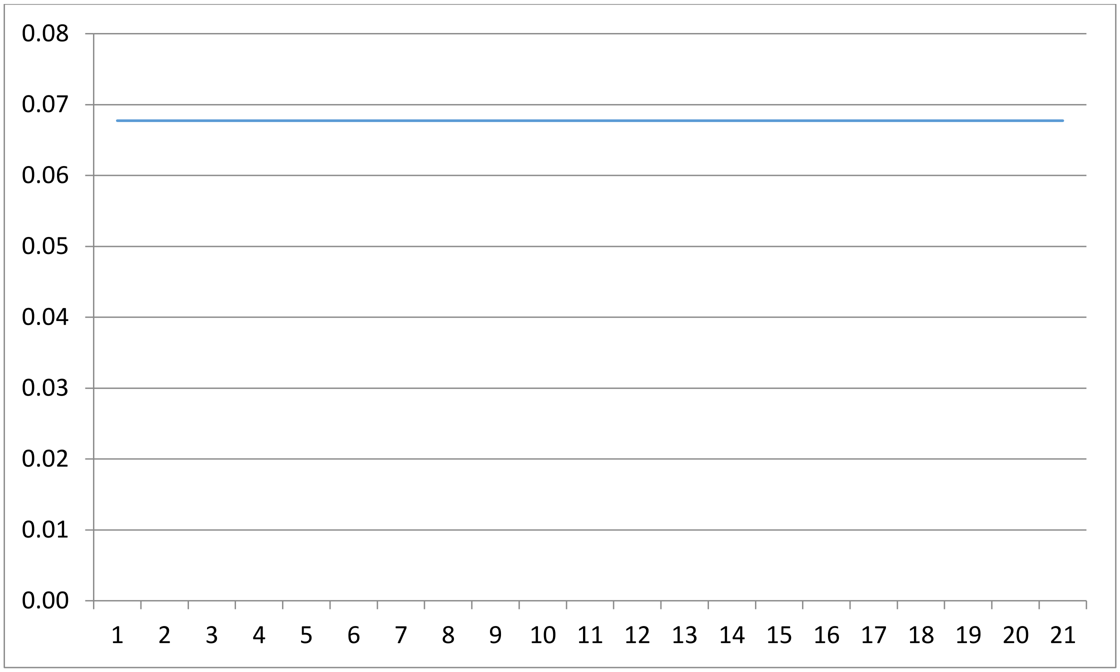
Fig 4.2: Representation of prey company 1 before and after interaction.

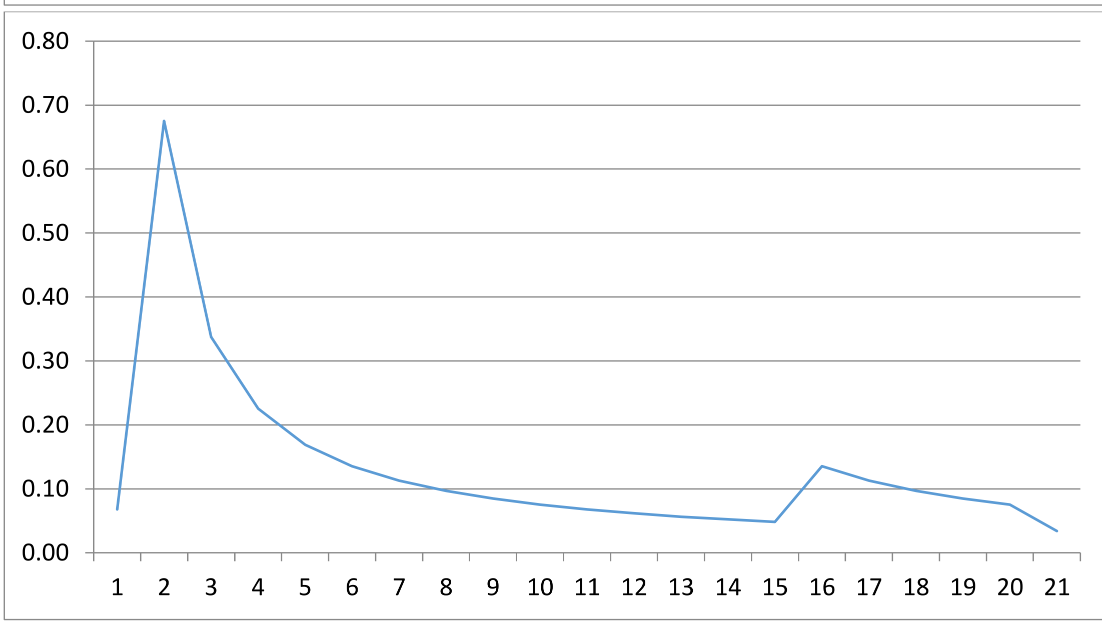




The Prey company 1 though not affected by the interaction experienced a stable trading activities during between the 14th and 15th trading weeks having achieved above average share between the 7th and 8th week of trading.

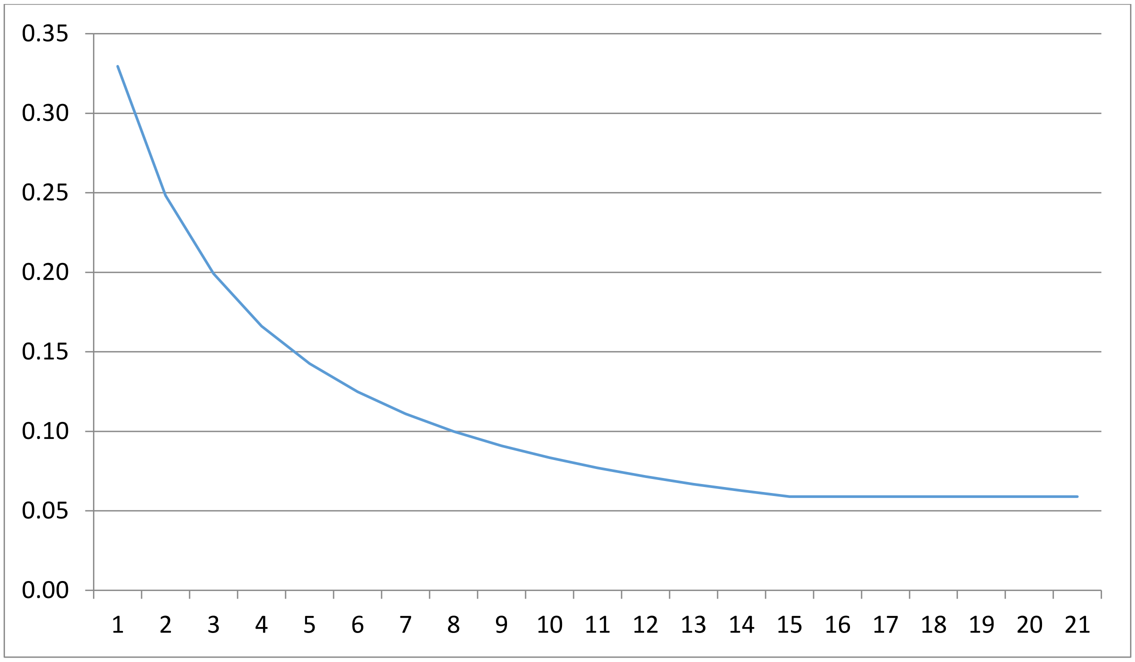
Fig 4.3: Representation of share values of Prey company 2 before and after interaction.

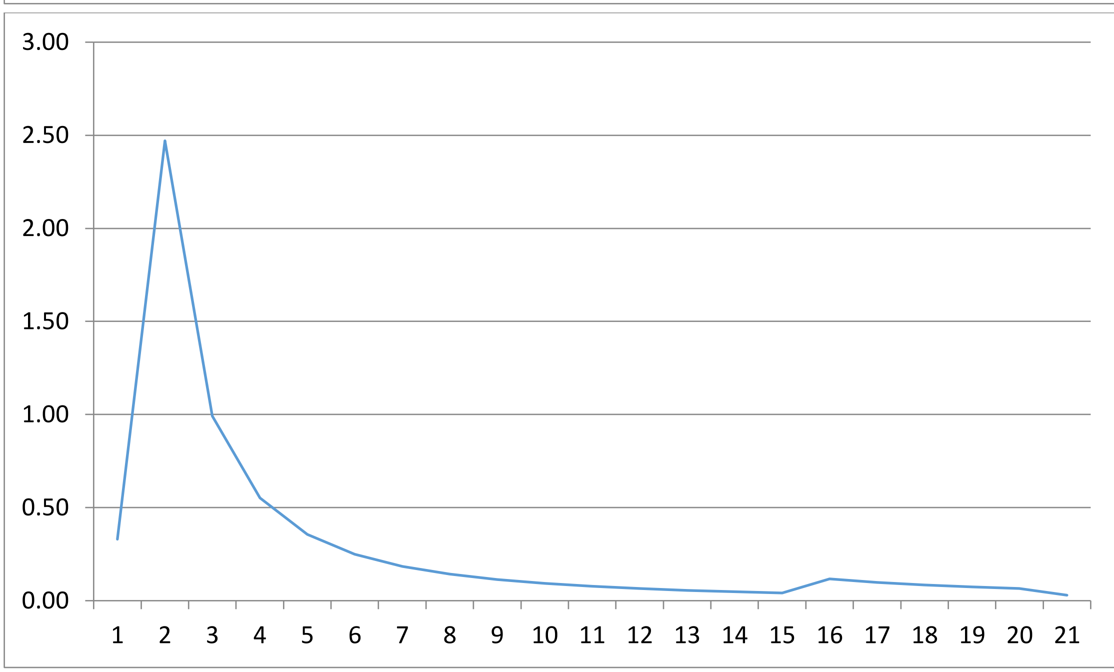




The above figure The prey company 2 had a relatively stable trading activity before the interaction which placed it on a smooth average of 0.067. The tide however changed with the interaction which gave it a rather chaotic trading experience spiking up to a maximum at the third trading week with a steady decline up to the 15th week when it gained again slightly.

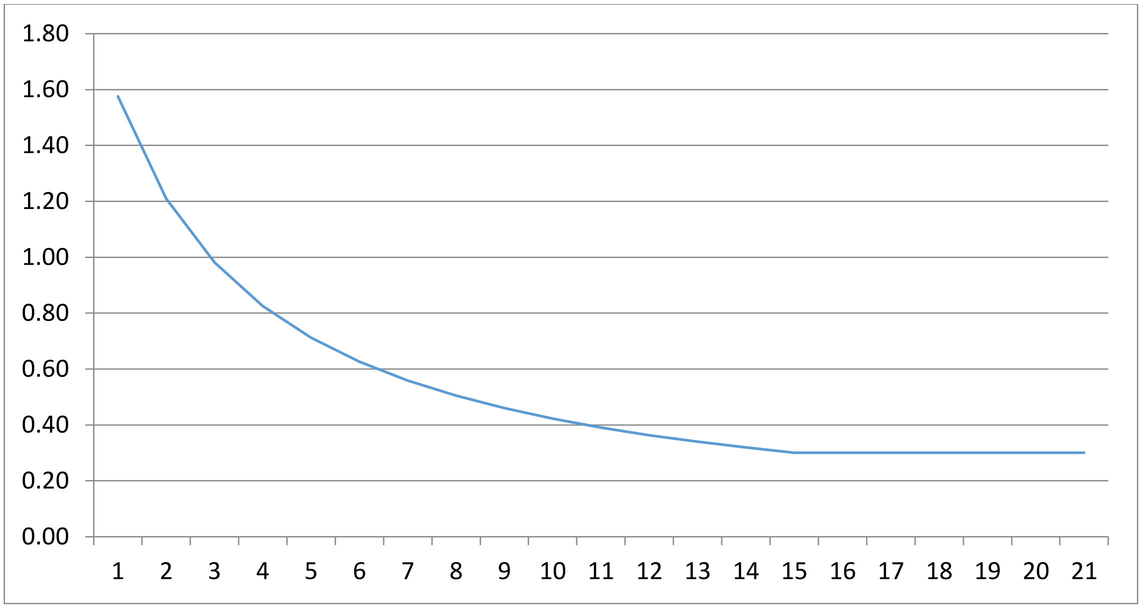
Fig 4.4: Representation of Predator Company before and after interacting with prey company 1

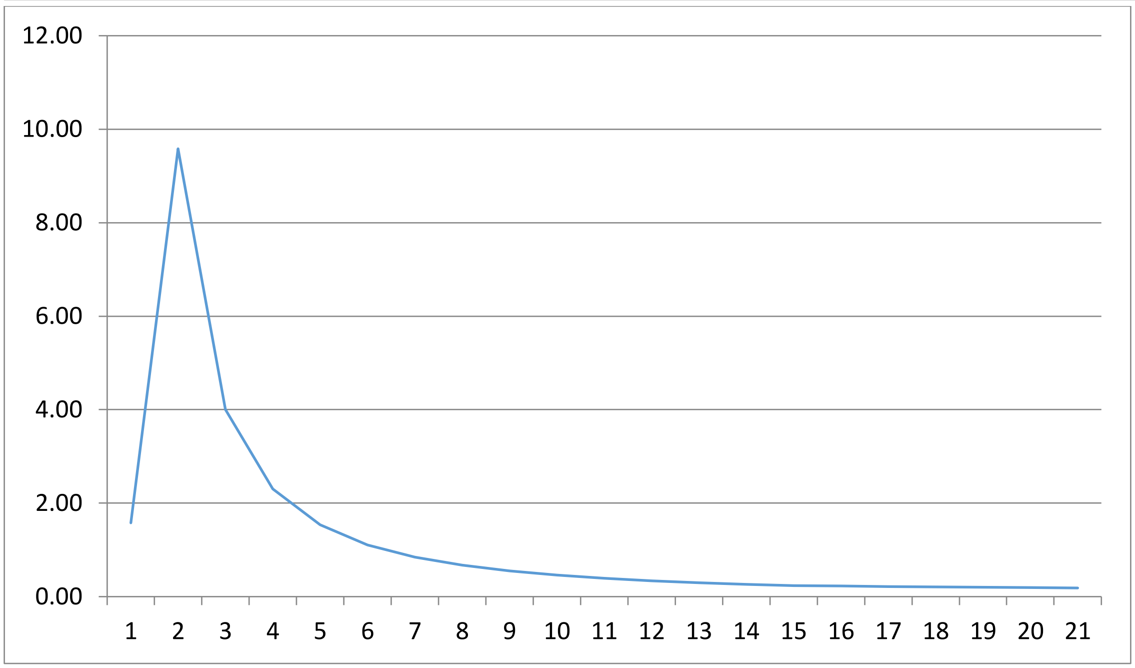




The predator company had enjoyed a stable trading activity during the 14th trading week after a decline earlier in the week before interaction with the predator company. This trend was distorted with a spike between the first and third month of trading and suddenly declined again. It struggled to maintain stability during the 11th week onward.

Fig 4.5: Representation of Predator Company before and after interaction with Prey company 2.





The predator company had enjoyed a stable trading activity during the 15th trading week onward after a decline earlier in the week before interaction with the predator company. After interaction the trend was distorted with a spike between the first and third week of trading and then maintained a gently declining trend starting from the 13th week of trading.

Discussion

When the death rates of the predator company share value is not very high, we witnessed a stable steady-state coexistence or oscillatory coexistence of prey and predator. This is obvious from the analysis of our novel financial predation model. The current price of the predator shares is a valuable variable which determines whether the strategy of merging with the smaller companies will be successful. The broad strategy calls for the trader to hold the target shares, convert them into predator shares, and then sell them to lock in a profit. The predator does not want to overpay for the target prey company and there is always financial risk involved.

Table 4.2a and b are the simulation of the share values of the prey and the predator companies before and after interaction respectively. The conversion rate c_i was varied while the rest parameters are kept constant to show the effects on the stability of the system. These financially, represent the conversion rates of prey company 1 to predator which is the price to earnings ratio of the prey company 1 stock. The harvesting rate of prey and anti-predator behavior of prey are respectively a_i and b_i parameters known as the predatory function taken to be the equity risk premium (excess returns based on investment in the stock market) and price volatility index (a measure of risk in terms of 'investor fear' in the stock market) respectively from the two prey stocks. The simulation table shows that these parameters affect the stability of the system in a large way. The simulation result shows that the stability region of these companies were impacted on when the conversion rates were varied, this is obvious from table 4.2b which supports the analytical work. According to our model, this means as the equity risk premium increases for prey company 2, the regions of stability for the price to earnings ratio for prey company 1 and prey company 2 both decrease slightly. Therefore, a predator company who wishes to invest in either of these companies can be advised that if the stock parameters of those companies are kept within the stability intervals, their investment would be stable and profitable. Otherwise, parameters outside of these intervals would cause the model to become unstable and therefore it would not be wise to invest. Figures 4.1 to 4.5 are graphic illustration of the impact on the stability of the share values of the companies as a result of the interactions. Table 4.2a and b showed on Fig 4.1 displays the oscillatory and quasi steady state or otherwise of the stability of the entire system model from our analysis. It can also be shown that the stable equilibrium which means that the relative prices per share of both prey companies and the predator company can co-exist with these parameters. This is an ideal situation for an investor since he would want to have a stable investment with minimal financial losses. These are supported by the Figures 4.1a and b. Fig 4.2: Representation of prey company 1 before and after interaction. The Prey company 1 though not affected by the interaction

experienced stable trading activities between the 14th and 15th trading weeks having achieved above average share between the 7th and 8th week of trading. The implication is that it is safe for the prey company 1 to invest within this region of stability for profit. The implication of fig 4.3 places investors of the Prey company 2 under a great risk of loosing their investment based on the initial stability they enjoyed before interaction with other investor. Thus, within this region of instability, the investor needs to be very thrift with their investment. We witness from fig 4.4 that investors of the Predator company will be taking a great risk of loosing their investment based on the initial stability they enjoyed before interaction with other investor. Thus, within this region of oscillatory stability produced in Figures 4.4 are situations investors would want to avoid since their investments would possible incur losses. Fig 4.5: can be interpreted to have enjoyed quasi stable trading activities weeks ahead before interacting with the prey company 2. Hiss could be a trap which might lure the predator company into making further heavy investment which could lead to loss of investment since the trend stability was lost after interaction. So the investor needs to exercise care in investing within this region of stability.
By carefully monitoring the changes in these parameters, the investor will be able to a stability region where it is safe to invest. These stability parameter intervals for these values would allow better decision-making especially when a large company is interested in investing in smaller companies. These novel ideas is not meant to replace existing financial models, but an attempt to apply mathematical toolst in financial decision-making process when making large investments on a weekly basis.

Conclusion

We have been able to establish that equilibrium analysis can be applied to stock market having shown that it behaves like an ecosystem and that understanding the ramifications of these mechanisms can provide an investor with insights that could yield competitive advantages in financial investment. This work has explored the idea of investing smaller companies coming into an established market and thriving side by side with fraud laden mature investors. Stability analysis eexposes investors to regions of stability where they can make risk free investment. Future work would dwell in using real-life data to ascertain regions of stability and inspect the causes of losses or gain in the investment represented by the data.

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