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QUALITATIVE BEHAVIOUR OF BETTLE INSECT SPECIES WITH AN INITIAL CONDITION(IC=0.5) OVER A LONGER DURATION OF TIME.

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ABSTRACT:
In this study, we have used analytically method to predict and observe the behaviour of hybrid selections (how characteristics D is passed on through generations) in beetle insects for some instance generations using the initial an conditions, $IC = 0.5$. Furthermore, a numerical scheme ODE45 computational approach was used to study the qualitative behaviour of the bettle population proportion over a period of 40 generation with an initial condition IC=0.5 and we observe that the population proportion ranging from the base year; $t = 0\ upto$ t=40th generation increases monotonically from the initial value of 0.5. We notice a drastic increase in the population proportion from the base year;$t = 0$ up to the $20th\ generation, t = 20$ and then we notice a slower rate of increase in the population proportion from the $20th\ generation; t = 20$ up to the time of convergence with a saturated value of 1.0000(million) which is from the $37th\ generation$ to the $40th\ generation$. The detail work and full results are presented and discussed in this study.

Keywords: *dynamical system, deterministic, convergence, hybrid.*

Introduction

In modelling hybrid selections, here we consider the population of beetle species to determine how quickly a particular characteristic (say D) will pass from one generation to the next. In biology, hybrid is the offspring resulting from combining the qualities of two organisms of different breeds, varieties, species or genera through sexual reproduction (wikipedia). Furthermore, Hybrids are not always intermediates between their parents (such as in blending inheritance) but can show hybrid vigour, sometimes growing larger or taller than either parent. The concept of a hybrid is interpreted differently in animals and plant breeding, where there is interest in the individual parentage. However, in genetics, attention is focused on the numbers of chromosomes associated with the change in character while in taxonomy; a key question is how closely related the parent species are.

In this scenario, species are reproductively isolated by strong barriers to hybridization, which include genetic and morphological differences, differing times of fertility, mating behaviours and cues, physiological rejection of sperm cells or the developing embryo while some act before fertilization and others after fertilization. In studying the characterization of a biological species over time, human impact on the environment has resulted in an increase in the interbreeding between regional species and the proliferation of introduced species worldwide has also resulted in an increase in hybridization. Following this scenario, this genetic mixing may threatens many species with extinction. In animal breeding, from the point of view of animal breeders, there are several kinds of hybrid formed from crosses within a species such as different breeds, single cross hybrids result from the cross between two true-breeding organisms which produces an F1 and hybrid (first filial generation).

Jatavand Dhar (2014). This study is on hybrid approach for pest control with impulsive releasing of natural enemies and chemical pesticides.In this paper, they proposed a three tropic level plant-pest-natural enemy food chain model with stage structure in natural enemy. Moreover, impulsive releasing of natural enemies and harvesting of pests are also considered.

*Tabashnik**, et al (2013).*** They worked on the study of the resistance of insect Resistance to Bacillus Thuringiensis (BT) Crops. The evolution of resistance in pest can reduce the effectiveness of insecticidal proteins from BT produced by transgenic crops. They analysed results of 77 studies from five continents reporting field monitoring data for resistance to BT crops, empirical evolution of factors affecting resistance or both. The results imply that proactive evaluation of the inheritance and initial frequency of resistance is useful for predicting the resistance and improving strategies to sustain the effectiveness of BT crops.

Zawbaa, et al (2018). They worked on a Hybrid Bio-Inspired Heuristic Approach.This work is on Large-dimensionality Small-Instance Set Feature selection.Selection of a representative set of features is still a crucial and challenging problem in machine learning. The complexity of the problem increases when any of the following situations occur: a very large number of attributes (large-dimensionality.

They proposed a hybrid of two methods which have the advantage of providing good learning from fewer examples and a fair selection of features from a really large set, all these while ensuring a high standard classification accuracy of the data.

Kirkpatrick and Ravigné (2002).They worked on speciation by natural and sexual selection - models and experiments. This paper attempts to unify the literature of a large number of mathematical models, showing how natural and sexual selection can cause prezygotic isolation to evolve by identifying five major elements that determine the outcome of specialization caused by selection: a form of disruptive selection, a form of a mating preference, a way to transmit the force of disruptive selection to the isolating mechanism, a genetic basis for increased isolation. These five elements appear to operate largely independent of each other and can be used to make generalization is most likely to happen.

Velliangiri, and Karthikeyan(2010).They worked on a hybrid optimization scheme for intrusion detection using considerable feature selection.The intrusion detection is an essential section in network security because of its immense volume of threats which bothers the computing systems. The real-time intrusion detection dataset comprises redundant or irrelevant features. The duplicate features make it quite challenging to locate the patterns for intrusion detection. Hybrid optimization scheme (HOS) is designed for combining adaptive artificial bee colony (AABC) with adaptive particle swarm optimization (APSO) for detecting intrusive activities.

Larson (2017).He studied a population of $30\ rabbits$that were introduced into a new region. It was estimated the maximum population that could be sustained in that region was$400\ rabbits$. After$1\ year$, the population of rabbits increased and was estimated to be $90\ rabbits$. By applying Gompertz Growth

Model, he was able to predict the increase in population of rabbits in a space of3 $year$to be an estimate of $244\ rabbits$.

Larson, and Edwards (2018).They studied a population of $8\ beavers$ that were introduced into a new wetland area. They estimated that the maximum population the wetlands could sustain was $60\ beavers$.In3 $year$, the population increased to$15\ beavers$. Through the application of Gompertz Growth model, they were able to estimate the population of beavers after a space of $10\ years$ to be $34\ beavers$.

Mathematical formulations

In genetics, a commonly used hybrid selection model is based on this differential equation, *(Larson 2017)*;

$$\frac{dy}{dt} = ky(1-y)(a-by)$$

In this model,

$\frac{dy}{dt}$ represents the rate of change of population with a specific characteristic D over time t.

$\boldsymbol{y}$ represents the portion of the population that has certain characteristics.

$\boldsymbol{t}$ represents the time(measured in generations).

$\boldsymbol{a}$ is a constant that represents a genetic characteristic.

$\boldsymbol{b}$ represents constant of genetic parameter.

$\boldsymbol{k}$ represents the genetic growth rate that depends on the genetic characteristic.

In this study, this model equation will be utilized in studying a population of beetles to determine how quickly characteristics D will pass from one generation to the next on the basis (model assumption) that at the beginning of our study, at $t = 0;\ 50\%$ of the population in millions have characteristic D and after four (4) generations, at $t = 4$; 80% of the population in millions have characteristic D to enable us obtain the genetic growth rate that must be used in this study.

Mathematical Analysis

In order to predict the characteristics of the population over time t, using the model equation *(Larson 2017)*

$$\boldsymbol{\frac{dy}{dt} = ky(1-y)(a-by)}$$

$where\ a = 2\ and\ b = 1$

$\frac{dy}{dt} = ky(1-y)(2-y)\ is\ a\ special\ case\ we\ are\ studying\ here,$

$using\ the\ following\ conditions:$

$$y(0) = 0.5, and\ y(4) = 0.8$$

$$\frac{dy}{dt} = ky(1-y)(2-y)$$

$y(0) = 0.5$

$y(4) = 0.8$

$\boldsymbol{using\ seperating\ variables}$

$$\frac{dy}{y(1-y)(2-y)} = kdt$$

$\boldsymbol{integrating\ both\ sides}$

$$\int \frac{dy}{y(1-y)(2-y)} = \int kdt$$

$\boldsymbol{from\ L.H.S, resolving\ into\ partial\ fraaction}$

$$\frac{1}{y(1-y)(2-y)} = \frac{A}{y} + \frac{B}{(1-y)} + \frac{C}{(2-y)}$$

$\boldsymbol{find\ L.C.M\ of\ R.H.S}$

$$\frac{1}{y(1-y)(2-y)} = \frac{A(1-y)(2-y) + By(2-y) + Cy(1-y)}{y(1-y)(2-y)}$$

$1 = A(1-y)(2-y) + By(2-y) + Cy(1-y)$

$\boldsymbol{at\ y = 0}$

$1 = A(1-0)(2-0) + B(0)(2-0) + C(0)(1-0)$

$1 = A(2)$

$\boldsymbol{A = \frac{1}{2}}$

$\boldsymbol{at\ y = 1}$

$1 = A(1-1)(2-1) + B(1)(2-1) + C(1)(1-1)$

$$1 = B(1)$$

$\boldsymbol{B = 1}$

$\boldsymbol{at\ y = 2}$

$1 = A(1-2)(2-2) + B(2)(2-2) + C(2)(1-2)$

$$1 = C(-2)$$

$$\boldsymbol{C = -\frac{1}{2}}$$

$$\frac{1}{y(1-y)(2-y)} = \frac{1}{2y} + \frac{1}{(1-y)} - \frac{1}{2(2-y)}$$

$$\int \left(\frac{1}{2y} + \frac{1}{(1-y)} - \frac{1}{2(2-y)}\right) dy = \int kdt$$

$$\int \frac{1}{2y} dy + \int \frac{1}{(1-y)} dy - \int \frac{1}{2(2-y)} dy = \int kdt$$

$$\frac{1}{2}\int \frac{1}{y} dy + (-1)\int (-1)\frac{1}{(1-y)} dy - \frac{1}{2}(-1)\int (-1)\frac{1}{(2-y)} dy = \int kdt$$

$$\frac{1}{2}\int \frac{1}{y} dy - \int \frac{-1}{(1-y)} dy + \frac{1}{2}\int \frac{-1}{(2-y)} dy = \int kdt$$

integrating both sides

$$\frac{1}{2}\ln y - \ln(1-y) + \frac{1}{2}\ln(2-y) = kt + c$$

multiplying through by 2

$$\ln y - 2\ln(1-y) + \ln(2-y) = 2kt + 2c$$

applying the law of indices

$$\ln\left(\frac{y(2-y)}{(1-y)^2}\right) = 2kt + 2c$$

$$e^{\ln\left(\frac{y(2-y)}{(1-y)^2}\right)} = e^{2kt} \cdot e^{2c}$$

$$\boldsymbol{where\ A = e^{2c}}$$

$$\frac{y(2-y)}{(1-y)^2} = Ae^{2kt}$$

to obtain the constant k, we apply the condition;

$$y(0) = 0.5$$

$$\frac{0.5(2-0.5)}{(1-0.5)^2} = Ae^{2k(0)}$$

$$\frac{0.5(1.5)}{(0.5)^2} = Ae^{0}$$

$$3 = A(1)$$

$\boldsymbol{A = 3}$

$\boldsymbol{putting\ A\ into\ the\ general\ solution}$

$$\frac{y(2-y)}{(1-y)^2} = 3e^{2kt}$$

$\boldsymbol{to\ obtain\ the\ constant\ k, we\ apply\ the\ condition};$

$$y(4) = 0.8$$

$$\frac{0.8(2-0.8)}{(1-0.8)^2} = 3e^{2(4)k}$$

$$\frac{0.8(1.2)}{(0.2)^2} = 3e^{8k}$$

$$24 = 3e^{8k}$$

$\boldsymbol{dividing\ through\ by\ 3}$

$$8 = e^{8k}$$

$$\ln 8 = 8k$$

$\boldsymbol{dividing\ through\ by\ 8}$

$$\frac{1}{8}\ln 8 = k$$

$$k = 0.2599$$

$$\boldsymbol{\frac{y(2-y)}{(1-y)^2} = 3e^{0.5199t}}$$

$$\frac{y(t)[2-y(t)]}{[1-y(t)]^2} = 3e^{0.5199t}$$

$\boldsymbol{we\ solve\ further\ to\ make\ y\ subject\ of\ the\ formular}$

$Set\ \alpha = 0.5199$

$$y(t)[2-y(t)] = 3e^{\alpha t}(1-y(t))^2$$

$$y(t)[2-y(t)] = 3e^{\alpha t}(y^2-2y+1)^2$$

$$y(t)[2-y(t)] = 3y^2e^{\alpha t} - 6ye^{\alpha t} + 3e^{\alpha t}$$

$$2y - y^2 = 3y^2e^{\alpha t} - 6ye^{\alpha t} + 3e^{\alpha t}$$

$$3y^2e^{\alpha t} - 6ye^{\alpha t} + 3e^{\alpha t} - 2y + y^2 = 3e^{\alpha t}$$

$\boldsymbol{collecting\ and\ factorizing\ like\ terms}$

$$(3e^{\alpha t}+1)y^2 - (6e^{\alpha t}+2)y + 3e^{\alpha t} = 0$$

$$(3e^{\alpha t}+1)y^2 - 2(3e^{\alpha t}+1)y + 3e^{\alpha t} = 0$$

$$\boldsymbol{diving\ through\ by}\ (\mathbf{3}\boldsymbol{e^{\alpha t}} + \mathbf{1})$$

$$\frac{(3e^{\alpha t}+1)}{(3e^{\alpha t}+1)}y^2 - 2\frac{(3e^{\alpha t}+1)}{(3e^{\alpha t}+1)}y + \frac{3e^{\alpha t}}{(3e^{\alpha t}+1)} = 0$$

$$y^2 - 2y + \frac{3e^{\alpha t}}{(3e^{\alpha t}+1)} = 0$$

$$y^2(t) - 2y(t) + \frac{3e^{\alpha t}}{(3e^{\alpha t}+1)} = 0$$

$$\boldsymbol{introducing\ the\ algebraic\ formular\ y} = \frac{-\boldsymbol{b} \pm \sqrt{\boldsymbol{b}^2 - 4\boldsymbol{ac}}}{2\boldsymbol{a}}$$

$$Set\ \boldsymbol{a} = \mathbf{1};\ \boldsymbol{b} = -\mathbf{2};\ and\ \mathbf{c} = \boldsymbol{p}(\boldsymbol{t}) = \frac{\mathbf{3}\boldsymbol{e^{\alpha t}}}{(\mathbf{3}\boldsymbol{e^{\alpha t}} + \mathbf{1})}$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4c}}{2}$$

$$y = \frac{2 \pm \sqrt{4 - 4c}}{2}$$

$$y = \frac{2 \pm \sqrt{4(1-c)}}{2}$$

$$y = \frac{2 \pm \sqrt{4} * \sqrt{1-c}}{2}$$

$$y = \frac{2 \pm 2\sqrt{1-c}}{2}$$

$$y = \frac{2\left(1 \pm \sqrt{(1-c)}\right)}{2}$$

$$y = 1 \pm \sqrt{(1-c)}$$

$$\boldsymbol{y} = \mathbf{1} \pm \sqrt{(\mathbf{1} - \boldsymbol{c})} \qquad (\boldsymbol{General\ Solution})$$

$$\boldsymbol{at\ t} = \mathbf{0};$$

$$c = p(0) = \frac{3e^{\alpha(0)}}{(3e^{\alpha(0)}+1)};\ recall\ \alpha = 0.5199$$

$$c = p(0) = \frac{3e^0}{(3e^0+1)} = \frac{3}{(3+1)} = \frac{3}{4} = 0.75$$

$y(0) = 1 \pm \sqrt{1 - 0.75} = 1 \pm \sqrt{0.25} = 1 \pm 0.5$

$y(0) = 1 + 0.5 \; and \; 1 - 0.5$

$\boldsymbol{y(0) = 0.5000 \; and \; 1.5000}$

$\boldsymbol{at\ t = 1}$;

$$c = p(1) = \frac{3e^{\alpha(1)}}{(3e^{\alpha(1)} + 1)}; recall\ \alpha = 0.5199$$

$$c = p(1) = \frac{3e^{0.5199}}{(3e^{0.5199} + 1)} = \frac{5.0456}{6.0456} = 0.8346$$

$y(1) = 1 \pm \sqrt{1 - 0.8346} = 1 \pm \sqrt{0.1654} = 1 \pm 0.4067$

$y(1) = 1 + 0.4067 \; and \; 1 - 0.4067$

$\boldsymbol{y(1) = 1.4067 \; and \; 0.5933}$

$\boldsymbol{at\ t = 2}$;

$$c = p(2) = \frac{3e^{\alpha(2)}}{(3e^{\alpha(2)} + 1)}; recall\ \alpha = 0.5199$$

$$c = p(2) = \frac{3e^{1.0398}}{(3e^{1.0398} + 1)} = \frac{8.4860}{9.4860} = 0.8946$$

$y(2) = 1 \pm \sqrt{1 - 0.8946} = 1 \pm \sqrt{0.1054} = 1 \pm 0.3247$

$y(2) = 1 + 0.3247 \; and \; 1 - 0.3247$

$\boldsymbol{y(2) = 1.3247 \; and \; 0.6753}$

Method of solution

We have computed the percentage of the population proportion of beetle species with initial condition value (IC=0.5) using analytical approach for some instance generation. This result shows various population proportions of beetle insects in millions that have adapted to certain characteristics (characteristics D). This model can be used to predict the percentage of the population of beetle species with a certain characteristic over time.

Over a long duration of time, analytical approach will be time consuming and we may not be able to predict how character evolves for several generations, hence we resort to computational approach using a more efficient and robust method of Matlab software numerical scheme of ODE45. This will be the main focus of our results and discussions.

Results and Discussions

Table 1

Predicting the impact of experimental time for an initial condition (IC=0.5) on the population proportion with characteristics D in beetle species over a period of the first 10 $generations$.

EXAMPLE	TIME	PP(IC=0.5)
1	0.0000	0.5000
2	1.0000	0.5933
3	2.0000	0.6752
4	3.0000	0.7441
5	4.0000	0.8001
6	5.0000	0.8445
7	6.0000	0.8796
8	7.0000	0.9070
9	8.0000	0.9281
10	9.0000	0.9444
11	10.0000	0.9571

In this study, from $Table$ 1 with 11 numerical examples ranging from the base year, $t = 0(Example\ 1)$ to the $10th\ generation,\ t = 10\ (Example\ 11)$. We observe the movement of the population proportion of beetles with characteristics D from $0.5000\ (in\ millions)$ at the base year, $t = 0$ to $0.9571(in\ millions)$ at the $10th\ generation, t = 10$ under the initial condition, $IC = 0.5$ which indicates an increase in the population of beetles with characteristics D.

Table 2

Predicting the impact of experimental time for an initial condition (IC=0.5) on the population proportion with characteristics D in beetle species over the period of the $11th$ to the $20th\ generation$.

EXAMPLE	TIME	PP(IC=0.5)
12	11.0000	0.9670
13	12.0000	0.9745

14	13.0000	0.9803
15	14.0000	0.9848
16	15.0000	0.9883
17	16.0000	0.9910
18	17.0000	0.9930
19	18.0000	0.9946
20	19.0000	0.9959
21	20.0000	0.9968

In this study, from $Table\ 2$ with 10 numerical examples ranging from the $11th\ generation, t = 11\ (Example\ 12)$to the $20th\ generation, t = 20\ (Example\ 21)$. We observe the movement of the population proportion of beetles with characteristics D from $0.9670\ (in\ millions)$ at the $11th\ generation,\ t = 11$ to $0.9968\ (in\ millions)$ at the $20th\ generation,\ t = 20$ under the initial condition, $IC = 0.5$ which indicates an increase in the population of beetles with characteristics D.

Table 3

Predicting the impact of experimental time for an initial condition (IC=0.5) on the population proportion with characteristics D in beetle species over the period from the $21st$ to the $30th\ generation$.

EXAMPLE	TIME	PP(IC=0.5)
22	21.0000	0.9975
23	22.0000	0.9981
24	23.0000	0.9985
25	24.0000	0.9989
26	25.0000	0.9991
27	26.0000	0.9993
28	27.0000	0.9995
29	28.0000	0.9996
30	29.0000	0.9997
31	30.0000	0.9998

In this study, from $Table\ 3$ with 10 numerical examples ranging from the $21st\ \ generation, t = 21\ (Example\ 22)$to the $30th\ \ generation, t = 30\ (Example\ 31)$. We observe the movement of the population proportion of beetles with characteristics D from $0.9975\ (in\ millions)$ at the$21st\ \ generation,\ t = 21$ to $0.9998\ (in\ millions)$ at the $30th\ \ generation,\ t = 30$ under the initial condition, $IC = 0.5$ which indicates an increase in the population of beetles with characteristics D.

Table 4

Predicting the impact of experimental time for an initial condition (IC=0.5) on the population proportion with characteristics D in beetle species over the period of the $31st$ to the $40th\ generation$.

EXAMPLE	TIME	PP(IC=0.5)
32	31.0000	0.9998
33	32.0000	0.9999
34	33.0000	0.9999
35	34.0000	0.9999
36	35.0000	0.9999
37	36.0000	0.9999
38	37.0000	1.0000
39	38.0000	1.0000
40	39.0000	1.0000
41	40.0000	1.0000

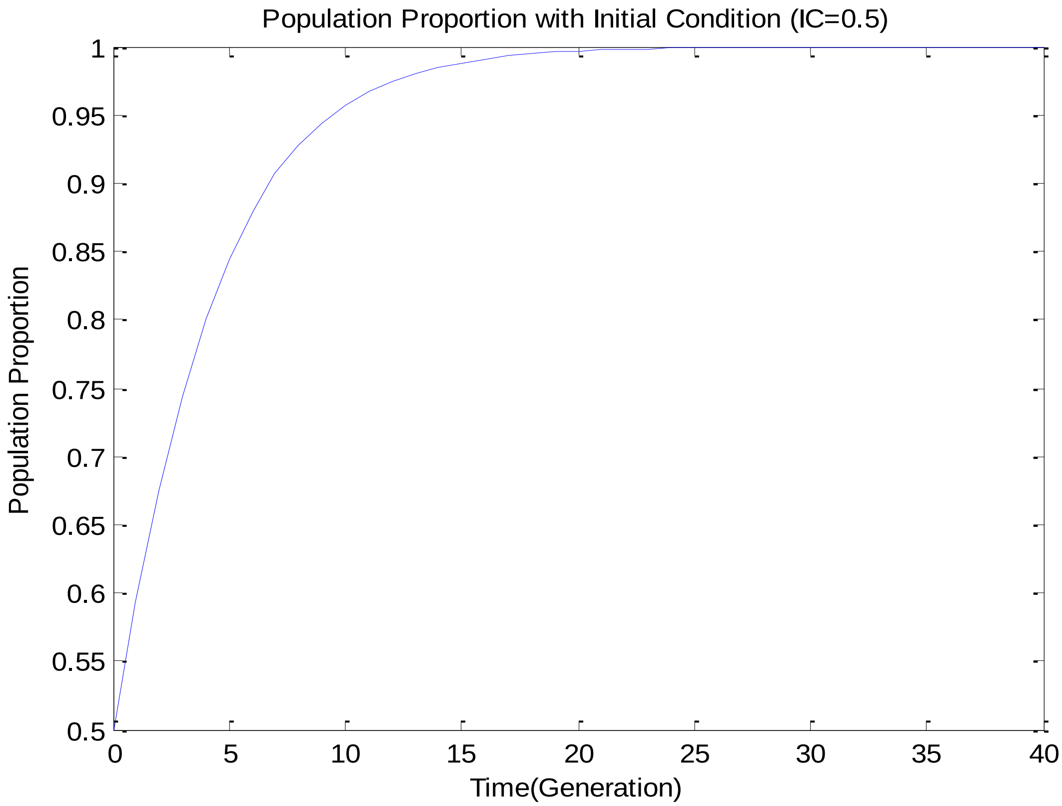
In this study, from $Table\ \ 4$ with 10 numerical examples ranging from the $31st\ \ generation, t = 31(Example\ 32)$ to the $40th\ \ generation, t = 40\ (Example\ 41)$. We observe the movement of the population proportion of beetles with characteristics D from $0.9998\ (in\ millions)$ at the$31st\ \ generation,\ t = 31$ to $1.0000\ (in\ millions)$ at the $40th\ \ generation,\ t = 40$ under the initial condition, $IC = 0.5$ which indicates an increase in the population of beetles with characteristics D.

We further observe a convergence of the initial conditions IC=0.5 at the $37th\ \ generation$ up to $40th\ \ generation$ which makes the population proportion $1.0000\ (in\ millions)$ the point of convergence (the carrying capacity).

From our analysis , ranging from $Table$ 1 to $Table$ 4 we conclude that the population proportion with the initial condition $IC = 0.5$ is monotone increasing having an upper bound of 1.0000 $(in\ millions)$

In this study, we have shown the trend in the characteristics behavior over time.

Figure 1



$Figure$ 1 shows a graph of the population proportion with initial condition $IC = 0.5$ ranging from the base year; $t = 0$ to the $40th\ generation$; $t = 40$. We observe that the population proportion increases from $0.5\ and\ saturated\ at\ 1.000\ (million)$ within the space of $40th\ \ generation$.

We notice a drastic increase in the population proportion from the base year; $t = 0$ up to the $20th\ generation, t = 20$ and then we notice a slower rate of increase in the population proportion from the $20thgeneration; t = 20$ up till the time of convergence to a saturated value of 1.0000(million) which is from the $37th\ generation$ to the $40th\ generation$.

CONCLUSION

Using the analytical method to study the qualitative characterization of the dynamical system is tasking and did not give an early insight of the expected results and also involved some errors due to approximation, we introduced the core method for this work which is the computational method. Through this method, we were able to predict the population proportion over a period of 40 generation using MatLab ODE45 numerical scheme. It was observed that there was an increase in the population proportion using initial condition, $IC = 0.5$ which further move to a convergence at 1.0000 (in millions) from the$37th\ generation$ up till the$40th\ generation$.

The differential equation model for hybrid selections used in this work is very effective in the determination and prediction of the behaviour of hybrid selections in beetle insects only over a period of time.

RECOMMENDATION

To researchers who may want to do similar works as this, we recommend that:

The impact of the growth rate should be check using computational method.

The population proportion at varying initial conditions should be predicted using computational method.

Similar method should be adopted to study the qualitative behavior other biological species with certain genetic traits or character over time.

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