BERKELEY PUBLICATION AND RESEARCH INTERNATIONAL
03.28.2022
Journal of Health, Metabolism & Nutrition Studies Vol. 19 No. 3

ESTIMATION OF LOW-ENERGY ON THE BASIC OF NEUTRON – PROTON AND PROTON – PROTON INTERACTIONS

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ABSTRACT

In this study, we established a qualitative understanding of the physical quantities which characterize nucleon-nucleon interaction and pion nucleon based on the fact that nuclear forces at low energies in the nucleon-nucleon system are mainly determined by the one-pion exchange mechanism. The result of the computation governed by Yukawa potential as recommended by meson field theory at lowest energies scattering length and effective range shows that the highest energy is obtained at the interaction of proton-proton where the scattering length is in the range of (2.85 – 2.90) g/cm while the effective range is recorded to low. On the other hand, high energy value was obtained at a value of (2.00 -2.05) g/cm of effective range interaction a while low energy was recorded in the case of

Introduction:

The fundamental physical characteristic for a strong nuclear interaction is the pion nucleon coupling constant. The Pion nucleon coupling plays an important role in the approximation of the nucleon-nucleon and pion-nucleon interactions. As a result of this, it is important to study their characteristics in other to refine their values. A detailed history of the development of the situation around the pion-nucleon coupling constant can be found in Hulthen L. *et, al.,* (1957), Bohr A. *et, al.,* (1969), (Ericson T, (1988),

ISSN: 1213-4406

scattering length at the same length. The charge pion –nucleon coupling constant is obtained at a value of $\frac{g^2_{\pi^\pm}}{4\pi} = 14.54$ this physical quantity expressed excellent agreement with the experimental value of $\frac{g^2_{\pi^\pm}}{4\pi} = 14.52$. This shows electromagnetic interaction associated with the correction will be likely added or removed from the experimental values of the nuclear – coulomb-low energy parameters of *pp* scattering.
PACS numbers: 13.75.Cs, 13.75.Gx, 14.20.Dh, 14.40.Be, 25.40.Cm, 25.40.Dn

Keywords: Nucleon – nucleon interaction, *pp* scattering, Yukawa potential and Energies

Miller *et, al.,* (1990), Macheleidl R. (2001).

In the present time, there is no strife about the neutral pion-nucleon coupling constant $\frac{g^2_{\pi^0}}{4\pi}$. The value of $\frac{g^2_{\pi^0}}{4\pi} = 13.51$ from (Naghdi M, 2014), said to be among the values that are determined experimentally just in the recent years, it has fit in with a perfect agreement with the earlier results from de Swart J. J *et, al.* (1998), $\frac{g^2_{\pi^0}}{4\pi} = 13.54$ Arndt R.A *et.al* (1995), and (Machleidt. R, 2000), $\frac{g^2_{\pi^0}}{4\pi} = 13.60$ de Swart J. J *et, al.* (1998), and its average value is given as $\frac{g^2_{\pi^0}}{4\pi} = 13.58$ as quoted in (Machleidl R. 2001), Rahm J. et, al., (2011). However, there is no agreement among members on the values of the charged pion- nucleon constant $\frac{g^2_{\pi^\pm}}{4\pi}$. Dumbrajs and his co-authors came up with their compilation given a value of $\frac{g^2_{\pi^\pm}}{4\pi} = 14.28$ (Bugg D.V 2004) obtained in Arndt R.A *et, al.* (1995), (Machleidt R. 2000) from the data on $\pi^\pm$ P scattering. On the basis basic of energy-dependent partial–wave analysis (PWA) of the data on nucleon-nucleon scattering, on the other hand, the Nijmegen group found the values of $\frac{g^2_{\pi^\pm}}{4\pi} = 13.54$ (6,15) Rahm J. et, al., (2011), for the charge pion – nucleon

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coupling constant. Their result was close to the coincident value with the coupling constant $\frac{g_{\pi^0}^2}{4\pi} = 13.54\,(13)$ determined for neutral pion. The values $\frac{g_{\pi^\pm}^2}{4\pi} \sim 13.69 \div 13.79$ of the charged pion-nucleon coupling constant, that is said to be close to the constant $\frac{g_{\pi^0}^2}{4\pi} \sim 13.59$ for neutral pion, were presently obtained based on data on $\pi_p^\pm$ interaction in some other research. In addition, the Uppsala neutron research group obtained much greater values for the charged pion – nucleon coupling constant, $\frac{g_{\pi^\pm}^2}{4\pi} = 14.62, \frac{g_{\pi^\pm}^2}{4\pi} = 14.52, and\ \frac{g_{\pi^\pm}^2}{4\pi} = 14.74$, which exceed substantially the mean value of the coupling constant for neutral pion, $\frac{g_{\pi^0}^2}{4\pi} = 13.59$. Thus, it is of great importance to address the problem of the charge dependence of the pion-nucleon coupling constant in a suitable manner, to evaluate whether the pion nucleon coupling constant for neutral and charged pion differ from each other.

Materials and Method

In this study, we examine the pion – nucleon coupling constants for neutral and charged pion based on data on the low energy nucleon-nucleon (NN) scattering semi-phenomenological one-boson-exchange potential models, with various exchange mesons, is frequently used to describe the (NN) interaction. In this method, the exchange of pion, which is light, determines primarily the long-range part of the NN potential, while the exchange of rho and Omega mesons, which are heavier, determines the interaction at intermediate and short distances, which is substantial at higher energies. In the situation where low energies, its effect extremely correspond to long distances. The use of the most effective and efficient one – pion-exchange potentials is optimal to describe (NN) interaction. In respect to this circumstance and under some basic assumption that Nuclear forces on the (NN) system at low energies is due to primarily to exchange the virtual pion. We make use of the well-known Yukawa Potential and then follow the meson field theory Hulthen L. *et, al.,* (1957), Bohr A. *et, al.,* (1969), (Ericson T, (1988), to describe the (NN) interaction. in view to that, we match the parameters of potential (that is, its depth V_0 and range R) with low-energy parameters of (NN))scattering in the $^1S_0$ spin-singlet state. By introducing the parameters of the Yukawa potential for (pp) and neutron-proton (np) interactions with the equation that relates changed was deduced $(f^2_{\pi^\pm})$ and neutral $(f^2_{\pi^0})$ pseudovector pion- nucleon coupling constants to each other.

The Meson Field Theory

According to meson field theory, virtual pion in the nucleon is largely exchanged due to the strong nuclear interaction which is primarily determined by the long-range part of the (NN) interaction and according to (NN) potential which follows the meson field theory is called the Yukawa Potential which is writing as

$$V_Y(r) = -V_0 \frac{e^{-\mu r}}{\mu r} \quad \ldots (3.1)$$

Where r is the distance between two nucleons and μ is the pion mass M_π given by the equation as.

$$\mu = \frac{M_\pi C}{\hbar} \quad \ldots (3.2)$$

Whereby C is the speed of light and $\hbar$ is the reduced Planck constant. The nuclear force range R is coincident with the Compton wavelength of the pion.

The pseudo vectors pion- nucleon coupling constant f_π by the simple equation (Ericson T. 1995)

$$V_0 = m_\pi C^2 f^2_\pi \quad \ldots (3.3)$$

Thus, the pion mass m_π and the pion – nucleon coupling constant f_π are basic characteristics of pion – nucleon interaction, which is of paramount importance in the studying of (NN) and pion – nucleon interactions Hulthen L. *et, al.,* (1957), Bohr A. *et, al.,* (1969), (Ericson T, (1988), Miller *et, al.,* (1990), Macheleidl R. (2001). The pseudo-vector coupling constant for neutral $(f^2_{\pi^0})$ and charged $(f^2_{\pi^\pm})$ pions are related to the pseudoscalar coupling constants $(g^2_{\pi^0})$ and $(g^2_{\pi^\pm})$ by the equation.

ISSN: 1213-4406

$g_{\pi^0}^2 = (\frac{2m_p}{m_{\pi^\pm}})^2 f_{\pi^0}^2$ ... (3.4)

$g_{\pi^\pm}^2 = (\frac{m_p+m_n}{m_{\pi^\pm}})^2 f_{\pi^\pm}^2$ ... (3.5)

Where m_p and m_n are the masses of proton and neutron respectively and $m_{\pi^\pm}$ is the charged pion mass.

The experimental values of the proton, neutron, pion mass, neutral pion mass and $\hbar C$is given as.

$m_p = 938.272046 MeV/C^2$ ... (3.6)

$m_n = 939565379 MeV/C^2$ ... (3.7)

$m_{\pi^0} = 134.9766 MeV/C^2$ ... (3.8)

$m_{\pi^\pm} = 139.57018 MeV/C^2$ ... (3.9)

$\hbar C = 197.3269718 MeV$ ... (3.10)

All the values in equation (3.6 – 3.10) were obtained from the compilation of the particle data group Alarcon J.M *et, al.,* (2013). Exchange of neutral pion led to two charged protons interaction according to equation (3.4) and (3.5), the parameters V_0^{PP} and μ_{pp} of the Yukawa potential Hulthen L. *et, al.,* (1957) are determined by the neutral – pion mass m_{π^0} and the coupling constantf_{π^0}. But for neutron-proton interaction, both neutral and charged pions are exchanged. In the potential can be determined by using (Matsinos E, 2013) then average pion mass values is given as

$\overline{m_\pi} = \frac{1}{3}(m_{\pi^0} + 2m_{\pi^\pm})$ ... (3.11)

And the pion nucleon coupling constant

$\overline{f_\pi^2} = \frac{1}{3}(f_{\pi^0}^2 + 2f_{\pi^\pm}^2)$ ... (3.12)

To determine the parameters of the potential in potential Hulthen L. *et, al.,* (1957), we will employ the data for the interactions of the two nucleons at the lowest energies in the $^1S_0$ spin-singlet state. Further, we evaluate proton-proton parameters μ_{pp} and V_0^{PP} based on the scattering length a_{p} and the effective range r_{pp}. In doing this, electromagnetic interaction associated with correction will be removed from the experimental values of the nuclear – coulomb-low energy parameters of pp scattering. After the removal of the corrections, the values of the purely nuclear scattering length a_{pp} and effective range r_{pp} for proton-proton scattering become (4).

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$a_{pp} = -17.3$ ... (3.13)
$r_{pp} = 2.85$ ... (3.14)

By employing the variable – phase approach, (Babenko V.A 2015) the value pp scattering parameters in Bergervoet J. R et, al., (1990) and Arndt R. A et, al., (1995) we obtain the following results for the parameter of the Yukawa Potential for proton-proton scattering.

$\mu_{pp} = 0.8393$ ... (3.15)
$V_0^{pp} = 44.8295 MeV$ ... (3.16)

By equation (2), 4, (14), and (15) the mass and the pion –Nucleon coupling constant for the neutral pion in the case of the Yukawa potential for proton-proton interaction.

$m_{\pi^o}^Y = 165.6108 MeV/C^2$ ... (3.17)
$(f_{\pi^o}^Y)^2 = 0.2707$ ... (3.18)

This is much greater than the experimental values

$m_{\pi^0} = 134.9766 MeV/C^2$ ... (3.19)
$\left(f_{\pi^0}^2\right) = 0.0749$ ... (3.20)

Thus, we have

$m_{\pi^o}^Y = C_{m\pi} m_{\pi^0}$ ... (3.21)
$(f_{\pi^0}^y)^2 = C_{f_\pi} f_{\pi^0}^2$ ... (3.22)
$C_{m_\pi} = 1.2270$ ... (3.23)
$C_{f_\pi} = 3.6142$ ... (3.24)

We can say that a similar relationship exists in equation (3.17) and (3.18) hold for the charged – pion mass and charged pion – nucleon coupling constants. And hence, for the average pion – mass. because of this, it can be shown that the average pin-nucleon coupling constant, i.e $C_{m\pi^0} = C_{m\pi^\pm} = C_{\bar{m}_\pi}$ and $C_{f\pi^0} = C_{f\pi^\pm} = C_{\bar{f}_\pi}$. It can be shown that the neutron-proton parameters μ_{np} and V_0^{np} of the potential in Hulthen L. *et, al.,* (1957) are related to an analogous proton-proton interaction parameter μ_{np} and V_0^{np} by the equation.

$\mu_{np} = \frac{\bar{m}_\pi}{m_{\pi^0}} \mu pp$ ... (3.25)

$$V_0^{np} = \frac{\bar{m}_\pi}{m_{\pi^0}} \frac{\bar{f}_\pi^2}{f_{\pi^0}^2} V_0^{pp} \quad \dots (3.26)$$

In accordance with equation (3.11) and (3.19), the ratio of the average pion-mass $\bar{m}_\pi$ to the neutral pion mass m_{π^0} is

$$\frac{\bar{m}_\pi}{m_{\pi^0}} = 1.0227 \quad \dots (3.27)$$

By using equation (3.19 – 3.25) and the experimental value of the singlet up scattering length for the $^1S_0$ state (Babenko, 2016) and (Arriola E.R 2016).

$$a_{np} = -23.71 \quad \dots (3.28)$$

The following values for the parameters

$$\mu_{np} = 0.8584 \quad \dots (3.29)$$

$$V_0^{np} = 48.0742MeV \quad \dots (3.30)$$

The effective range r_{np} found np scattering the Yukawa potential with the parameter values in (Arriola E.R 2016) and (Matsinos E, 2017).

$$r_{\square p} = 2.70 \quad \dots (3.31)$$

Experimental value

$$r_{np} = 2.70 \quad \dots (3.32)$$

The values of the singlet np scattering length in (Babenko, 2016) and effective range in (Matsino E, 2017) are in very good agreement with the values $a_{np} = -23.7154$ and $r_{np} = 2.706$ that we obtained in (Arriola E.R 2016) and (Matsinos E, 2017). By using the experimental values of the np-scattering cross-section and the experimental values of the deuteron characteristics. Thus, the use of experimental values of the neutral and charged –pion masses in (Blomgren J. 1999) leads to a consistent description of experimental data on proton-proton and neutron-proton scattering in the region of the low energies.

From equations (3.3) and (3.26) we can obtain an important equation that relates the pseudo-vector changed and neutral pion-nucleon coupling constants.

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$$f_{\pi^{\pm}}^2 = \frac{1}{2}\left(\frac{3V_0^{np}}{V_0^{pp}}\frac{\mu_{pp}}{\mu_{np}} - 1\right)f_{\pi^0}^2 \quad \text{... (3.33)}$$

The neutral pion –nucleon coupling constant together with the parameter of proton-proton scattering in (Machleidt R. 2000) and (Linkaisong, V. 2001) and the parameters of neutron –m proton scattering in (Arriola E.R 2016) and (Matsino E, 2017), with the pseudo-vector charged pion nucleon coupling constant obtain given as

$$f_{\pi^{\pm}}^2 = 0.0804 \quad \text{... (3.34)}$$

From equation (3.4) and (3.5) and taking into account the pseudo-vector coupling constants for the neutral pion in Baru V. et*, al.,* (2011) and for the charged pion in (Bjorken J.D 1964), the proton and neutron masses in (Blomgren J, 1999) and the charged pion-mass in (Naghdi M, 2014) the value of pseudoscalar pion nucleon coupling constants.

$$\frac{g_{\pi^0}^2}{4\pi} = 13.49 \quad \text{... (3.35)}$$

$$\frac{g_{\pi^{\pm}}^2}{4\pi} = 14.54 \quad \text{... (3.36)}$$

As the value in (SIiv L.A (1974) the pseudo-scalar charge pion nucleon coupling constant is found to be perfectly in agreement with the experimental coupling constant.

$$\frac{g_{\pi^{\pm}}^2}{4\pi} = 14.52 \quad \text{... (3.37)}$$

Uppsala neutron research group

The values obtained in the recent study for the pion-nucleon coupling constant sizably differed from the value of the neutral pion nucleon coupling constant (Dysom F.J 1948). There is a 6.95% difference which is indicatively the substantial charge dependence of the pion nucleon coupling constant.

The nuclear forces in the nucleon-nucleon system at low energies primarily to exchange virtual pion, we have established relations between quantities that characterize the pion – nucleon and nucleon-nucleon interactions. We have deduced the equation that relates pseudo-vector charged and mental pion-nucleon coupling constant to the depth of the potentials that describe the np and pp interactions by using the values of $a_{pp} = 17.2$ and $r_{pp} =$

2.849 currently recommended for the purely nuclear pp Scattering length and effective range. The experimental value of $a_{pp} = -23.71$ for the singlet np scattering length and the values of $\frac{g^2_{\pi^0}}{4\pi}$=13.56 for the neutral pion – nucleon coupling constant pseudo-scalar. The values obtained are $\frac{g^2_{\pi\pm}}{4\pi} = 14.54$ for the charged pion nucleon coupling constant. The values are closely the same as the experimental values as $\frac{g^2_{\pi\pm}}{4\pi} = 15.51$ and $\frac{g^2_{\pi\pm}}{4\pi} = 17.73$ for the case of neutron-proton and with the experimental values of $\frac{g^2_{\pi\pm}}{4\pi} = 14.52$ and $\frac{g^2_{\pi\pm}}{4\pi} = 14.74$ and with the average value of $\frac{g^2_{\pi\pm}}{4\pi} = 14.28$. at the same time, the value we obtained is an odd value with $\frac{g^2_{\pi\pm}}{4\pi} = 13.54$ it was found by using (NN) scattering data and the value of $\frac{g^2_{\pi\pm}}{4\pi} = 13.76$ computed from data on πN scattering. It is worth noting, however, a small value of pion nucleon coupling constant as $\frac{g^2_{\pi^0}}{4\pi} = 14.49$ results in the weakening of the neutron-proton potential and to the absolute value of the singlet np scattering length which is underestimated as compared with the experimental value of $|a_{pp}| = 23.70$.

The **graphical figures** present the relationship between pion-nucleon coupling constant based on purely nuclear scattering length a_{pp}, a_{np} and effective range $r_{pp},\ r_{np}$ on the Yukawa potential.

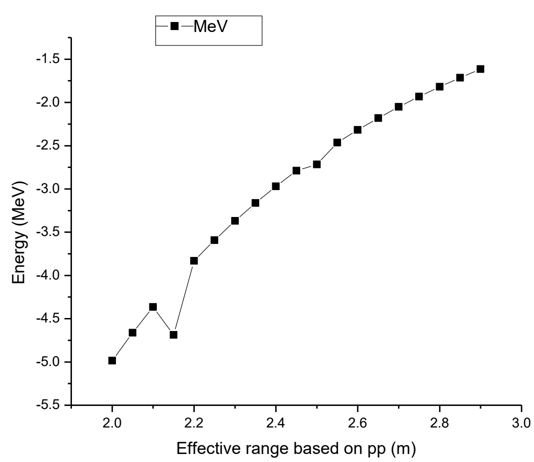

Figure 1. The plot Energy and Effective Range based on Proton-Proton.

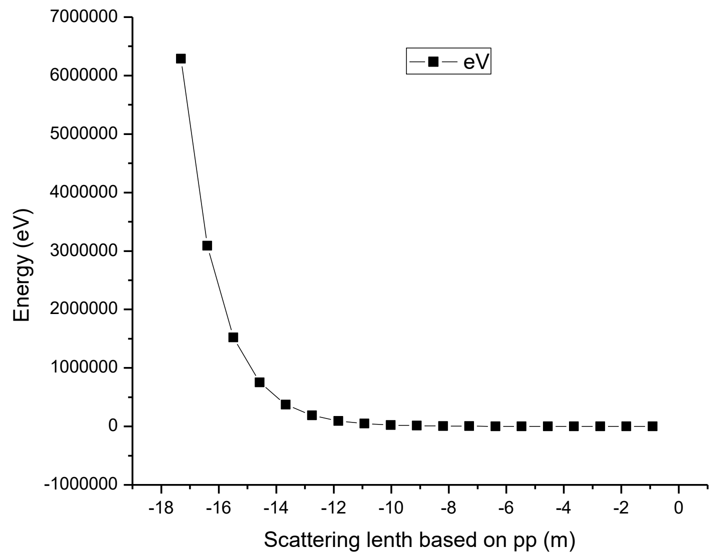

Figure2. The plot of energy and pure nuclear scattering length.

Result

The result obtained in the study signified a substantial charged dependence on the pion –nucleon coupling constant, the low energies pion nucleon and nucleon-nucleon parameters result in the following conclusions that the desecration of the charge independence of nuclear forces is due to primarily to the difference in mass and pion – nucleon coupling constants between the neutral and charge pion and lead to a violation of the charge independence of nuclear force can be found, from the obtained result shows that the in pion – nucleon coupling constants have a value of 165.6108 $^{MeV}/_{C^2}$ and 0.2707 with an average Pion – mass of 1.0227.

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