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COMPUTATIONAL ANALYSIS OF THE EFFECT OF VARYING THE GROWTH RATES OF INTERACTING DYNAMICAL SYSTEMS ON THE MANHATTAN'S DISTANCE MEASURE: ECOSYSTEM FUNCTIONING IMPLICATIONS

J. I. GALADIMA[1], E. N. EKAKA-A[2], B. O. NWALA[3]
[1]Department of Mathematics, IBB University, Lapai, Minna, Niger State, Nigeria. [2]Department of Mathematics, Faculty of Science, Rivers State University, Nkpolu, Port Harcourt, Nigeria. [3]Department of Mathematics and Statistics, Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt, Nigeria

Abstract
Of the three classifications of the real line metric function, the Manhattan's distance function is a significant type of a metric function specification. Calculating its value analytically can take a longer computational time to obtain an approximate value. To circumvent this problem, we have applied the ODE 45 computationally efficient numerical method to compute the effect of varying the growth rates of interacting yeast species continuous nonlinear real time-dependent dynamical systems on the Manhattan's distance measure. We would expect our present results to provide a further insight on an aspect of ecological modeling called the ecosystem functioning. The novel results that we have obtained which we have not seen elsewhere are fully presented and discussed quantitatively.

Introduction
In the theory of the real number metric function; there are three types of the metric functions namely the Manhattan's distance, the Hamming's distance and the Euclidean distance [1]. These distinct types of metric measures have measurable scientific benefits in the discipline of ecosystem functioning [2].

Since biodiversity scenario depends on the pairs of data sets, it is important to find out if the properties of a symmetry property of a metric space is satisfied from which the Manhattan's distance will be computed based on the increasing independent variable hereby called time t in the unit of days following the growth of two competing yeast species [3]. Other similar aspects of the dynamical system modeling of two competing populations can be found in the research contributions of [3-10] in which the variation of the growth rates effect on the Manhattan's distance have not been studied.

Mathematical Formulations

Following Pielou [3], we have considered the following continuous dynamical system of nonlinear first order ordinary differential equations with the following Lotka-Volterra mathematical structure

$$\frac{dy_1(t)}{dt} = y_1(t)[a - by_1(t) - cy_2(t)] \qquad (1)$$

$$\frac{dy_2(t)}{dt} = y_2(t)[d - ey_1(t) - fy_2(t)] \qquad (2)$$

For the purpose of this study, the notations $y_1(t)$ and $y_2(t)$ specify the limiting biomasses of the yeast species 1 and yeast species 2 under the simplifying assumption that the positive starting biomass for the yeast species 1 is $y_1(0)$ while the positive starting biomass for the yeast species 2 is $y_2(0)$. In this present study, we have used the unit of the grams per area of grass cover for the biomass. The growth rate parameter values are a = 0.1 and d = 0.08, the intra-species competition parameter values are b = 0.0014 and f =0.001 whereas the inter-species competition parameter values are c = 0.0012 and e = 0.0009.

Method of Solution

In this study, we have applied the ordinary differential equations numerical method of order 45 [4] to compute the Manhattan's distance measure which we have not previously researched into.

In the characterization space of the time-dependent solution trajectories or solution maps, the Manhattan's distance is defined by

$$d(x, y) = \max|x - y| \qquad (3)$$

where x and y are called the elements of the yeast species 1 biomass and the yeast species 2 biomass respectively. That is, the disparity between the yeast species 1 biomass and the yeast species 2 biomass is measured by the metric function being the absolute value of each pair of data of which the maximum of these time-dependent metrics is called the Manhattan's distance measure. Mathematically, the concept of a real line metric and hence the Manhattan's distance measure are the functions of two time-dependent data which can be numerically determined using their initial condition boundaries. For this reason, if a few model parameter values are fixed and the growth rates are either decreased or increased with a specified initial condition, the metric and the Manhattan's distance measure will be affected.

Looking at the Table 1results, column one (1) specifies the independent variable t that represents the length of the growing season in the unit of days ranging from t = 0 which defines the initial condition time to t = 95 days which defines the harvesting time. The second column data represent the simulated weight of the first yeast species which converges from the initial value of 4.00 to a finite limit value of 82.4564 at time t = 95 days whereas the third column data represent the simulated weight of the second yeast species which converges from the initial value of 4.00 to a finite limit value of 76.3385 at time t = 95 days. These two sets of data have the same dimension having equal number of elements as shown in the second and the third columns. From the theory of functional analysis, for each pair of data, the estimated metric is calculated on the application of equation (3). One of the properties of a metric space is called the symmetric property. For example, for the first row pair of data values (see Table 1) for t = 0, the metric value of d(4.0, 4.0) is equal to zero because the absolute value of zero is zero; for the second row pair of data values (see Table 1) for t = 5 days, the calculated metric value of d(6.8214, 6.3533) is equal to 0.4581 approximately since all the calculated metric values are approximated to four (4) decimal real numbers; for the third row pair of data values (see Table 1) for t = 10 days, the calculated metric value of d(11.6173, 9.9195) is equal to 1.6979 approximately since all the calculated metric values are approximated to four (4) decimal real numbers and also on the fact that the simulated data values where formatted as long decimals from which the Matlab function computes the metric value and estimated the metric value for the pair of data (11.6173, 9.9195) to be 1.6979 (which is different from 1.6978 being the analytical value). The same procedure was used to calculate the metric value for the fourth row pair of data up to the last row pair of data corresponding to t =

95 days. The fourth column of data specifies the metric values of the second column data and the third column data. By using the same procedure of calculating the estimated metric between the third column data and the second column data, we have presented the metric row calculated values as shown in column five (5). Next, from the theory of functional analysis, when the metric values between two non-empty sets of data are equal, the mathematical concept of a symmetry property is satisfied

As displayed in Table 1, all the metric calculated values are the same which prove the property the symmetry property of a metric space. In particular, the symmetry metric values as shown in the fourth and the fifth columns increase positively monotonically from the value of zero for t =0 to the value of 28.8565 for t = 45 and start to decrease negatively monotonically until the value of 11.1180. A closer look at the metric values shows that the maximum value of the metric value is 28.8565 provided the growth rate is decreased by five (5) percent. This observation is consistent with the theoretical definition of the Manhattan's distance as defined in equation three (3) and computationally obtained and displayed as shown in Table 1-Table 6.

Since the computing of these metrics and hence the Manhattan's distance measure deals with bigger time-dependent data with varying initial condition, it makes sense to apply a numerical method to calculate the Manhattan's distance measure for a specified numerical simulation. From a biological perspective, the ecosystem supplies ecosystem services that sustain several types of living organisms within the chosen environment. Ecosystem functioning is an aspect of the ecosystem that benefits from the ecosystem services [11]. If the metric between interacting yeast species affects their accessibility to ecosystem services, it would affect the ecosystem functioning characterization to an extent. Therefore, the smaller the value of the metric and hence the Manhattan's distance measure, the lesser its impact on the ecosystem services and ecosystem functioning and the bigger the value of the metric and hence the Manhattan's distance measure, the bigger its impact on the ecosystem services and ecosystem functioning. In this present study, the Manhattan's distance measure is being affected by the experimental time and the specified initial condition.

Data Nomenclature

For the purpose of consistent interpretation, column one specifies the variation of the length of the growing season in the unit of days; column two specifies the biomass of the first yeast species due to the changing experimental time for a specified percent variation of the growth rates together; column three specifies the biomass of the second yeast species due to the changing experimental time for the equal initial condition boundaries of 4.0 and 4.0 and in the sequel for the (10.0, 10.0) initial condition ; columns four and five specify the computed symmetry property of a metric function from which the Manhattan's distance was computed for a chosen sample space.

Results

The results of this present study are presented as displayed in Table 1-Table 4.

Table 1: Computing the Manhattan's distance measure due to the initial condition (4.0, 4.0) using ODE 45 numerical method when the experimental time ranges from t = 0 to t = 95 in the unit of days using a five (5) percent severe decrease of the growth rates

0.0000	4.0000	4.0000	0.0000	0.0000
5.0000	6.8214	6.3633	0.4581	0.4581
10.0000	11.6173	9.9195	1.6979	1.6979
15.0000	19.4689	15.0022	4.4667	4.4667
20.0000	31.3015	21.7704	9.5311	9.5311
25.0000	46.6126	29.9689	16.6437	16.6437
30.0000	62.5535	38.8383	23.7151	23.7151
35.0000	75.4044	47.3364	28.0680	28.0680
40.0000	83.4366	54.5801	**28.8565**	**28.8565**
45.0000	87.2942	60.1655	27.1286	27.1286
50.0000	88.5010	64.1447	24.3562	24.3562f
55.0000	88.3406	66.8287	21.5119	21.5119
60.0000	87.5841	68.5649	19.0192	19.0192
65.0000	86.6465	69.6653	16.9812	16.9812
70.0000	85.7136	70.3477	15.3659	15.3659
75.0000	84.8626	70.7695	14.0932	14.0932
80.0000	84.1166	71.0270	13.0896	13.0896
85.0000	83.4740	71.1845	12.2895	12.2895
90.0000	82.9249	71.2802	11.6447	11.6447
95.0000	82.4564	71.3385	11.1180	11.1180

The following results correspond to the equal initial condition boundaries of 10.0 and 10.0 on the Manhattan's distance based on the column specifications we have defined above.

Table 2: Computing the Manhattan's distance measure due to the initial condition (10.0, 10.0) using ODE 45 numerical method when the experimental time ranges from t = 0 to t = 95 in the unit of days using a five (5) percent severe decrease of the growth rates

0.0000	10.0000	10.0000	0.0000	0.0000
5.0000	13.7329	14.0540	0.3211	0.3211
10.0000	18.1693	19.2993	1.1300	1.1300
15.0000	22.8420	25.7376	2.8956	2.8956
20.0000	26.8720	33.1504	6.2784	6.2784
25.0000	29.2412	41.0825	11.8412	11.8412
30.0000	29.4246	48.9293	19.5047	19.5047
35.0000	27.8268	56.1144	28.2876	28.2876
40.0000	25.3750	62.2410	36.8661	36.8661
45.0000	22.8310	67.1562	44.3253	44.3253
50.0000	20.5666	70.9092	50.3426	50.3426
55.0000	18.6814	73.6699	54.9885	54.9885
60.0000	17.1514	75.6426	58.4912	58.4912
65.0000	15.9183	77.0264	61.1081	61.1081
70.0000	14.9194	77.9815	63.0622	63.0622
75.0000	14.1033	78.6360	64.5327	64.5327
80.0000	13.4296	79.0802	65.6506	65.6506
85.0000	12.8677	79.3812	66.5136	66.5136
90.0000	12.3942	79.5840	67.1898	67.1898
95.0000	11.9917	79.7207	**67.7290**	**67.7290**

The following results correspond to the effect of a ten percent severe decrease of the growth rates on the Manhattan's distance measure using the initial condition boundaries of 4.0 and 4.0.

Table 3: Computing the Manhattan's distance measure due to the initial condition (4.0, 4.0) using ODE 45 numerical method when the experimental time ranges from t = 0 to t = 95 in the unit of days using a ten (10) percent severe decrease of the growth rates

0.0000	4.0000	4.0000	0.0000	0.0000
5.0000	4.3332	4.0877	0.2455	0.2455
10.0000	4.6753	4.1748	0.5005	0.5005
15.0000	5.0243	4.2612	0.7631	0.7631
20.0000	5.3775	4.3467	1.0308	1.0308
25.0000	5.7324	4.4313	1.3011	1.3011
30.0000	6.0864	4.5149	1.5715	1.5715
35.0000	6.4367	4.5974	1.8393	1.8393
40.0000	6.7807	4.6787	2.1020	2.1020
45.0000	7.1158	4.7588	2.3571	2.3571
50.0000	7.4399	4.8375	2.6024	2.6024
55.0000	7.7507	4.9149	2.8358	2.8358
60.0000	8.0465	4.9908	3.0557	3.0557
65.0000	8.3258	5.0652	3.2606	3.2606
70.0000	8.5875	5.1381	3.4494	3.4494
75.0000	8.8308	5.2094	3.6214	3.6214
80.0000	9.0551	5.2791	3.7760	3.7760
85.0000	9.2603	5.3471	3.9131	3.9131
90.0000	9.4464	5.4135	4.0329	4.0329
95.0000	9.6138	5.4782	**4.1356**	**4.1356**

The following results correspond to the effect of a ten percent severe decrease of the growth rates on the Manhattan's distance measure using the initial condition boundaries of 10.0 and 10.0.

Table 4: Computing the Manhattan's distance measure due to the initial condition (10.0, 10.0) using ODE 45 numerical method when the experimental time ranges from t = 0 to t = 95 in the unit of days using a ten (10) percent severe decrease of the growth rates

0.0000	10.0000	10.0000	0.0000	0.0000
5.0000	9.7234	9.9029	0.1795	0.1795
10.0000	9.4530	9.8114	0.3584	0.3584
15.0000	9.1881	9.7251	0.5370	0.5370
20.0000	8.9282	9.6435	0.7153	0.7153
25.0000	8.6730	9.5665	0.8935	0.8935
30.0000	8.4223	9.4936	1.0712	1.0712

35.0000	8.1761	9.4246	1.2485	1.2485
40.0000	7.9343	9.3592	1.4249	1.4249
45.0000	7.6972	9.2973	1.6001	1.6001
50.0000	7.4648	9.2386	1.7738	1.7738
55.0000	7.2374	9.1828	1.9455	1.9455
60.0000	7.0152	9.1299	2.1147	2.1147
65.0000	6.7985	9.0796	2.2811	2.2811
70.0000	6.5875	9.0318	2.4443	2.4443
75.0000	6.3825	8.9864	2.6039	2.6039
80.0000	6.1835	8.9431	2.7596	2.7596
85.0000	5.9909	8.9020	2.9111	2.9111
90.0000	5.8046	8.8628	3.0581	3.0581
95.0000	5.6249	8.8255	**3.2006**	**3.2006**

The following results correspond to the effect of a 101 percent increase of the growth rates on the Manhattan's distance measure using the initial condition boundaries of 4.0 and 4.0.

Table 5: Computing the Manhattan's distance measure due to the initial condition (4.0, 4.0) using ODE 45 numerical method when the experimental time ranges from t = 0 to t = 95 in the unit of days using a 101 percent increase of the growth rates

0.0000	4.0000	4.0000	0.0000	0.0000
5.0000	6.8559	6.3946	0.4614	0.4614
10.0000	11.7350	10.0140	1.7210	1.7210
15.0000	19.7585	15.2057	4.5528	4.5528
20.0000	31.8793	22.1332	9.7462	9.7462
25.0000	47.5397	30.5232	17.0165	17.0165
30.0000	63.7346	39.5749	24.1597	24.1597
35.0000	76.6365	48.2028	28.4337	28.4337
40.0000	84.5712	55.5061	**29.0651**	**29.0651**
45.0000	88.2959	61.0938	27.2021	27.2021
50.0000	89.3933	65.0430	24.3504	24.3504
55.0000	89.1602	67.6869	21.4733	21.4733
60.0000	88.3617	69.3849	18.9768	18.9768
65.0000	87.4017	70.4543	16.9474	16.9474

70.0000	86.4588	71.1133	15.3455	15.3455
75.0000	85.6045	71.5184	14.0861	14.0861
80.0000	84.8588	71.7643	13.0945	13.0945
85.0000	84.2183	71.9140	12.3043	12.3043
90.0000	83.6721	72.0044	11.6678	11.6678
95.0000	83.2068	72.0592	11.1476	11.1476

The following results correspond to the effect of a 105 percent increase of the growth rates on the Manhattan's distance measure using the initial condition boundaries of 10.0 and 10.0.

Table 6: Computing the Manhattan's distance measure due to the initial condition (10.0, 10.0) using ODE 45 numerical method when the experimental time ranges from t = 0 to t = 95 in the unit of days using a 105 percent increase of the growth rates

0.0000	10.0000	10.0000	0.0000	0.0000
5.0000	13.7854	14.1086	0.3232	0.3232
10.0000	18.2979	19.4426	1.1447	1.1447
15.0000	23.0558	26.0063	2.9504	2.9504
20.0000	27.1467	33.5729	6.4262	6.4262
25.0000	29.5167	41.6677	12.1509	12.1509
30.0000	29.6405	49.6608	20.0203	20.0203
35.0000	27.9644	56.9562	28.9918	28.9918
40.0000	25.4527	63.1498	37.6971	37.6971
45.0000	22.8760	68.0939	45.2179	45.2179
50.0000	20.5983	71.8488	51.2505	51.2505
55.0000	18.7103	74.5962	55.8859	55.8859
60.0000	17.1826	76.5491	59.3665	59.3665
65.0000	15.9535	77.9121	61.9586	61.9586
70.0000	14.9589	78.8482	63.8893	63.8893
75.0000	14.1470	79.4867	65.3397	65.3397
80.0000	13.4771	79.9181	66.4411	66.4411
85.0000	12.9184	80.2092	67.2908	67.2908
90.0000	12.4479	80.4045	67.9566	67.9566
95.0000	12.0479	80.5355	**68.4876**	**68.4876**

Discussion of Results

For Table 1 when the initial condition boundaries are 4.0 and 4.0, the maximum metric otherwise called the Manhattan's distance is specified by 28.8565 approximately at time t = 40 days whereas for the initial condition boundaries (10, 10), the Manhattan's distance is specified by 67.7290 at time t = 95 days as presented in Table 2 . Therefore, the inclusion of the two distinct initial conditions indicates a bigger disparity of the Manhattan's distance which can be found at two different time values. In particular, for the (4.0, 4.0) initial condition, the metric increases from t = 0 to t=40 up to the value of 28.8565 and thereafter decreases from t = 45 to t = 95 up to the value of 11.1180 approximately. In this instance, the metric indicates a point of inflexion between t = 40 and t = 45 due to the five (5) percent decreased variation of the growth rates together. Hence, the Manhattan's distance measure is associated with the time point of inflexion. Under the same variation of the growth rates, for the initial condition (10.0, 10.0), the metric increases positively monotonically from the value of zero for t = 0 up to the value of 67.7290 for t = 95, hence the Manhattan's distance measure has occurred for t = 95 (Table 2). These two observations about the characteristic of the Manhattan's distance measure is only unique to the specified model parameters and the type of interaction between two yeast species of the [3] model parameterization and its experimentally determined model parameter values.

In the context of a ten (10) percent decreased variation of the growth rates together (Table 3), when the initial condition is (4.0, 4.0), the time-dependent metric increases positively monotonically from t = 0 to t = 95 in which the maximum value of the metric called the Manhattan's distance value is 4.1356 at time t = 95 days. In contrast, when the initial condition is (10.0, 10.0), the Manhattan's distance value is 3.2006 at time t = 95 days as presented in Table 4. Therefore, we have found the Manhattan's distance to have occurred at the time instance t = 95 days but differs in the choice of the two different initial condition boundaries. That is, the boundary condition (10.0, 10.0) is associated with a smaller value of the Manhattan's distance measure.

In the context of 101 percent increase of the growth rates together (Table 5), when the initial condition is (4.0, 4.0), the Manhattan's distance value is 29.0651 at time t = 40 days. In contrast, when the initial condition is (10.0, 10.0), the Manhattan's distance value is 68.4876 at time t = 95 days as presented in Table 6 when the two growth rates were varied by 105 percent. Therefore, a small increase of the growth rates is associated with a relatively bigger value of the Manhattan's distance value at an earlier time instance when the initial condition is (4.0, 4.0) whereas at a higher increase of the growth rates when the initial condition is (10.0, 10.0), the Manhattan's distance is more than twice bigger.

In summary, by using the method of a numerical simulation that is indexed by the ODE 45 which is computationally more efficient, we have found a fundamental link between the initial condition, the instance of experimental time in the unit of days and the Manhattan's distance measure. Since the metric and the Manhattan's distance measure depend on the time changing biomass of the interacting yeast species which also depend on the initial condition from which the concepts of ecosystem services and the ecosystem functioning can be derived, it follows that the effect of the growth rates on the Manhattan's distance measure can have an impact on the ecosystem functioning scenario. This present novel result is a major cutting-edge contribution to the knowledge base in the computational ecological theory which complements the earlier experimental results in the work of [3] which we have not seen elsewhere.

Conclusion and Further Research

By a variation of the two growth rates together under different initial conditions, we have applied the ODE 45 numerical method to compute the Manhattan's distance. Depending on the choice of the time instance, we have found that the Manhattan's distance changes. In particular, when the growth rates are decreased by 10 percent under the choice of the initial condition (10.0, 10.0), we have found the smallest value of the Manhattan's distance to be equal to 3.2006 at the optimal experimental time of ninety-five (95) days. That is, at the harvesting time of the competing yeast species for limited resources, the corresponding Manhattan's distance is small under the equal initial condition boundaries assumption in the deterministic ODE 45 numerical simulation which has not previously been discovered to the best of our knowledge.

From the biological context, the initial condition (4.0, 4.0) with a smaller value of the Manhattan's distance measure for $t = 40$ days is associated with a lesser vulnerability to the loss of ecosystem services and hence on the ecosystem functioning scenario which is consistent with the idea of [11] provided the two growth rates are decreased together by five (5) percent. In the situation when the two growth rates are decreased by ten (10) percent, the initial condition (10.0, 10.0) with smaller value of the Manhattan's distance measure for t = 95 days is associated with a lesser vulnerability to the loss of ecosystem services and ecosystem functioning which is again consistent with the biological idea of [11]. In contrast, the 101 percent increased variation of the growth rates together for the initial condition (4.0, 4.0) with a smaller value of the Manhattan's distance measure for t = 40 days is linked with a lesser vulnerability to the loss of ecosystem services and hence on the ecosystem functioning [11].

These results would provide some useful insights into the theory of ecosystem functioning and management. The present numerical method calculation of the Manhattan's distance can be extended to other variations of the model parameter values which we did not investigate in this present study.

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