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EFFECT OF RADIATION AND HEAT SOURCE/SINK ON THREE DIMENSIONAL MAGNETOHYDRODYNAMIC (MHD) CASSON FLUID FLOW OVER AN EXPONENTIAL STRETCHING SHEET

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Abstract

The effect of radiation and heat source/sink on three dimensional Magnetohydrodynamics (MHD) casson fluid flowover an exponentially stretching sheet is investigated. The governing partial differential equations were reduced to ordinary differential equations using similarity transformation. The reduced non-linear ordinary differential equations were solved analytically and the results obtained presented graphically. It was observed that increase in casson parameter and magnetic parameter decreased velocity profiles while thermal grashof number enhanced velocity profile, heat source/sink and radiation parameter enhanced the temperature profile while prandtl number and unsteadiness parameter decreased the temperature profile.

Keywords: *MHD, Casson fluid, Stretching sheet, Thermal radiation, Unsteadiness, heat source/sink.*

Introduction

Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear.A fluid in which the viscous stresses arising from its flow at every point are linearly proportional to the rate of change in its deformation over time is called Newtonian fluid. This means that in a

Newtonian fluid, the relationship between the shear stress and the shear rate are linear with the proportionality constant referred to as the coefficient of viscosity. On the other hand, a fluid whose flow properties are different in any way from that of the Newtonian fluid is called a non-Newtonian fluid. That is to say, in a non-Newtonian fluid, the relationship between the shear stress and the shear rate is nonlinear (Emmanuel *et al*., 2015).
Many researchers have developed and studied the transport properties of Casson fluid over the last few decades these authors, among others include Wahiduzzaman*et al.* (2014), they examined three-dimensional steady MHD casson fluid flow past a non-isothermal porous linearly stretching sheet, the governing equations were solved numerically using Nactsheim-swigert shooting iteration technique together with runge-kutta sixth order iteration.Hussanan*et al*. (2016) investigated the effects of Newtonian heating and inclined magnetic field on two-dimensional flow of a Casson fluid over a stretching sheet. Reddy *et al*. (2016) investigated the influence of radiation absorption on unsteady MHD free convective and mass transfer flow, a heat generating Casson fluid past an oscillating vertical plate embedded in a porous medium in the presence of constant wall temperature.
Mahanta and Shaw (2015) investigated the mixed convection stagnation point flow of an incompressible non-Newtonian fluid over a stretching sheet with magnetic field under convective boundary condition. The resulting partial differential equations were converted into ordinary differential equations by the suitable transformations. The velocity, temperature and concentration profiles were computed by employing the homotopy analysis method. Mythili*et al*. (2015) investigated the study of unsteady free convective Casson fluid flow over a vertical cone saturated with porous medium in presence of non-uniform heat source/sink, high order chemical reaction and cross diffusion effects. The governing equations were solved numerically using finite difference method of Crank-Nicolson type.
Mukhopadhyay (2013) investigated the analysis of non-Newtonian fluid flow and heat transfer over a nonlinearly stretching surface. The governing partial differential equations were transformed by using suitable transformations into ordinary differential equations and the numerical solutions were obtained with the shooting method.

Suresh *et al*. (2016) investigated the radiative convection of an unsteady flow and heat transfer of Casson fluid with variable thermal conductivity, viscous dissipation and heat source/sink past a stretching sheet. The governing partial differential equations were transformed into ordinary differential equations by suitable similarity transformations and the resultant equations were solved numerically using Matlab 45 solver via shooting technique. Mohammed*et al*. (2020) worked investigated an unsteady MHDcasson fluid flow over an exponentially stretching sheet with effect of radiation. This paper presents the unsteady case of Mohammed*et al*. (2020)

Mathematical Formulation

We consider three dimensionalunsteady incompressible flows over an exponentially stretching sheet. The sheet is stretched along the xy plane, while the fluid is placed along the z - axis; the sheet $z=0$, the uniform magnetic field is applied in z - direction that is perpendicular to the flow direction. Here, we assumed that the sheet was stretched with velocities $U_w = \frac{U_0 e^{\frac{x+y}{L}}}{(1-ct)}$ and $V_w = \frac{V_0 e^{\frac{x+y}{L}}}{(1-ct)}$ along the xy-plane respectively, $T_w = T_\infty + \frac{T_0 e^{\frac{x+y}{L}}}{(1-ct)} e^{\frac{x+y}{L}}$, where U_0, V_0 and T_0 are constants. A heat source/sink placed within the flow to allow for heat generation or absorption effects.

The rheological equation of state for an isotropic flow of casson fluid as stated by Kumar and Gangadhar(2015). can be expressed as

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_z}{\sqrt{2\pi}}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \frac{p_z}{\sqrt{2\pi_c}}\right)e_{ij}, \pi < \pi_c \end{cases} \quad (1)$$

In the above equation $\pi = e_{ij}e_{ij}$ and e_{ij} denotes the $(i,j)^{th}$ components of the deformation rate, π is the product of the deformation rate itself, π_c is the critical value of this product based on the non-Newtonian fluid model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid and p_z is the yield stress of the fluid. From (3.1) we obtain $\mu_B = \frac{1}{2}\frac{\tau_{ij}}{e_{ij}} - \frac{p_z}{\sqrt{2\pi}}$, $\upsilon = \frac{\mu_B}{\rho}$ and $\beta = \frac{\sqrt{2\pi_c}}{p_z}\mu_B$

The radiatve heat term, using the Roseland approximation as in Yusuf *et al.*[2016] is given by, $q_r = -\frac{4\sigma_1}{3k_1}\frac{\partial\left(T^4\right)}{\partial z}$ (2)

where σ_1 and k_1 are the Stefan Boltzmann constant and mean absorption coefficient respectively.

The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \upsilon\left(1 + \frac{1}{\beta}\right)\left[\frac{\partial^2 u}{\partial z^2}\right] - \frac{\sigma B^2}{\rho}u + g\beta_T\left(T - T_\infty\right) \tag{4}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \upsilon\left(1 + \frac{1}{\beta}\right)\left[\frac{\partial^2 v}{\partial z^2}\right] - \frac{\sigma B^2}{\rho}v \tag{5}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\left[\frac{\partial^2 T}{\partial z^2}\right] + \frac{Q_1}{\rho c_p}\left(T - T_\infty\right) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial z} \tag{6}$$

With the initial and boundary conditions

$$\left.\begin{array}{l} u(z,t) = 0, v(z,t) = 0, w = 0, T(z,t) = T_\infty,\ for\, t = 0\ for\ all\ z \\ u = U_w, v = V_w, T = T_w, at\ z = 0 \\ u \to 0, v \to 0, T \to T_\infty,\ at\ z \to \infty \end{array}\right\} \tag{7}$$

Where

u, v and w are the velocity component in the direction of x, y and z respectively, β is the casson fluid parameter, υ is the kinematic viscosity, B is the magnetic induction, B_0 is constant, T is temperature of the fluid, β_T is the coefficient of volume expansion for temperature differences, β_{T_0} is constants, Q_1 is heat generation term ($Q_1 > 0$) or absorption ($Q_1 < 0$) coefficient, Q_0 is a constant, k thermal diffusivity , δ is the density of the fluid, g is acceleration due to gravity, σ is the electrical conductivity, c_p is the specific heat capacity at constant pressure, T_∞ is the free stream temperature, k is the Boltzmann constant k_0 is constant

METHODOLOGY OF SOLUTION

Introducing the following similarity variables:

$$\left.\begin{aligned}&\eta=\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x+y}{2L}}z,u=\frac{U_0}{(1-ct)}e^{\frac{x+y}{L}}f'(\eta),v=\frac{U_0}{(1-ct)}e^{\frac{x+y}{L}}g'(\eta),T=T_\infty+\frac{T_0}{(1-ct)}e^{\frac{x+y}{L}}\theta(\eta),\\&\beta_T=\frac{\beta_{T_0}}{(1-ct)}e^{\frac{x+y}{L}},B=\frac{B_0}{(1-ct)^{\frac{1}{2}}}e^{\frac{x+y}{2L}},Q=\frac{Q_0}{(1-ct)}e^{\frac{x+y}{L}}\end{aligned}\right\} \quad (8)$$

The transformed equations and the boundary conditions are:

$$\left(1+\frac{1}{\beta}\right)f'''(\eta)+(f+\eta f'+g+\eta g')f''(\eta)-2(f'+g')\left(f'+\frac{\eta}{2}f''\right)-Mf' \\ -\frac{a}{R_e}\left(f'+\frac{\eta}{2}f''\right)+G_{r_\theta}\theta(\eta)=0 \quad (9)$$

$$\left(1+\frac{1}{\beta}\right)g'''(\eta)+(f+\eta f'+g+\eta g')g''(\eta)-\frac{a}{R_e}\left(g'+\frac{\eta}{2}g''\right)-2(f'+g')\left(g'+\frac{\eta}{2}g''\right) \quad (10) \\ -Mg'=0$$

$$\frac{1}{P_r}\theta''(\eta)+\frac{R}{P_r}\theta''(\eta)+(f+\eta f'+g+\eta g').\theta'(\eta)-2(f'(\eta)+g'(\eta))\left(\theta(\eta)+\frac{\eta}{2}\theta'(\eta)\right) \\ -\frac{a}{R_e}\left(\theta(\eta)+\frac{\eta}{2}\theta'(\eta)\right)+Q_h\theta(\eta)=0 \quad (11)$$

$$\left.\begin{aligned}&f(0)=0,\quad g(0)=0,\ f'(0)=1,\quad g'(0)=\alpha,\quad \theta(0)=1,\\&f'\to 0\ as\ \eta\to\infty,\ g'\to 0\ as\ \eta\to\infty,\ \theta\to 0\ as\ \eta\to\infty\end{aligned}\right\} \quad (12)$$

Where,

$$G_{r_\theta}=\frac{2Lg\beta_{T_0}T_0}{U_0^2},\frac{a}{R_e}=\frac{2Lc}{U_0e^{\frac{(x+y)}{L}}},M=\frac{2L\sigma B_0^2}{\rho U_0},\alpha_h=\frac{k_h}{\rho c_p},$$

$$,Q_h=\frac{2LQ_0}{\rho c_pU_0},R=\frac{16T_\infty^3\sigma_1}{3k_1k_h},\frac{1}{P_r}=\frac{k_h}{\rho c_p\upsilon}$$

The transformed non-linear ordinary differential equations (9) to (12) were solved using iteration perturbation method as used in (Mohammed *et al*. 2015,Olayiwola2016andMohammed *et al*. 2020).

The solution of (9) to (12) gives:

$$f_0'(\eta) = e^{-c_5\eta} \tag{13}$$

$$g_0'(\eta) = \alpha e^{-c_5\eta} \tag{14}$$

$$\theta_0(\eta) = e^{-c_6\eta} \tag{15}$$

$$\left.\begin{array}{l} f_1' = c_7 e^{-2c_5\eta} + c_8 e^{-c_6\eta} - c_9 e^{-(b+c_5)\eta} - \dfrac{c_{15}}{c_5} e^{-c_5\eta} - c_{10}\eta e^{-c_5\eta} - c_{11} e^{-c_5\eta} + c_{12}\eta^2 e^{-c_5\eta} \\ + c_{13}\eta e^{-c_5\eta} + c_{14} e^{-c_5\eta} \end{array}\right\} \tag{16}$$

$$\left.\begin{array}{l} g_1' = c_{16} e^{-2c_5\eta} - c_{17} e^{-(b+c_5)\eta} - \dfrac{c_{23}}{c_5} e^{-c_5\eta} - c_{18}\eta e^{-c_5\eta} - c_{19} e^{-c_5\eta} + c_{20}\eta^2 e^{-c_5\eta} + \\ c_{21}\eta e^{-c_5\eta} + c_{22} e^{-c_5\eta} \end{array}\right\} \tag{17}$$

$$\theta_1 = -c_{24} e^{-(b+c_6)\eta} - c_{25}\eta e^{-c_6\eta} - c_{26} e^{-c_6\eta} + c_{27} e^{-(c_5+c_6)\eta} + c_{12}\eta^2 e^{-c_6\eta} + c_{13}\eta e^{-c_6\eta} + c_{14} e^{-c_6\eta} - \frac{c_{28}}{c_6} e^{-c_6\eta} \tag{18}$$

Where,

$$c_5 = \frac{b}{c_2},\ c_6 = \frac{b}{c_4},\ c_7 = \frac{1}{c_5^2 c_2}(1+\alpha),\ c_8 = \frac{G_{r_\theta}}{c_2 c_6 (c_5 - c_6)}, c_9 = \frac{c_5}{b^2 c_2 (b + c_5)}(1+\alpha),$$

$$c_{10} = \frac{1}{c_2 c_5}\left(\frac{c_5}{b} + \frac{\alpha c_5}{b} - bc_5 + M + c_3\right), c_{11} = \frac{1}{c_2 c_5^2}\left(\frac{c_5}{b} + \frac{\alpha c_5}{b} - bc_5 + M + c_3\right), c_{12} = \frac{c_3}{4c_2},$$

$$c_{13} = \frac{c_3}{2c_2 c_5}, c_{14} = \frac{c_3}{2c_2 c_5^2},\ c_{15} = (c_7 + c_8 - c_9 - c_{11} + c_{14})c_5, c_{16} = \frac{\alpha}{c_5^2 c_2}(1+\alpha), c_{17} = \frac{c_5 \alpha}{b^2 c_2 (b + c_5)}(1+\alpha),$$

$$c_{18} = \frac{\alpha}{c_2 c_5}\left(\frac{c_5}{b} + \frac{\alpha c_5}{b} - bc_5 + M + c_3\right),\ c_{19} = \frac{\alpha}{c_2 c_5^2}\left(\frac{c_5}{b} + \frac{\alpha c_5}{b} - bc_5 + M + c_3\right),$$ **4**

$$c_{20} = \alpha\frac{c_3}{4c_2},\ c_{21} = \alpha\frac{c_3}{2c_2 c_5},\ c_{22} = \alpha\frac{c_3}{2c_2 c_5^2}, c_{23} = (c_{16} - c_{17} - c_{19} + c_{22})c_5,\ c_{24} = \frac{1}{bc_4 (b + c_6)}\left(\frac{c_6}{b} + \frac{\alpha c_6}{b}\right),$$

$$c_{25} = \frac{1}{c_4 c_6}\left(\frac{c_6}{b} + \frac{\alpha c_6}{b} - bc_6 - Q_h + c_3\right), c_{26} = \frac{1}{c_4 c_6^2}\left(\frac{c_6}{b} + \frac{\alpha c_6}{b} - bc_6 - Q_h + c_3\right),$$

$$c_{27} = \frac{2}{c_4 c_5 (c_5 + c_6)}(1+\alpha),\ c_{28} = (c_{27} + c_{30} - c_{24} - c_{26})c_6,$$

Results and discussion

The graphical illustrations of the velocity profiles and and the temperature profile for different physical parameters are shown in figures below. The computations were done using MAPLE 17.

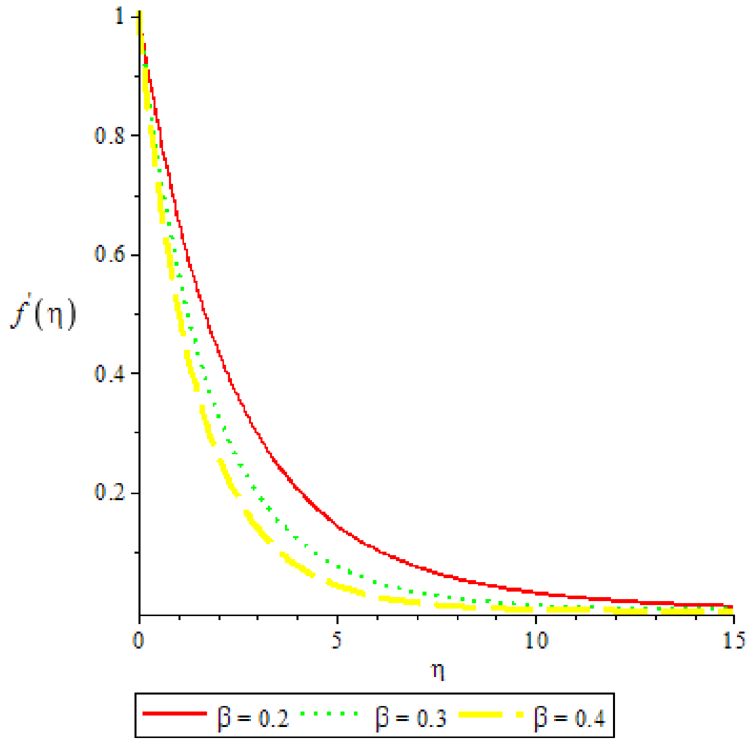

Figure 4.1: Effect of Casson parameter on velocity profile along x- direction

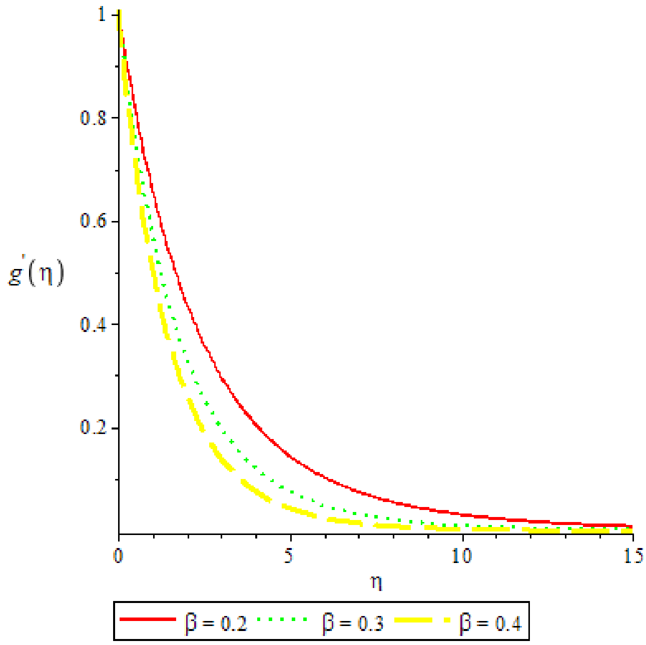

Figure 4.2: Effect of Casson parameter on velocity profile along y- direction

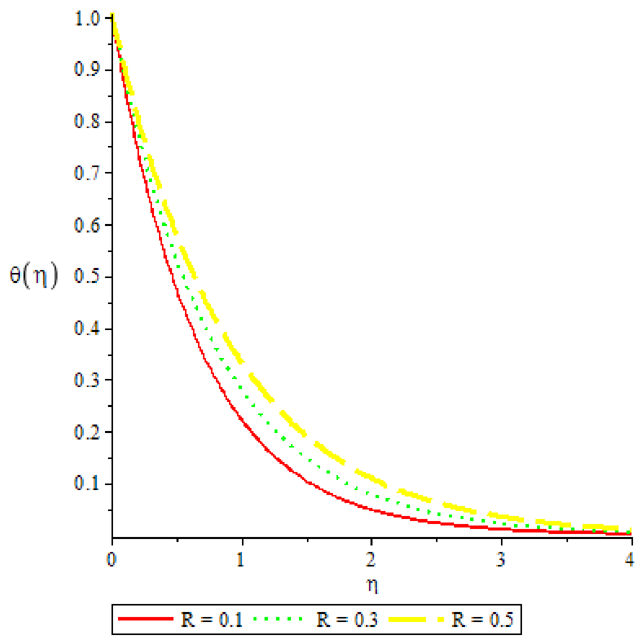

Figure 4.3: Effect of Radiation parameter on temperature profile

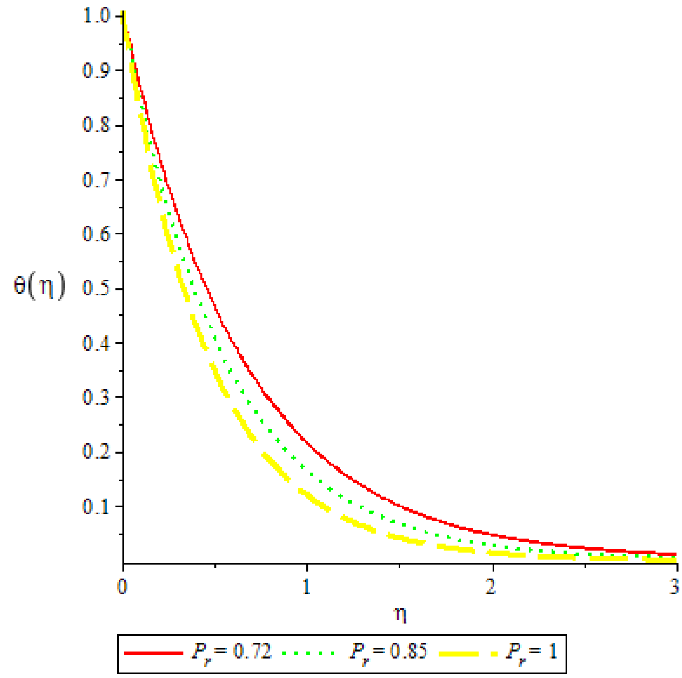

Figure 4.4: Effect of prandtl number on temperature profile

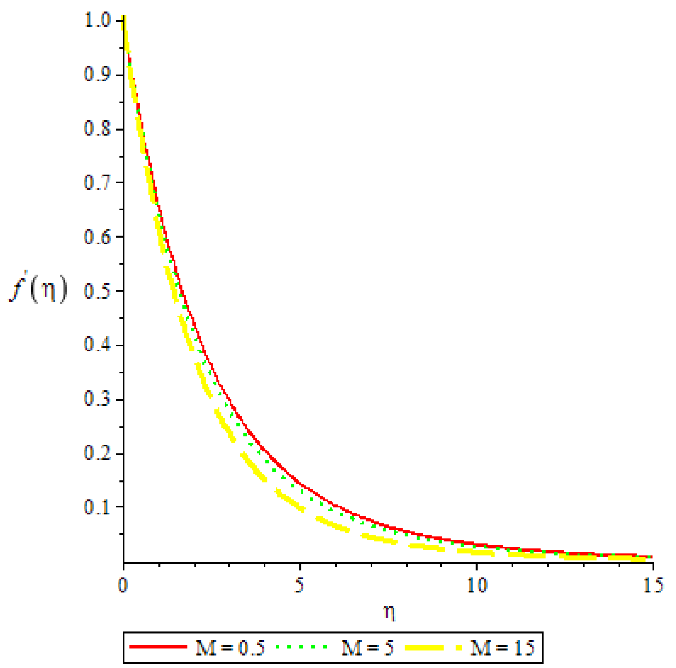

Figure 4.5: Effect of Magnetic parameter on velocity profile along x- direction

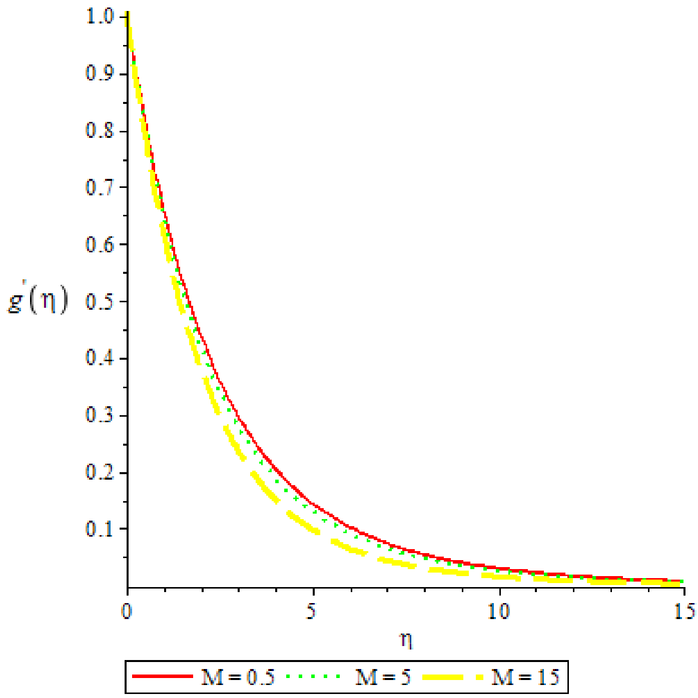

Figure 4.6: Effect of Magnetic parameter on velocity profile along y- direction

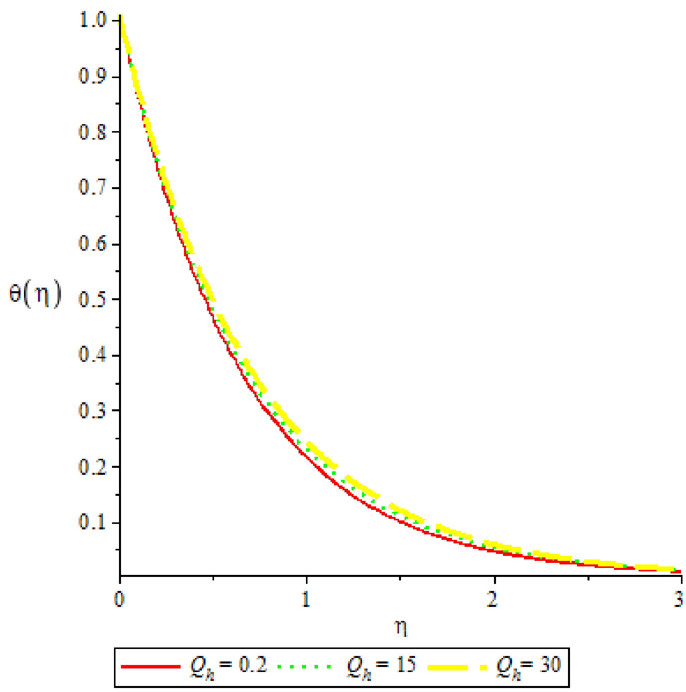

Figure 4.7: Effect of Heat source on temperature profile

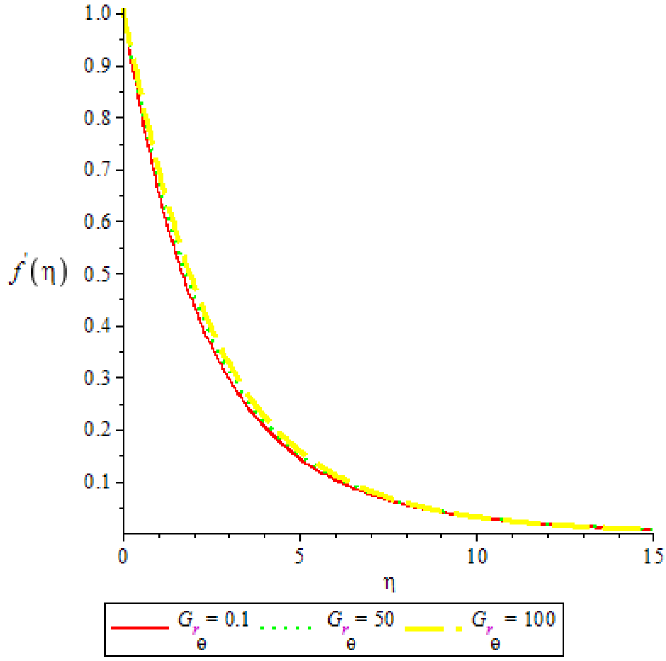

Figure 4.8: Effect of Grashof number on velocity profile

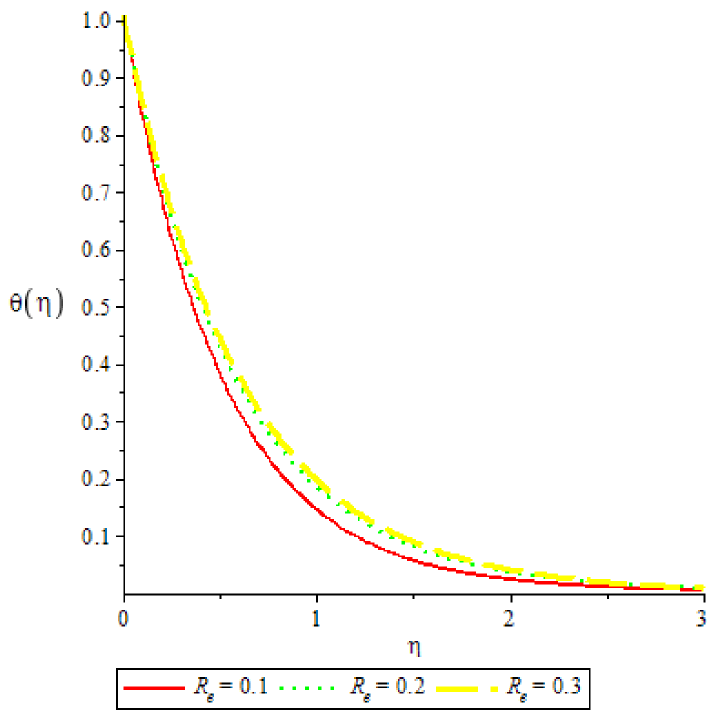

Figure 4.9: Effect of Renold number on temperature profile

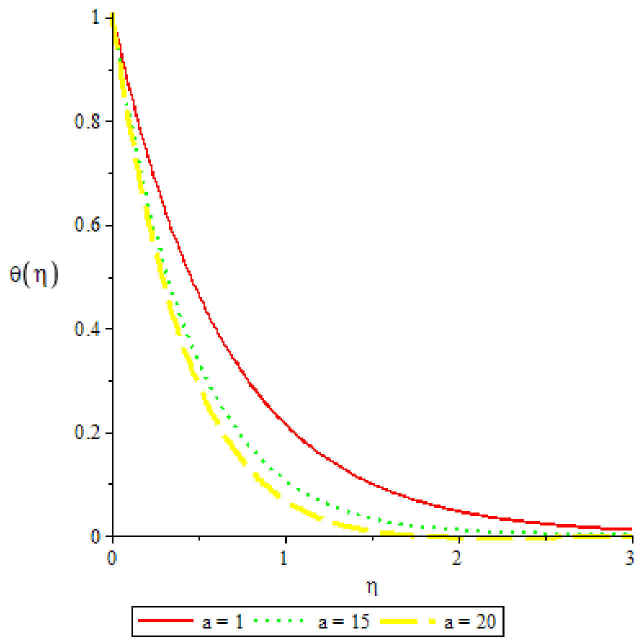

Figure 4.10: Effect of Unsteadiness parameter on temperature profile

Figures 4.1 and 4.2 show the velocity profiles against the similarity variable η for different values of casson parameter β. It was observed from these figures that as casson parameter increased, the fluid velocity distribution decreased inside the boundary layer. Figure 4.3 and 4.4 depicts the effects of radiation parameter R and prandtl number P_r on the temperature profile. It was observed that increase in radiation parameter increased the temperature profile while increase in prandtl number decreased the temperature profile. In figure 4.5 and 4.6, it was observed that increase in magnetic parameter decreased the velocity profiles and heat source enhanced the temperature profile as shown in figure 4.7. From figures 4.8 to 4.10, we observed that increase in thermal grashof numberenhanced the velocity profile whileincrease in reynold numberenhanced the temperature profileand unsteadiness parameter decreased the temperature profile.

Conclusion

From the graphical illustration above, we observed and conclude as follows

- Casson parameter decreased the velocity profiles

- Magnetic parameter decreased the velocity profiles along x and y directionsrespectively
- Heat source and Radiation parameter enhanced the temperature profile while Prandtl number decreased the temperature profile
- Thermal grashof number enhancedthe velocity profile
- Increase in unsteadiness parameter decreased the temperature profile

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