FACTORS OF DESIGN WHICH AFFECT THE FATIGUE STRENGTH OF POWER TRANSMISSION TAPERED SHAFT.

ISHAQ T. MUHAMMAD, TANIMU G. IBRAHIM AND ABUBAKAR J. ABDULLAHI
Department of Mechanical Engineering, Kaduna Polytechnic, Kaduna, Nigeria.

ABSTRACT
A shaft fatigue failure almost always occurs at a notch, hole, keyway, shoulder, or other discontinuity where the effective stresses have been amplified. These are the obvious locations where the shaft should be first analyzed, particularly in regions of high stress. This paper discuss some service factors that affects fatigue strength of a power transmitting shaft and relates the diameter and area of the tapered region of a shaft with the extension produced when subjected to loading.

Keywords: *Shaft, tapered, Fatigue, Stress concentration, Elongation.*

INTRODUCTION
The term "shaft" applies to rotating machine members used for transmitting power or torque. The shaft is subject to torsion, bending, and occasionally axial loading. Stationary and rotating members, called axles, carry rotating elements, and are subjected primarily to bending. Transmission or line shafts are relatively long shafts that transmit torque from motor to machine. Countershafts are short shafts between the driver motor and the driven machine. Head shafts or stub shafts are shafts directly connected to the motor. (Stuart H. Loeewenthal 1984) A shaft is a rotating member, usually of circular cross-section for transmitting power. It is supported by bearings and supports two flywheels. It is subjected to torsion, and bending in combination. Generally shafts are not of uniform diameter but are stepped, keyways, sharp corners etc. The stress on the shaft at a particular point varies with rotation of shaft there by introducing fatigue. Even a perfect component when repeatedly subjected to loads of sufficient magnitude, will eventually propagate a fatigue crack in some highly stressed region, normally at the surface, until final fracture occurs.(R. A. Gujar)The

reliable design of power transmitting shafts 1s predicated on several major elements. First, the fatigue (stress-life) characteristics of the given shaft 1n Its expected service environment must be established. This can be accomplished from full-scale component fatigue test data or approximated, using test specimen data. Some of the Influencing factors to be considered are the surface condition of the shaft, the presence of residual stress or points of stress concentration and certain environmental factors such as temperature or a corrosive atmosphere. Secondly, the expected load-time history of the shaft must be obtained or assumed from field service data and then properly simulated analytically. The effects of variable amplitude loading, mean stress and load sequence are potential Important factors to Include 1n a description of the loading history. Finally, a reliable mathematical model 1s needed which rationally considers both the fatigue characteristics of the shaft and Its loading history to arrive at the proper shaft diameter for the required service life and reliability. One last step 1s to check shaft rigidity and critical speed requirements, since these and other non strength factors can occasionally dictate an Increase 1n shaft diameter. This 1s often the case for lightweight, high speed machinery(H. A. Rothbart, ed., Second)

METODOLOGY

DESIGN CONSIDERATION

If a shaft is acted upon by a pure torque T about its polar axis, shear stresses will be set up in directions perpendicular to the radius on all transverse section (fig.1).

The complementary shear stress on longitudinal planes will cause a distortion of filaments which were originally in the longitudinal direction. It will be assumed that points lying on a radius before twisting will remain on a radius, the angle of twist being θ over a length l of shaft. This assumption is justified by the symmetry of the cross- section.

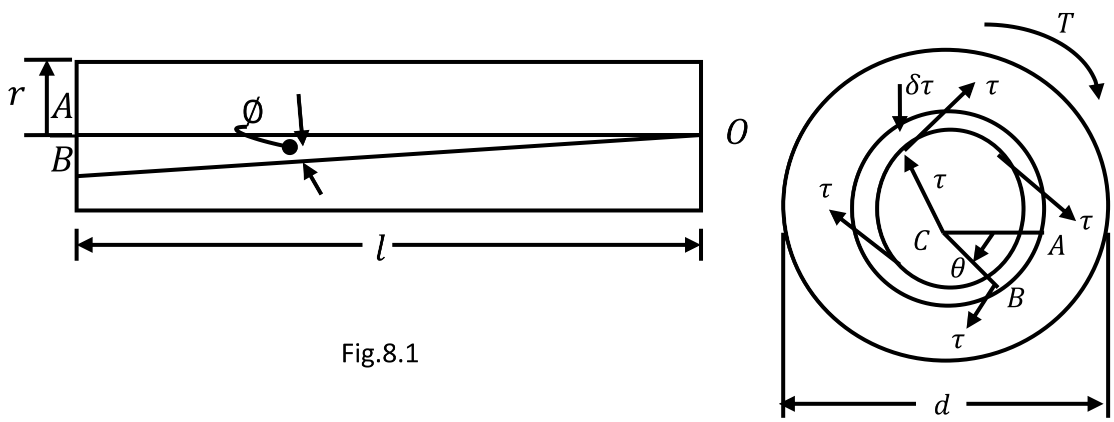


Fig.8.1

The left- hand figure shows the shear strain $\emptyset$ of elements at a distance from the axis ($\emptyset$ is constant for constant T), so that a line organically OA twists to OB, and <ACB= θ, the relative angle of twist of cross-section a distance l apart.
Arc AB= r$\theta = l\,\emptyset$ approx
But $\emptyset$=$T/_{G,}$ where *G* is the modulus of rigidity, By substitution and rearranging

$$T/_{r,} = G\theta/_{l}$$

The torque can be equated to the sum of the moments of the tangential stresses on the elements $2\pi r\delta r$,i.e
T=$\int \tau\,(22\pi r\delta r)r$
=$(G\theta/_{l})\int(22\pi r\delta r)r^2$from (1)
=$(G\theta/_{l})f$
Where f is called the polar moment of inertia
Combining (1) and (2)
T/J=$\tau/_{r} = G\theta/_{l}$
Showing that, for a given torque, the shear stress is proportional to the radius.
For a solid shaft
$J=\pi\, D4/_{32}$
And the maximum stress
f=$16T/_{\pi D^3,}$ at r=D/2
For a hollow shaft:
f=$(\pi/32)(D^4 - d^4)$
And f=$\frac{16D.T}{(D^4-d^4)}$ at r=D/2
Torsional stiffness $R = T/_{\theta} = Gf/l$

2.2 FATIGUE FAILURE

Ductile machine elements subjected to repeated fluctuating stresses above their endurance strength but below their yield strength will eventually fall from fatigue. The Insidious nature of fatigue Is that 1t occurs without visual warning at bulk operating stresses below plastic deformation. Shafts sized to avoid fatigue will usually be strong enough to avoid elastic failure, unless severe transient or shock overloads occur(Stuart H. Loewenthal 1984)

FACTORS OF DESIGN WHICH AFFECT THE FATIGUE STRENGTH :

1. Stress concentrations caused by sudden changes in cross- section and features such as screw threads and keyways. Fatigue failures are found to start from cracks at these points of stress concentration, very little redistribution of stress being possible even in ductile materials. However, the stress concentration factors under fatigue conditions is found to be rather less than under static conditions.
2. Surface treatment. Considerable improvement in the fatigue strength of manufactured parts can be achieved by surface hardening (e.g carburizing) or by work hardening processes. Cold rolling and shot peening have been found to give increases of up 20% in the endurance limit, due to surface hardening and to the residual compressive stresses set up which resist the formation of fatigue cracks.
3. Surface finish. The highest fatigue strength in obtained with smooth surfaces, particularly in the case of high-tensile steels,.
4. The frequency of stress reversals also influences the fatigue limit, which is higher for increased frequency.

The most satisfactory empirical formula embodying the experimental results for steels is due to Gerber, which may be written:

$\sigma =$ R/2 +$\sqrt{(}\sigma_\mu^2 nR\sigma\mu$

Where σ is the maximum stress during each cycle at the fatigue limited,

R is the stress range,

$\sigma\mu$ is the normal ultimate tensile stress and n is a constant for one material

For mild steel, n=1.5, for high- tensile steel, n= 2.0 applied to the particular cases previously mentioned:

Reversed stresses

$$\sigma = R/2$$

And is can easily be shown that

$\sigma = \sigma_u/2n = \sigma_u/3$ for mild steel

1. Repeated stresses

$$\sigma = R$$

And solving the equation gives

$\sigma 0.61\sigma_u$ for mild steel

Noting that $\sigma - R/2$=M, Gerber's formula can be re-arranged to give

$$R=\frac{\sigma_u}{n}\left(1-\frac{M^2}{\sigma_u^2}\right)$$

and Goodman Suggested a simpler straight line law relating the stress range and the mean stress, i.e.

$$R=\frac{\sigma_u}{n}\left(1-\frac{M}{\sigma\mu}\right)$$

Note that in both formulae $f\,{}^{u}/_{n}$ is the stress range for reversed stress conditions *(i.e M=0)* and Fig. 1 shows the variation of *R* with *M*

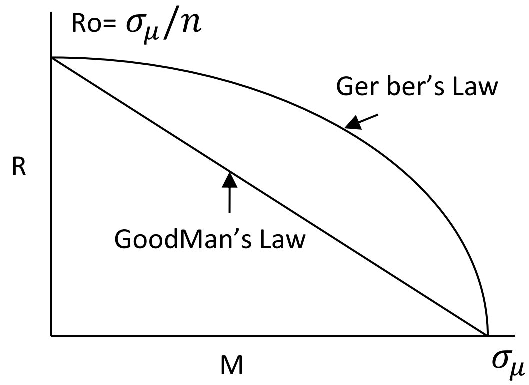


Fig. 1.0

According to Gerber and Goodman. In practice, if the values of σ_u and R_0 (=σ_u/n) for a given material are found by experiment, the fatigue limits under other conditions can be determined from this diagram.

Service factors that affects fatigue strength are discussed as follows

(1) Surface Factor

Since the shaft surface is the most likely place for fatigue cracks to start, the surface condition significantly affects the fatigue limit, This figure is based on a compilation of test data from several investigations for a variety of ferrous metals and alloys

Ssurface decarburization, which often accompanies forging, can severely reduce fatigue strength. Most, if not all, of the strength reduction due to surface condition can be recovered by cold rolling, shot peening, and other means of inducing residual compressive stress into the surface.

(2) The size factor

The size effect is attributed the greater volume of material under stress and thus the greater likelihood of encountering a potential fatigue-initiating defect in the material's metallurgical structure. Also the heat treatment of large parts can produce a metallurgical structure that neither is as uniform nor has as fine a grain structure as that obtained with smaller parts. Another factor is that smaller shafts have a higher stress gradient; that is, the rate of stress change with depth is greater .

(3)Temperature factor

At lower temperatures (to -129" C, or -200*F) carbon and alloy steels possess significantly greater bending fatigue strength. *As* the temperature is increased to approximately 427* C (800* F),m carbon steels actually show a small improvement in fatigue strength relative to room-temperature values.

(4)Fatigue Stress Concentration Factor

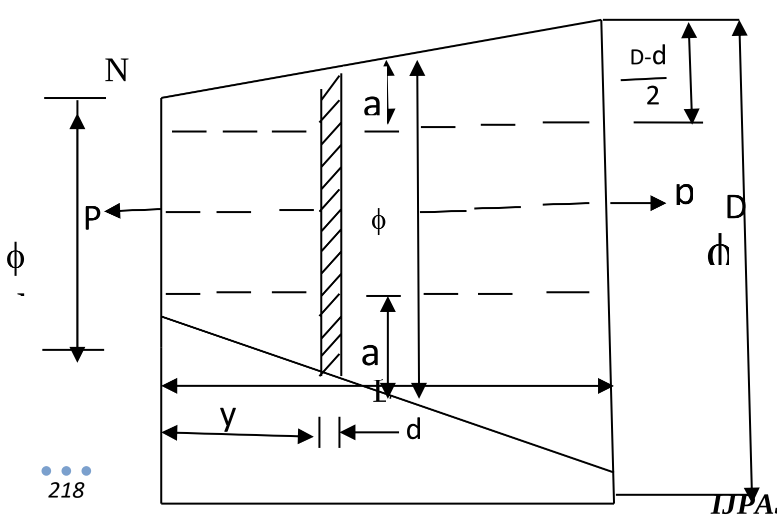
Experience has shown that a shaft fatigue failure almost always occurs at a notch, hole, keyway, shoulder, or other discontinuity where the effective stresses have been amplified. These are the obvious locations where the shaft should be first analyzed, particularly in regions of high stress. The effect of a stress concentration on the fatigue limit of the shaft is represented by the fatigue stress concentration factor .

(5) Corrosion fatigue factor.

The formation of pits and crevices on the surface of shafts due to corrosion, particularly under stress, can cause a major loss 1n fatigue strength. Exposed shafts on outdoor and marine equipment as well as those 1n contact with corrosive chemicals are particularly vulnerable. Corrosion fatigue cracks can even be generated 1n stainless steel parts where there may be no visible signs of rusting.

2.4 Elongation produced by the tapered region of the shaft when subjected to loading.

Consider a tapered shaft below:



Diameter the at one end to diameter D at another end, and D > d, for an element of length FY at distance Y from the small end of the bar, the elongation of element due to load P,

δx = $\frac{p\delta y}{AE}$ ………….. (1)

Area A = $\frac{\pi s^2}{4}$ where δ = diameter of element

From fig (i) S = d + 2a (by construction)

From fig (ii) $\frac{a}{y}$ = $\frac{D-d}{2L}$ (similar Triangles)

$$a = \left(\frac{D-d}{2L}\right)y$$

Hence

$$s = d + 2\left(\frac{D-d}{2L}\right)y$$

$$= d + \left[\frac{D-d}{L}\right]y$$

Let c = $\frac{D-d}{L}$

S = d + c y

Substitute S into A=$\frac{\pi S^2}{4}$

We have

A= $\frac{\pi}{4}$ (d + cy)^2

Put A into equ (1)

δx = $\frac{pdy}{\frac{\pi}{4}(d+cy)E^2}$--------------(3)

Elongation of the tapered bar

$$\int dx = \int \frac{pdy}{\frac{\pi}{4}(d+cy)^2} E$$

$$x = \frac{4p}{\pi E}\int_o^L (d+cy)^{-2}\mathrm{E}$$

Let Z = d + cy

$$\frac{dz}{dy} = \mathrm{C} \quad dy = \frac{dz}{c}$$

Hence

$$\mathrm{X} = \frac{4pL}{\pi E\,(2R)\,(2r)}$$

$$\mathrm{X} = \frac{pl}{\pi E\ Rr}$$

This is the extension produced from the tapered part of the shaft.

3.0 CONCLUUSION

Machines and equipment have shaft as major component to transmit power, torque and accommodate members like gear, bearing and pulley. These machines in developing countries are designed and fabricated locally for processing agricultural products and manufacturing purposes, This paper will help manufacturers to understand the reason for machine failure as attributed failure in shaft design and produce efficient components with longer life span.

REFERENCES

R.S. khurmi and J.K Gupta, 2004, Theory of Machine, Eurasia Publishing House (PVT) LTD, Ramnagar, New Delhi-110055

Robert L. Norton (2000): Machine Design- An Integrated Approach, 2nd Edition, Pretice Hall

Emmanuel Adeleke Fagbemi International Journal of Agricultural Sciences and Natural Resources 2014; 1(5): 103-106

Rotary Motion Flexible Shafts, Seventh ed., S. S. White Industrial Products, Di_. of Pennwalt, New Jersey, 1977.

Liquid Rocket Engine Turbopump Shafts and Couplings1972.. NASA

Sadowy, M.: Shafts, Couplings, Keys, etc. Mechanical Design and Systems Handbook, H. A. Rothbart, ed., McGraw-Hill Book Co., 1964, pp. 27-1 to 27-29.

Lowell, C. M.: "A Rational Approach to Crankshaft Design," ASME Paper 55-A-57, Nov. 1955.