International Journal of Pure and Applied Science
Published by Cambridge Research and Publications
Vol. 21 No.9
June, 2021.

INFLUENCE OF VAN HIELE PHASE DESCRIPTORS ON STUDENTS' DEVELOPMENT OF LEVELS OF GEOMETRIC UNDERSTANDING AND COGNITIVE ACHIEVEMENT IN GEOMETRY: FOCUS ON SENIOR PUBLIC SECONDARY SCHOOLS IN TARABA STATE

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ABSTRACT
This investigation was prompted by the tension that became apparent between students' geometric thinking levels (i.e. stages in the understanding of geometry) and their achievement in geometry being reported in the research literature. Literature reports do not only convey varying degrees of geometric understandings by students as they learn school geometry, they also try to explain why the differences in understanding. The most inspiring explanation is that which relates achievement in geometry to geometric thinking levels. It explains that in their learning of school geometry, students' thinking levels occur in stages and that the stages are in a hierarchy. Unfortunately, our classroom practices are such that students receive instructions on the same geometric content as if they belong to the same geometric thinking levels at the same time. This study explored the influence of Van Hiele phase descriptors on students' development of levels of geometric thinking or understanding and on their achievement in geometry in Taraba State. The quasi-experimental design was employed. A sample of one hundred and eighty Senior Secondary two (SS II) Mathematics students randomly drawn from three clusters of schools was used. Ten research questions guided the study and three null hypotheses were tested at the 0.05 level of significance. Two instruments known as Geometric Thinking Levels Attainment Test (GTLAT) and General Geometry Test (GGT) were used to collect data. The data collected were analyzed using mean, standard deviation and t-test statistic. The results revealed that the Van Hiele phase descriptors fostered the movement of students from one thinking level to

the next higher one and students in the Van Hiele checklist-based classrooms achieved significantly better than their counterparts in the conventional lecture class.

Keywords: *Geometry; geometric understanding; geometric thinking levels; phase descriptors; cognitive achievement*

INTRODUCTION

Any nation striving to develop economically must make corresponding advances in science and technology. Nigeria through its current vision plan called Nigeria Vision 20-20, desires to become one of the twenty most developed economies of the world by the year 2020. A section of the vision document reads, "By 2020, Nigeria will be one of the 20 largest economies in the world, able to consolidate its leadership role in Africa and establish itself as a significant player in the global economic and political arena" (Federal Ministry of Education, 2005).

To be among the world's 20 top-most economies, some of the conditions Nigeria needs to have on the ground, according to Ayagi (2008), include, among other things:

- having a reliable and a steady source of power generation, transmission and distribution;
- having a strong industrial base; and
- having improvements in agriculture, health, education, mining and other vital sectors of the national economy. Clearly, all these conditions are hardly met without the continuous application of science and technology.

Developments in science and technology are facilitated by having a mathematically literature citizenry. Wilson, (2005) observed that the subject mathematics does not only aid the intellectual development of an individual, it is also the foundation upon which the much needed scientific and technological development of the individual's country stands. Mathematics is an effective tool for developing the capacities of individuals for clear logical thinking with a view to finding scientific solutions to problems. A branch of mathematics involving lots of critical and logical thinking is geometry. Hence the focus of this study on geometric thinking levels of students and their achievement in geometry.

The frequency with which the West African Examinations Council (WAEC) Sets questions on topics in geometry is high, thus attracting investigations into the subject area of geometry. For instance, 35%, 38%, 40%, 42% and 37% of the multiple choice items prepared by WAEC for 2012, 2013, 2014, 2015 and 2016 respectively were all drawn from geometry. Further, WAEC chief examiner's report for the May/June 2015 West African School Certificate Examination (WASCE) included geometry among areas of weakness of candidates. WAEC's act of setting questions frequently in geometry (WAEC 2012, 2013, 2014, 2015 and 2016) and its declaration of geometry as a difficult subject and one in which learners are generally weak (WAEC Chief Examiner, 2015) are sources of inspiration for the researcher to focus on geometry.

To think means to use one's mind or to consider plans in one's mind (De Jager-Haum, 2000). Thinking therefore refers to using one's mind or considering plans in one's mind. Thinking is in levels according to Van Hiele (1986). Levels of thinking are the stages at which thinking occurs in different sophistications. That is to say that thinking at level n is not as sophisticated as it is at level n+1. Or thinking at level n+1 is more sophisticated than it is at level n. This brings us to hierarchy of the thinking levels. Hierarchy of levels explains that levels of thinking are in a fixed sequence with some of the levels being lower or higher than others. For example, a child cannot operate at thinking level n+1 without first having gone through level n. Or a child cannot be at thinking level n without first having been at level n-1, etc. Pierre Marie Van Hiele, a great Dutch Mathematics Educator, propounded and developed the geometric thinking level theory. He said, in their learning of school geometry, students' thought processes occur in stages which he called levels of thinking. He identified five of these levels but the current research concentrated on only four because even Van Hiele himself, the proponent of the thinking level theory, in his later writings, discarded the fifth level. The 4 levels are:

1. Level 1 (Also called recognition level or visual level).
2. Level 2 (Also called analysis level or descriptive level).
3. Level 3 (Also called order level or informal deduction level).
4. Level 4 (Also called formal deduction level). A prominent feature of these thinking levels is that a student operating at level n, for instance, may not learn the geometry that is associated with thinking level n+1. This investigation explored the influence of the Van Hiele phase

descriptors on students' development or attainment of geometric thinking levels and their cognitive achievement in geometry.

The learning of school geometry in Nigeria has for a long time been based on the formal axiomatic geometry which Euclid created over 2000 years ago (Adele, cited in Atebe and Schafer, 2010). In his era, Euclid's logical construction of geometry with its axioms, postulates, definitions, theorems and proofs was, indeed, an admirable achievement (Van Hiele, 1999). However, Van Hiele (1999), expressed the view that school geometry presented and learnt in the traditional Euclidean fashion assumes that school children also think at the formal deduction level (level 4). Van Hiele's argument here can be understood better in the light of the following statement: For students to learn school geometry in the Euclidean fashion, they should strive to not only understand but also translate axioms, postulates, definitions and theorems into constructible geometry. But not all the students are at this formal deduction level. The thought processes injected into the learning of this kind of geometry may not be the same for all the students at the same time. Worried about students' weakness in their understanding of school geometry as reported by WAEC Chief Examiner (2015) and by their poor performance in the subject (WAEC, 2010, 2011), the current researcher embarked on the study of the influence of Van Hiele phase descriptors on students' development of levels of geometric understanding and their cognitive achievement in geometry.

PURPOSE OF THE STUDY

The purpose of this study was generally, to determine the influence of the Van Hiele phase descriptors on students' development of levels of geometric understanding and on their cognitive achievement in geometry. Specifically, the study sought to determine the influence of these phase descriptors on:

1. Students' attainment or development of geometric thinking levels
2. Students' cognitive achievement in the General Geometry Test (GGT)
3. Male students' cognitive achievement in the General Geometry Test (GGT) and
4. Female students' cognitive achievement in the General Geometry Test (GGT).

Research Questions

The following Ten research questions guided the conduct of the investigation:

1. What are the Van Hiele geometric thinking levels of Senior Secondary two (SS2) students in the classrooms that used the check-list of the Van Hiele phase descriptors in teaching?
2. What are the Van Hiele geometric thinking levels of Senior Secondary two (SS2) students in the classrooms that used the lecture method in teaching?
3. What is the mean score of Senior Secondary two (SS2) students in the classrooms that used the check-list of the Van Hiele phase descriptors in teaching?
4. What is the mean score of Senior Secondary two (SS2) students in the classrooms that used lecture method in teaching?
5. What are the Van Hiele geometric thinking levels of male and female students in the treatment classroom?
6. What are the Van Hiele geometric thinking levels of male and female students in the control classrooms?
7. What is the mean score of male SS2 students in the classrooms that used the checklist of the Van Hiele phase descriptors in teaching?
8. What is the mean score of male SS2 students in the classrooms that used lecture method in teaching?
9. What is the mean score of female SS2 students in the classrooms that used the checklist of the Van Hiele phase descriptors in teaching?
10. What is the mean score of female SS2 students in the classrooms that used lecture method in teaching?

Research Hypotheses

H_{o1}. No significant difference exists between the achievement of Senior Secondary two (SS2) students in the classrooms that used check-list of Van Hiele phase descriptors and that of students in the classrooms that used the conventional (lecture) method.

H_{o2}. There is no significant difference between the achievement of male SS2 students and female SS2 students in the classrooms that used the checklist of Van Hiele phase descriptors in teaching.

H_{o3}. Male SS2 students' achievement in the GGT does not differ significantly from that of female SS2 students in the classrooms that used the conventional (lecture) method.

Method

A quasi-experimental design was used to investigate the influence of the Van Hiele phase descriptors on students' attainment or development of levels of geometric thinking and on their cognitive achievement in geometry. A sample of 180 SS2 students was randomly taken and used. Also, a sample of 3 out of 118 Senior Secondary Schools was used. Cluster random sampling technique was used for selecting the schools. Senior Secondary Schools in the state of Taraba were clustered into three i.e. Schools in Northern Educational Zone, Central and Southern. From each of these clusters, one school was selected through randomization by balloting and were named schools 1, 2 and 3. For the selection of research participants, two intact classes of SS2 students were chosen from each of the selected schools using simple random sampling. All students in the chosen classes participated in the study.

Table 1 below shows the distribution of students according to the three schools selected for the study:

Table 1: Distribution of students across the 3 selected schools

Schools							
		1		2		3	
Classes	A	B	C	D	E	F	Total
Males	18	17	19	18	17	14	103
Females	13	12	11	12	15	14	77
Total	31	29	30	30	32	28	180

Further, these intact classes were randomly assigned to experimental groups or control groups. Accordingly, the intact classes B, D and E were randomly assigned to the experimental groups while A, C and F had their random assignment done to the control group. Two tests, known as Geometric Thinking Levels Attainment Test (GTLAT) and General Geometry Test (GGT) formed instruments for data collection. The GTLAT was an instrument which explored the attainment of geometric thinking levels by participating SS2 students while the GGT was used to determine their achievement in geometry. The two instruments were validated by seasoned science educators, one each in Modibbo Adama University (MAU), Yola, Taraba State University (TASU), Jalingo and Federal University of Agriculture (FUA), Makurdi. Test-retest reliability approach was used to estimate reliability of the two tests. Pearson's Products Moment Correlation Coefficient Method (PPMC) was used to get reliability coefficients of 0.89 and 0.85 for GTLAT and GGT respectively.

Experimental Procedure

To obtain information on which investigational results and conclusions were based, participating students were given two different kinds of instruction. Participants in the experimental classrooms (EC) received instructions in which the Van Hiele phase descriptors were used. On the other hand, participants in the control classrooms (CC) received instructions through the conventional lecture. Teachers in the experimental or treatment classrooms were groomed, enabled and advised to use the checklist of Van Hiele phase descriptors in their teaching. For the sake of emphasis, the checklist is presented here:

I. Teacher introduces the topic by recognizing and building on learners' prior knowledge
II. Teacher delays the introduction of formal vocabulary and condones learners' use of common informal terms in the ensuing discussion
III. Teacher asks questions that seek to clarify students' imprecise terminology and gradually introduces formal mathematical language
IV. Teacher creates an interactive learning environment and encourages learners to challenge, contest and negotiate meanings and solutions to mathematical problems
V. Teacher asks questions that steer students' thought toward the central idea being developed.
VI. Teacher uses open-ended questions and encourages learners to seek their own solution strategies
VII. Teacher encourages learners to elaborate on their responses and
VIII. Teacher uses questions that encourage learners to reflect on, refine and summarize their ideas about the concepts learnt.

To properly handle this treatment, the researcher organized a two-week training exercise for participating teachers. The research conditions were properly explained. The framework for thinking about the tasks (the Van Hiele phase descriptors) were exhaustively explained to the teachers.

At the end of four weeks of instructions with participating SS2 students, the tests were administered. Students' scripts were marked and scored using the marking guide or scheme. Data collected were analyzed using the mean and standard deviation for answering the research questions and the t-test statistic for testing the significance of the stated hypotheses.

Results

This section is concerned with presentation and statistical analysis of results. The results are presented in tables. A research question precedes each table and the content of the table answers the research question. A hypothesis follows the research question for testing at the 5% level of significance. The t-test statistic was used for the analysis. A statement follows the summary of each result, rejecting or accepting the stated null hypothesis.

Research Questions 1 and 2

1. What are the Van Hiele geometric thinking levels of Senior Secondary two (SS2) students in the classrooms that used the Van Hiele phase descriptors?
2. What are the Van Hiele geometric thinking levels of Senior Secondary two (SS2) students in the classrooms that used the lecture method in teaching?

Table 2 below indicates the Van Hiele levels of geometric thinking of participating Senior Secondary two students in both treatment and control classrooms;

Table 2: Van Hiele Levels of Geometric Thinking of SS2 students

Levels Attained Groups	1	2	3	4	Total
Treatment	21	32	22	16	91
Control	22	49	11	7	89
Total	43	81	33	23	180

From table 2 (above) it is seen that 21 students in the treatment group attained the lowest Van Hiele geometric thinking level i.e. level 1 while 22 control students attained the same level (level 1). The next lowest level (level 2) was attained by only 32 students in the treatment group while up to 49 students in the control group attained level 2. Up to 22 treatment students attained level 3 but only 11 control students attained this level (level 3). The highest Van Hiele thinking level (level 4) was attained by 16 students in the treatment group while only 7 students in the control group climbed to this highest level. On the whole, a total of 53 treatment students operated on lower Van Hiele geometric thinking levels (i.e. levels 1 and 2). In comparison, 71 control students operated on lower Van Hiele geometric thinking levels (1 and 2). On the other hand, 38 students in the treatment group operated on higher thinking levels (levels 3 and 4). In contrast, merely 18 students managed to reach higher thinking levels (Levels 3 and 4). There are more control students operating on lower Van Hiele levels than there are treatment students. Conversely, there are less control students operating on higher Van Hiele levels than there are treatment students.

Questions 3 and 4:

3. What is the mean score of Senior Secondary two (SS2) students in the classrooms that used the check-list of the Van Hiele phase descriptors in teaching?
4. What is the mean score of Senior Secondary two (SS2) students in the classrooms that used lecture method in teaching?

Table 4: Mean and Standard Deviation for Treatment and Control GPS

Groups	N	Mean	Standard Deviation
Treatment	91	3.96	2.4
Control	89	3.09	1.84
Difference		0.87	0.56

Table 4 (above) shows that the mean score of the treatment students is 3.96 with their standard deviation being 2.4. On the other hand, the mean score of the control students is 3.09 with their standard deviation being 1.84. This gives a mean difference of 0.87 and a difference in standard deviation of 0.56, indicating the superiority of the treatment group over the control group in achievement in the General Geometry Test (GGT).

Questions 5 and 6

5. What are the Van Hiele geometric thinking levels of male and female students in the treatment classroom?
6. What are the Van Hiele geometric thinking levels of male and female students in the control classrooms?

Table 3: Van Hiele Levels of Geometric Thinking for Male and Female SS2 students

Levels Attained							
Groups	Sex	N	1	2	3	4	Total
Treatment	Male	52	8	17	16	11	52
	Female	39	13	15	6	5	39
Control	Male	51	13	24	9	5	51
	Female	38	9	25	2	2	38
	Total	180	43	81	33	23	180

From table 3 (above), it can be seen that only 8 male treatment students operated on level 1 (the lowest thinking level) while up to 13 female treatment students were on the lowest level (level 1). Also, 17 male treatment students attained level 2 but 15 female treatment students were on this level (level 2). With this, 25 male treatment students operated on lower Van Hiele thinking levels (levels 1 and 2) while 28 female treatment students operated on levels 1 and 2. This means that there were more female treatment students on lower Van Hiele thinking levels (1 and 2) than there were male treatment students. For the control group, 13 male control students operated on the lowest Van Hiele level (level 1) while 9 female control students were on this level (level 1). 24 and 25 male and female control students respectively were on the second lowest Van Hiele thinking level (level 2). On the whole, 37 male control students operated on lower geometric thinking levels (levels 1 and 2) while 34 female control students operated on levels 1 and 2. This means that there were more male control students on the lower levels than there were females. For the higher levels, there were 16 male treatment students on level 3 and 11 male treatment students on level 4. A total of 27 male treatment students operated on higher levels (levels 3 and 4). For control group, 9 female control students were on level 3 while only 5 female control students were on level 4. A total of 14 female control students only were on higher Van Hiele levels (levels 3 and 4). This means there were more male treatment students (27) operating on higher Van Hiele levels than there were female treatment students (11 of them) on higher levels. Also, there were more male control students on higher levels (14 of them) than there were female control students on higher levels (4 0f them).

Questions 7, 8, 9 and 10

7. What is the mean score of male SS2 students in the classrooms that used the checklist of the Van Hiele phase descriptors in teaching?
8. What is the mean score of male SS2 students in the classrooms that used lecture method in teaching?
9. What is the mean score of female SS2 students in the classrooms that used the checklist of the Van Hiele phase descriptors in teaching?
10. What is the mean score of female SS2 students in the classrooms that used lecture method in teaching?

Table 5: Mean and Standard Deviation for Male and Female Treatment Group

Groups	Sex	N	Mean	Standard Deviation
Treatment	Male	52	4.83	2.21
	Female	39	2.5	2.13
	Difference		2.27	
Control	Male	51	3.76	1.93
	Female	38	2.18	3.32
	Difference		1.58	

Table 5 shows that the mean and standard deviation scores for male and female treatment students are 4.83 and 2.21; 2.56 and 2.13 respectively. This gives a mean difference of 2.27. Also, table 5 shows that the mean and standard deviation scores for male and female control students are 3.76 and 1.93; 2.18 and 3.32. A mean difference of 1.58 is noticed. In both groups (treatment and control, it is evident that the male students are superior to the females as disclosed by differences in mean scores.

H_{o1}: No significant difference exists between the achievement of SS2 students in the classrooms that used checklist of Van Hiele phase descriptors and that of students in the classrooms that used the conventional (lecture) method.
Table 6 below is the summary of t-test Analysis of SS2 students' Achievement in the Treatment and Control Groups

Table 6: Summary of t-test Analysis of SS2 students' Achievement in the Treatment and Control Groups

Group	N	Mean	Standard Deviation	t-Cal	t-tab.	df	Inference
Treatment	91	3.96	2.4				
Control	89	3.09	1.84	25.88	1.96	178	Significant

From table 6, it is evident that students in the treatment (experimental) group whose instructions were based on the checklist of Van Hiele phase descriptors have higher mean score than the control students who received instructions through lecture. Also, the computed t-Value of 25.88 far exceeds the critical or tabulated t-Value of 1.96. The stated null hypothesis is therefore rejected. This confirms the fact that there is significant difference between the achievement of students who received instruction through Van Hiele phase descriptors and that of students on the conventional (lecture) group.
H_{o2}. There is no significant difference between the achievement of male SS2 students and female SS2 students in GGT in the classrooms that used the checklist of Van Hiele phase descriptors in teaching.

Table 7: t-test Analysis of Male and Female SS2 students' GGT Achievement within the Treatment Group

Gender	N	Mean	Standard Deviation	t-Calculated	t-tabulated	df	Inference
Male	52	4.83	2.21				
				33.31	1.96	89	
Female	39	2.56	2.13				Significant

Result in table 7 indicates that the male students have much higher mean score than the female students within the experimental group. Again, the calculated t (33.31) is by far greater than the critical t (1.96). The stated null hypothesis is therefore rejected. There is significant difference in achievement between male and female students within the treatment group.

H_{o3}. Male SS2 students' achievement in the GGT does not differ significantly from that of female SS2 students in the classrooms that used the conventional (lecture) method.

Table 8: t-test Analysis of Male and Female SS2 students' GGT Achievement within the Control Group

Gender	N	Mean	Standard Deviation	t-Calculated	t-Critical	df	Inference
Male	51	3.76	1.93				
				18.11	1.96	87	
Female	38	2.18	3.32				Significant

A similar situation to table 7 occurred in table 8. Male students' mean score is greater than that of females within the control group. Further, the calculated value of t largely exceeds that of critical t (18.11 > 1.96). Decision taken is that of rejection of the stated null hypothesis. Thus, the existence of a significant difference in achievement between male and female students within the control group is confirmed.

Summary of Findings

1. There were more control students operating on lower Van Hiele thinking levels than there were treatment students but there were less control students operating on higher Van Hiele thinking levels than there were treatment students.
2. Students in classrooms that used phase descriptors (treatment group) were superior in achievement to students in the lecture classrooms (control group).
3. There were more female treatment students on lower Van Hiele thinking levels (1 and 2) than there were males but there were more male control students on lower thinking levels than there were females.
4. There were more male treatment students (27) operating on higher Van Hiele thinking levels than there were female treatment students (11). Also, there were more male control students on higher thinking levels

(14) than there were female control students on higher thinking levels (4).

5. In both groups, (treatment and control), it is evident that the male students were superior to the females as disclosed by differences in mean scores
6. There is significant difference in achievement between students who received instructions through Van Hiele phase descriptors (treatment) and students in the conventional (lecture) group (control).
7. There is significant difference in achievement between male and female students within the treatment group.
8. A significant difference exists between the achievement of male and female students within the control group.

Discussion

It was discovered from this study that greater number of control students (students in the lecture classrooms), 71 out of 89, operated on lower Van Hiele geometric thinking levels. It was also discovered that fewer treatment students (students in Hiele's descriptors' classrooms), 53 out of 91, operated on lower Van Hiele thinking levels. This means that majority of students in the studied sample (124 out of 180) were lower level geometric thinkers. In terms of percentage, 68.89% of participating SS2 students operated as lower level geometric thinkers. Only 56 out of 180 participants (31.11%) were higher level geometric thinkers. A further disturbing discovery made by this study was that the geometric thinking levels were not only the determinants of the geometry content students could learn, they were also the determinants of the level of achievement these students could make in the investigational test (the GGT). Balasa (2020), made a similar discovery. His report, "Majority of students in the study area (i.e. 796 students, representing 68.03%) operated on lower Van Hiele geometric thinking levels and could only learn content of geometry meant for those lower levels (levels 1 and 2). But are classroom practices in the study area a reflection of these discovered facts? Not at all. Instructions in classrooms in this area are such that students are taught the same content as if they belong to the same geometric thinking levels. This has far reaching implications for the teacher.

Again, the finding that there were more lower level geometric thinkers in the control group than there were in the treatment group explained the influence of instructions on thinking levels. Students in the treatment group where instructions were based on phase descriptors moved to higher thinking levels more than students in the control group (where instructions were based on lecture). This confirms the argument made by Van Hiele (1999) that instructions can be used to facilitate movement from one level of geometric thinking to the next higher one, thus he proposed his phase descriptors.

Another discovery made by the study was that of the existence of statistically significant difference between the achievement of treatment students and that of control students. Students in classrooms with instructions based on phase descriptors (treatment) were superior in achievement to students in classrooms with instructions based on lecture (control). Usiskin, cited in Atebe and Schafer (2010) had similar finding.

A further discovery was that there were more female treatment students on lower Van Hiele thinking levels than there were males but there were more male control students on lower Van Hiele thinking levels than there were females. Again, there were more male treatment students on higher Van Hiele thinking levels than there were females. At the same time, there were more male control students on higher thinking levels than there were female control students on higher thinking levels. This means that gender was a factor in movement from one thinking level to another. In a previous finding, Senk (2009) detected the existence of a positive correlation between Van Hiele thinking levels of students and their proof-writing abilities but found no correlation between gender and proof-writing. Senk's finding thus disagrees with current

Yet another important discovery was made of the existence of significant difference between the achievement of male and female students within both the treatment and control groups. In both cases (treatment and control cases), male achievements were found to be superior to those of females. Tieng and Kwan Eu (2015) and Atebe and Schafer (2010) had similar findings.

In the General Geometry Test (GGT) results, we see evidence of association between lower scores and lower geometric thinking levels and between higher scores and higher geometric thinking levels. That is to say that students that attained lower Van Hiele thinking levels (levels 1 and 2) had low scores in the GGT while students that attained higher geometric thinking levels (levels 3 and

4) had high scores in the GGT. This discovery stands as a confirmation of an earlier claim by Van Hiele (1986) that thinking in the learning of geometry is in levels and that the levels are in a hierarchy. This result is in line with that of Aydin and Halat (2009) in a study on the impacts of undergraduate mathematics courses on college students' geometric reasoning stages, which investigated possible effects of different college level mathematics courses on college students' Van Hiele levels of geometric understanding. The results showed that students taking logic/proof based courses attained higher reasoning stages than students taking other college level mathematics courses, suggesting reasoning levels and hierarchy of the levels.

Educational Implications of the Findings

The findings of this research have far-reaching implications for education. They have implications for students, teachers, educators and curriculum planners. In the first place, it was found from the research that majority of the sampled students (68.89%) merely acquired lower thinking levels (levels 1 and 2). Only 31.11% acquired higher thinking levels (levels 3 and 4). It was equally noted from the literature that hierarchy of geometric thinking levels is such that a student operating on level 1, for instance, may not learn the geometry content meant for level 2. Similarly, a student on level 2, may not learn the geometry content at level 3, etc. The implication of this for the classroom is that teaching and learning of school geometry should be level specific. In other words, geometry content should match level of thinking. That is to say that students on thinking level 1 should strictly learn content meant for this level. The same thing is expected for all other levels. Unfortunately, the practice in the study area is such that students are taught the same content of geometry in total disregard for their levels of thinking. In other words, students are made to learn the same geometric content as if they belong to the same geometric thinking levels. Learning problems can be expected from such a practice as some learners are made to learn content for which they are not ready.

Many students in the study had the ability to recognize shapes only in some standard orientations. By implication, these learners did not understand that simple geometric shapes are defined by their properties and not by their orientations or positions in space. Teachers need to provide learners with activities for exploring the properties of geometric shapes not only in many but also in varied orientations.

Further, the findings debunked the general feeling among the generality of our people that our secondary school students can hardly solve difficult problems in geometry. Contrary to this feeling, many of the responses to such difficult problems (attempted by students on high geometric thinking levels, especially whose thinking levels matched the difficult problems), showed mature, systematic, step-by-step presentation and analytical reasoning. The meaning of this is that our students have the capacity to solve difficult geometrical problems provided that such problems are level specific.

CONCLUSION

It was concluded from the study that:

1. There were more control students operating on lower Van Hiele thinking levels than there were treatment students but there were less control students operating on higher Van Hiele thinking levels than there were treatment students. Majority of students in the study were lower level geometric thinkers.
2. Students in classrooms that used Van Hiele phase descriptors (treatment group) were superior in achievement to students in the lecture classrooms (control group).
3. There were more female treatment students on lower Van Hiele thinking levels (1 and 2) than there were males but there were more male control students on lower thinking levels than there were females.
4. There were more male treatment students (27) operating on higher Van Hiele thinking levels than there were female treatment students (11). Also, there were more male control students on higher thinking levels (14) than there were female control students on higher thinking levels (4).
5. In both groups, (treatment and control), it is evident that the male students were superior to the females as disclosed by differences in mean scores.
6. There is significant difference in achievement between students who received instructions through Van Hiele phase descriptors (treatment) and students in the conventional (lecture) group (control).

7. There is significant difference in achievement between male and female students within the treatment group.
8. A significant difference exists between the achievement of male and female students within the control group.

RECOMMENDATIONS

Based on findings, the following recommendations were made:

1. Teachers are to ensure that content of geometry to be learnt by students matches their geometric thinking levels
2. Teachers should determine their students' levels of geometric thinking before the commencement of academic programs so as to ensure that the students learn level appropriate content of geometry
3. Curriculum planners should consider replanning the curriculum such that content and sequencing will fit the geometric thinking levels of students
4. In designing and delivering instructions, teachers should not only teach the terminology associated with a given content area in geometry, they should also ensure that learners who have a correct verbal description of a geometric concept, also have a correct concept image associated with that concept and vice versa
5. Teachers need to prepare and present a variety of activities that create numerous opportunities for learners to explore the properties of simple geometric shapes in many and varied orientations
6. Students should use the findings of this study to adjust their Van Hiele geometric thinking levels so as to enhance their learning and achievement in school geometry.
7. Geometry educators should use the findings of this study to properly select, organize and present materials to geometry teachers who in turn will be better informed as to how to select level-specific content and suitable instructional methods.

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