Sub-Sahara African Academic Research Publications
December, 2020, Editions

JPAS
Journal of Pure and Applied Science
Vol..17, No.9, ISSN 2318-9315

MATHEMATICAL MODEL OF HIV /AIDS ON POPULATION DYNAMICS OF SULEJA GEOGRAPHICAL ENCLAVE OF NIGER STATE

[1]BAWA, M.; & [2]JIYA, S. N.

[1]Department of Mathematical Sciences, Ibrahim Badamasi Babangida University, Lapai [2]Department of Geography, Ibrahim Badamasi Babangida University, Lapai

ABSTRACT

This research work proposed and analysed a mathematical model for the dynamics of HIV/AIDS in the Suleja enclave. In particular, a model for the interaction of $S(t), L_A(t), L_C(t)$ *and* $I(t)$ *is developed using a system of ordinary differential equations. The work establishes the existence of the disease free equilibrium and obtain the endemic equilibrium state as well as the basic reproduction number* (R_0)*. The result revealed that* $R_0 < 1$*, which signifies that the disease can be controlled. Numerical simulation was carried out using the data generated from the field. The sensitivity analysis was done using the sensitivity software and the result shows that* τ_a *is the most influential parameter that can determine the magnitude of* $S(t)$ *and hence holds great promise to reduce the HIV/AIDS prevalence in the enclave. Therefore, increased test and treat as well as suppression of viral loads of infected members in community clinics and hospitals should be scaled up.*

Keywords*: Disease Free Equilibrium, Endemic Equilibrium, Mathematical Model, Human Immunodeficiency Virus, Acquired Immune Deficiency Syndrome.*

Introduction

Human immunodeficiency Virus (HIV) leads to Acquired Immune Deficiency Syndrome (AIDS). According to UNAIDS (2016), since the start of HIV/AIDS epidemic 35 years ago, 35 million people have died from AIDS

related illnesses globally with an estimated 78 million people to have become infected with HIV. In 2015, half of new infections among children between the ages of 0 to 14 years, occurred in only six countries; Nigeria, India, Kenya, Mozambique, Tanzania and South Africa. This calls for concerted efforts from innovators, communities, scientists, religious leaders and donors to close the prevention gab. HIV is transmitted through unprotected sex with someone who is infected, injection or transfusion of contaminated blood or blood products. Other ways of transmitting the virus include, sharing unsterilized injection, equipment and from infected mother to her child, this occurs during pregnancy, at birth and through breastfeeding (Avert, 2015).

According to NAIIS (2019), results released by the government of Nigeria indicate a national HIV prevalence in Nigeria of 1.4% among adults aged 15-49 years. Previous estimates had indicated a national HIV prevalence of 2.8%. So far, there is no cure for HIV infections. However, effective anti-retroviral drugs (ARV) can control the virus and help prevent onward transmission to the people. According to WHO (2020), at the end of 2019, an estimated 81% of people living with HIV knew their status, 67% were receiving anti-retroviral therapy (ART) and 59% had achieved suppression of the HIV virus with no risk of infecting others.

The Suleja Geographical Enclave

Suleja is located on Longitude.70 08'to 70 14'E and Latitude.90 05'to 90 17'N (Figure.1.1). Suleja is boarded by Gurara L.G.A by the North and West and by the East Tafa L.G.A and by the South, Abuja (Federal capital territory). The landmass of Suleja which can be accessed through major roads covers the emirate's wooded savanna area of about 2,980 square kilometers (1,150 sq mi) originally included four small Koro chiefdoms that paid tribute to the Hausa kingdom of Zazzau. Suleja metropolis covers 7 districts/wards e.g. Iku district, Wambai district, Magajiya district, Hashimi district, Bagama district, Zuma district, Madala district etc.

Climate

Suleja lies within the middle belt of Nigeria, with sub-humid type of climate classified as the tropical wet and dry (AW) by Koppen 1971. Suleja share two seasons and is dependent on the two prevailing air masses over the

country at different times of the year, the dry tropical continental air masses originating from the Atlantic Ocean and the Mediterranean.

The two air masses, nearly opposite in direction meet at a zone of discontinuity called the Inter-tropical Divergence Zone (ITDZ). It migrate North wards and South wards following the earth revolution. It reaches the Southern limit at the peak of rainy season at latitude, 5oN. In January, its Northern limit is in the variety of latitude 2o2"N in August. The ITD as explained above reaches the area at latitude 9o35"N between March and April and it recedes in October.

The town is influenced by two principal seasons, the rainy season and the dry season. The rainy season starts from the month of May and ends in October. As rivers and streams flood their banks with temperature being lower, during this period the soil moisture content is usually high which encourages plant growth. Most of the agricultural activities are carried out within rainy seasons (yekeen, 2005).

The dry season which starts in November and ends in April on the other hand is characterized by high temperature, except during the period of harmattan. During this period, the relative humidity slow, while all season river and stream dry up to form pools of stagnant water bodies which form good basis for local rice irrigation cultivation in the lower land of Niger state.

Hydrology

Suleja is a land locked local government area, with no major water body like ocean, or sea bordering it and thus precipitation is not as high as the southern parts of the country. The annual mean rainfall distribution in the state ranges from less than 900mm to 1200mm, a sizable amount of the rain water flows as runoff into rivers and streams from these surface detention, surface water bodies and vegetation, some of the rain water is also lost to the atmosphere through evaporation.

People and Culture

The predominant ethnic groups are the Hausas, Gbaris, Koros and Fulanis. The occupations of the people are mostly farming and civil service. Suleja is a commercial and administrative town. Yam, Guinea corn and Maize are

the main agricultural products of the town. Local trade among the predominantly Gbari (Gwari) population is mainly in sorghum, yams, corn (maize), millet, peanuts, shea nuts, tobacco, indigo, kola nuts, cattle, goats, chickens. The economy also support cattle trading, brewing, shea nut processing and gold mining.

Traditional industries and crafts in Suleja includes Gabri pottery, cotton weaving, dyeing with locally grown indigo and mat making. The town is mostly known for it commercial activities such as trading in agricultural products, dyed cotton cloth, metalworking and pottery. Modern industry includes a Pottery centre and a mini-NNPC depot. Since Suleja is near Abuja it supports the Federal Capital with agricultural products and affordable houses. Its has government secondary school (Academy), a hospital, Dorben polytechnic campus. It also has a radio broadcasting center for the town (Zuma Fm) and is a hub for Federal roads serving the Southern part of the state.

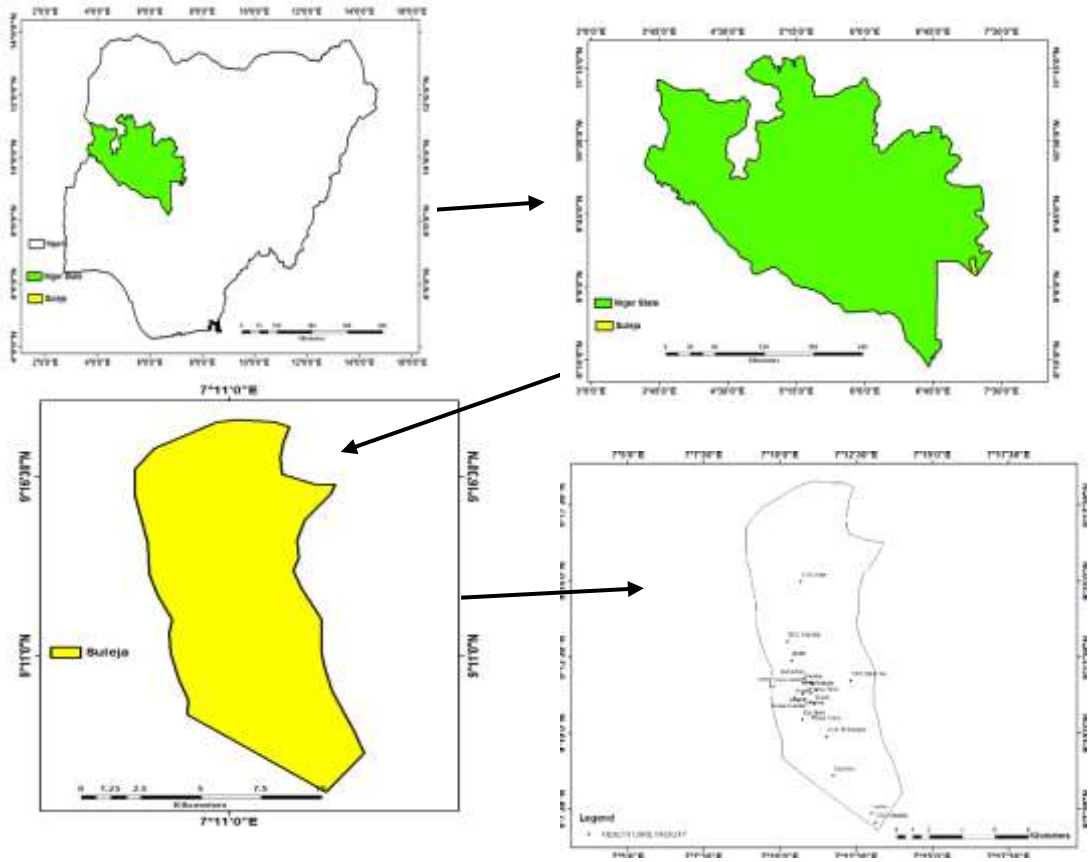


Fig. 1 : Map of the Study Area.

Review of HIV/AIDS Mathematical Models

Mathematical Modeling over the years has been beneficial in examining various disease dynamics such as Dengue, Ebola and HIV/AIDS. It also plays an essential role in the understanding of epidemiological designs for disease control.

In 1995, a simple model for Human Immunodeficiency Virus by David et al (1995), revolutionized our understanding of the disease. This work was the first to show that the infection pathogenesis was a rapidly varying dynamical process during which about twelve billion viral particles per day were being produced in infected individuals. David et al (2001) examine the impact of condom usage on the sexual transmission of HIV/AIDS amongst a homogeneously mixing male homosexual population. They first derive a multi-group SIR-type model of HIV/AIDS transmission where the homosexual population is split into subgroups according to frequency of condom use. Both Susceptible and infected individuals can transfer between the different groups. An important special case of their model was discussed which includes two risk groups and perform an equilibrium and stability analysis for this special case.

The University of California-Los Angeles researchers (UCLA), (2002) led by Blower, calculated the impact of a range of variables of HIV infection in San Francisco. The researchers assumed that anti-retroviral treatments lower the amount of virus contained in the bloodstream by at least half and possibly up to 100-fold, meaning that widespread anti-retroviral use will make it more difficult for HIV-positive individuals to transmit the virus to sexual partners.

An excellent review of pre-1990 models is reported in Palloni and Glicklich (1991), the review cites references from most leading epidemiological, medical, public health and statistical journals. In their review, they classified existing models according to three criteria; the type of outcomes the models are designed to evaluate, the data they require and the nature of the assumptions (behavioural, epidemiological, medical) on which they rest. They further added that increase sophistication in modeling techniques alone would not yield very useful results without

corresponding improvements in the quantity and quality of the data required by the models.

Materials and Methods

The Model

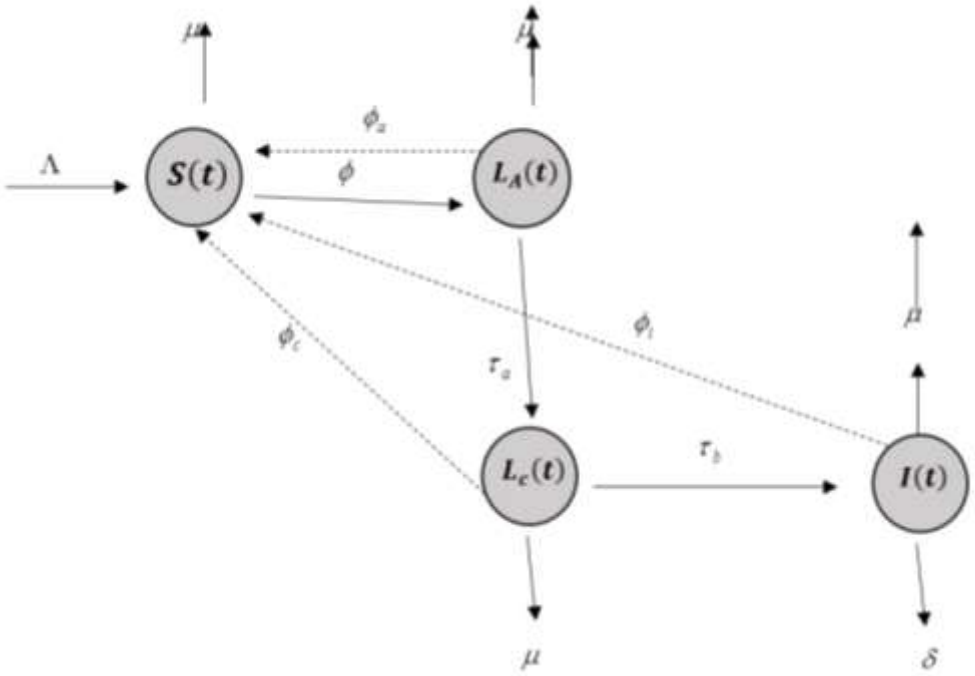
The population was partitioned into four compartments; susceptible compartment $S(t)$, latent class of acute level $L_A(t)$, latent class of chronic level $L_C(t)$ and class of those that have the manifestation of the AIDS symptoms of infection.

The population of the susceptible class increases with constant recruitment Λ, the population of this class decreases when individuals move from the susceptible class into the latent class of acute level via interaction with the other three classes and further decreases with natural death at the rate μ.

The population of the latent class of acute level increases with individuals coming in from the susceptible class via interaction of the susceptible individuals with the other three classes and decreases as a result of flow to $L_C(t)$ and natural death.

The latent class of chronic level increases when individuals from the latent class of acute level progress to the latent class of chronic level and decreases as members move to the class of those that have the manifestation of the AIDS symptoms of infection. This group also suffers natural death.

The class of those that have the manifestation of the AIDS symptoms of infection increases with flow of members from the latent class of chronic level and decreases as a result of natural death and death resulting from infection, that is μ and δ respectively.



Conceptual Framework of the Model
Fig 2: Schematic Representation of the HIV/AIDS Model.

$S(t)$ is the susceptible class in which members are virus free but are prone to infection through interaction with members that have the HIV virus.
$L_A(t)$ is the latent class of acute level who are already infected by HIV and can still perform daily activities like the susceptible. It is assumed that members in this class do not receive ART since they do not realise that they have HIV.
$L_C(t)$ is the latent class of chronic level which include members who have shown the symptoms of HIV infection. This class of people know their HIV status and the ART is implemented in this class.
$I(t)$ is the class of those that have the manifestation of the AIDS symptoms of infection. Members of this class are generally weak and inactive. Their condition is complicated by other diseases due to the decline of immunity in their body.
$N_x(t)$ is the total population.

The Model Equations

$$\frac{dS(t)}{dt} = \Lambda - \frac{\Phi_a S(t) L_A(t)}{N_x(t)} - \frac{\Phi_c(t) S(t) L_C(t)}{N_x(t)} - \frac{\Phi_i S(t) I(t)}{N_x(t)} - \mu S(t) \quad (1)$$

$$\frac{dL_A(t)}{dt} = \frac{\Phi_a S(t) L_A(t)}{N_x(t)} - \frac{\Phi_C S(t) L_C(t)}{N_x(t)} - \frac{\Phi_i S(t) I(t)}{N_x(t)} - \tau_a L_A(t) - \mu L_A(t) \quad (2)$$

$$\frac{dL_C(t)}{dt} = -\mu L_C(t) + \tau_a L_A(t) - \tau_b L_C(t) \quad (3)$$

$$\frac{dI(t)}{dt} = -\mu I(t) - \delta I(t) + \tau_b L_C(t) \quad (4)$$

Where

Λ = Recruitment rate of individuals

Φ_a = Rate of successful infection of $L_A(t)$

Φ_c = Rate of successful infection of $L_C(t)$

Φ_i = Rate of successful infection of $I(t)$

μ = Natural death rate

δ = Death rate resulting from AIDS

τ_a = Transition rate from $L_A(t)$ to $L_C(t)$

τ_b = Transition rate from $L_C(t)$ to $I(t)$

$$N_x(t) = S(t) + L_A(t) + L_C(t) + I(t) \quad (5)$$

In the biological feasible region

$$\Omega = (S(t), L_A(t), L_C(t), I(t)) \in \Re_+^4$$
$$S(t) \geq 0, L_A(t) \geq 0, L_C(t) \geq 0, I(t) \geq 0 \quad (6)$$

Existence of Equilibrium States

At the equilibrium state the rate of change of each variable is equal to zero, hence

$$\frac{dS(t)}{dt} = \frac{dL_A(t)}{dt} = \frac{dL_C(t)}{dt} = \frac{dI(t)}{dt} = 0 \quad (7)$$

At the disease free equilibrium (DFE) state, all the infected classes will be zero and the entire population will comprise of only susceptible individuals.

Let

$$\left(S(t), L_A(t), L_C(t), I(t) = S^*(t), L^*{}_A(t), L^*{}_C(t), I^*(t)\right) \quad (8)$$

Hence

$$\Lambda - \frac{\Phi_a S^*(t) L^*_A(t)}{N_x(t)} - \frac{\Phi_c(t) S^*(t) L^*_C(t)}{N_x(t)} - \frac{\Phi_i S^*(t) I^*(t)}{N_x(t)} - \mu S^*(t) = 0 \quad (9)$$

$$\frac{\Phi_a S^*(t) L^*_A(t)}{N_x(t)} - \frac{\Phi_C S^*(t) L^*_C(t)}{N_x(t)} - \frac{\Phi_i S^*(t) I^*(t)}{N_x(t)} - \tau_a L^*_A(t) - \mu L^*_A(t) = 0 \quad (10)$$

$$-\mu L^*_C(t) + \tau_a L^*_A(t) - \tau_b L^*_C(t) = 0 \quad (11)$$

$$-\mu I^*(t) - \delta I^*(t) + \tau_b L^*_C(t) = 0 \quad (12)$$

From equation (11)

$$\tau_a L^*_A(t) - (\mu + \tau_b) L^*_C(t) = 0 \quad (13)$$

From equation (12)

$$\tau_b L^*_C(t) - (\mu + \delta) I^*(t) = 0 \quad (14)$$

From equation (13)

$$L^*_A(t) = \frac{(\mu + \tau_b) L^*_C(t)}{\tau_a} \quad (15)$$

From equation (14)

$$I^*(t) = \frac{\tau_b L^*_C(t)}{\mu + \delta} \quad (16)$$

Substitute equations (15) and (16) into (9) and (10) to get

$$\Lambda - \frac{\phi_a S^*(t)\left(\frac{(\mu + \tau_b) L_c(t)}{\tau_a}\right)}{N_x(t)} - \frac{\phi_c S^*(t) L^*_c(t)}{N_x(t)} - \frac{\phi_i S^*(t)\left(\frac{\tau_b L^*_c(t)}{\mu + \delta}\right)}{N_x(t)} - \mu S^*(t) = 0 \quad (17)$$

$$\frac{\phi_a S^*(t)\left(\frac{(\mu + \tau_b) L^*_c(t)}{\tau_a}\right)}{N_x(t)} + \frac{\phi_c S^*(t) L^*_c(t)}{N_x(t)} + \frac{\phi_i S^*(t)\left(\frac{\tau_b L^*_c(t)}{\mu + \delta}\right)}{N_x(t)} - \tau_a\left(\frac{(\mu + \tau_b) L^*_c(t)}{\tau_a}\right) - \frac{\mu(\mu + \tau_b) L^*_c(t)}{\tau_a} = 0 \quad (18)$$

From equation (18)

$$\frac{\phi_a(\mu + \tau_b) S^*(t) L^*_c(t)}{\tau_a N_x(t)} + \frac{\phi_c S^*(t) L^*_c(t)}{N_x(t)} + \frac{\phi_i \tau_b S^*(t) L^*_c(t)}{(\mu + \delta) N_x(t)} - (\mu + \tau_b) L^*_c(t) - \frac{\mu(\mu + \tau_b) L^*_c(t)}{\tau_a} = 0 \quad (19)$$

That is

$$\left[\frac{\phi_a(\mu+\tau_b)S^*(t)}{\tau_a N_x(t)}+\frac{\phi_c S^*(t)}{N_x(t)}+\frac{\phi_c\tau_b S^*(t)}{(\mu+\delta)N_x(t)}-(\mu+\tau_b)-\frac{\mu(\mu+\tau_b)}{\tau_a}\right]L_c^*(t)=0 \quad (20)$$

That is

$$L_c^*(t)=0 \quad (21)$$

Or

$$\left[\frac{\phi_a(\mu+\tau_a)S^*(t)}{\tau_a N_x(t)}+\frac{\phi_c S^*(t)}{N_x(t)}+\frac{\phi_c\tau_b S^*(t)}{(\mu+\delta)N_x(t)}-\frac{\mu(\mu+\tau_b)}{\tau_a}-(\mu+\tau_b)\right]=0 \quad (22)$$

Substitute equation (21) into equations (15), (16) and (17) to get

$$L_A^*(t)=L_c^*(t)=I^*(t)=0 \quad (23)$$

And

$$\Lambda-\mu S^*(t)=0 \quad (24)$$

That is

$$S^*(t)=\frac{\Lambda}{\mu} \quad (25)$$

Hence, the DFE state exists at

$$\begin{bmatrix}S^*(t)\\L_A^*(t)\\L_C^*(t)\\I^*(t)\end{bmatrix}=\begin{bmatrix}\frac{\Lambda}{\mu}\\0\\0\\0\end{bmatrix} \quad (26)$$

While the EE state exists at

$$\begin{bmatrix}S^o(t)\\L_A^o(t)\\L_C^o(t)\\I^o(t)\end{bmatrix}=\begin{bmatrix}\frac{\mu N_x(\mu+\tau_a)(\mu+\tau_b)(\delta+\mu)}{\mu(\delta\mu\phi_a+\delta\phi_a\tau_b+\delta\phi_c\tau_a+\mu^2\phi_a+\mu\phi_a\tau_b+\mu\phi_c\tau_a+\phi_i\tau_a\tau_b)}\\ \frac{\mu N_x(\mu+\tau_b)(\mu+\delta)\left[\begin{matrix}\Lambda(\delta\mu\phi_a+\delta\phi_a\tau_b+\delta\phi_c\tau_a+\mu^2\phi_a+\mu\phi_a\tau_b+\mu\phi_c\tau_a+\phi_i\tau_a\tau_b)\\-\mu N_x(\mu+\tau_a)(\mu+\tau_b)(\delta+\mu)\end{matrix}\right]}{\mu N_x(\mu+\tau_a)(\mu+\tau_b)(\delta+\mu)\left[\delta\mu\phi_a+\delta\phi_a\tau_b+\delta\phi_i\tau_a+\mu^2\phi_a+\mu\phi_a\tau_b+\mu\phi_c\tau_a+\phi_i\tau_a\tau_b\right]}\\ \frac{\mu\tau_b N_x(\mu+\delta)\left[\begin{matrix}\Lambda(\delta\mu\phi_a+\delta\phi_a\tau_b+\delta\phi_c\tau_a+\mu^2\phi_a+\mu\phi_a\tau_b+\mu\phi_c\tau_a+\phi_i\tau_a\tau_b)\\-\mu N_x(\mu+\tau_a)(\mu+\tau_b)(\delta+\mu)\end{matrix}\right]}{\mu N_x(\mu+\tau_a)(\mu+\tau_b)(\delta+\mu)\left[\delta\mu\phi_a+\delta\phi_a\tau_b+\delta\phi_i\tau_a+\mu^2\phi_a+\mu\phi_a\tau_b+\mu\phi_c\tau_a+\phi_i\tau_a\tau_b\right]}\\ \frac{\tau_a\tau_b\mu N_x\left[\begin{matrix}\Lambda(\delta\mu\phi_a+\delta\phi_a\tau_b+\delta\phi_c\tau_a+\mu^2\phi_a+\mu\phi_a\tau_b+\mu\phi_c\tau_a+\phi_i\tau_a\tau_b)\\-\mu N_x(\mu+\tau_a)(\mu+\tau_b)(\delta+\mu)\end{matrix}\right]}{\mu N_x(\mu+\tau_a)(\mu+\tau_b)(\delta+\mu)\left[\delta\mu\phi_a+\delta\phi_a\tau_b+\delta\phi_i\tau_a+\mu^2\phi_a+\mu\phi_a\tau_b+\mu\phi_c\tau_a+\phi_i\tau_a\tau_b\right]}\end{bmatrix} \quad 27$$

The Basic Reproduction Number (R_o)

The basic reproduction number (R_o) is used to measure the ability of a disease to invade the entire population. It is the number of secondary infections produced by a typical case of an infection in a population where all individuals are susceptible and no one has immunity against the disease. It can therefore be measured by counting the number of secondary cases following the introduction of an infection into a totally susceptible population.

The basic reproduction number as described in Bawa et al (2013) and Diekmann et al (2010), also affirmed this definition. The basic reproduction number of the system (1) to (4) which is the spectral radius of the next generation matrix was obtained using the approach described by Diekmann and Heeterbek (2000) and subsequently analysed by Van de Driesches and watmough (2002).

The matrix F is the matrix of the new infection terms and V the matrix of the transition terms. Both matrices are obtained from the infected compartments (i.e $L_A(t), L_C(t)$ and $I(t)$) at DFE. The DFE of the model exists at the point

$$\begin{bmatrix} S^*(t) \\ L_A^*(t) \\ L_C^*(t) \\ I^*(t) \end{bmatrix} = \begin{bmatrix} \frac{\Lambda}{\mu} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, we have

$$F = \begin{bmatrix} \phi_a & \phi_c & \phi_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad 28$$

And

$$V = \begin{bmatrix} \tau_a + \mu & 0 & 0 \\ -\tau_a & \mu + \tau_b & 0 \\ 0 & -\tau_b & \mu + \delta \end{bmatrix} \qquad 29$$

Hence,

$$R_o = \frac{\Lambda\left(\delta\mu\phi_a + \delta\phi_a\tau_b + \delta\phi_c\tau_a + \mu^2\phi_a + \delta\phi_a\tau_b + \mu\phi_c\tau_a + \phi_i\tau_a\tau_b\right)}{\mu N_x\left(\mu+\tau_a\right)\left(\mu+\tau_b\right)\left(\delta+\mu\right)}$$

30

$$R_o = \frac{\left(\delta\mu\phi_a + \delta\phi_a\tau_b + \delta\phi_c\tau_a + \mu^2\phi_a + \delta\phi_a\tau_b + \mu\phi_c\tau_a + \phi_i\tau_a\tau_b\right)}{\left(\mu+\tau_a\right)\left(\mu+\tau_b\right)\left(\delta+\mu\right)}$$

31

Results and Discussion

For the numerical results, the following values for variables and parameters applicable to Suleja enclave are considered:

$N_x = 162{,}135$ Geonames: worldpopulationreview.com 2019

$\Lambda = 0.024236522$ knoema.com/atlas/Nigeria/death-rate

$\phi_a = 0.0048046381$ (field,2019)

$\phi_c = 0.0180713603$ (field,2019)

$\phi_i = 0.004292719$ (field,2019)

$\tau_a = 0.7899703424$ (field,2019)

$\tau_b = 0.1580022701$ (field,2019)

$\mu = 0.0119$ knoema.com

$\delta = 0.000863478$ (field,2019)

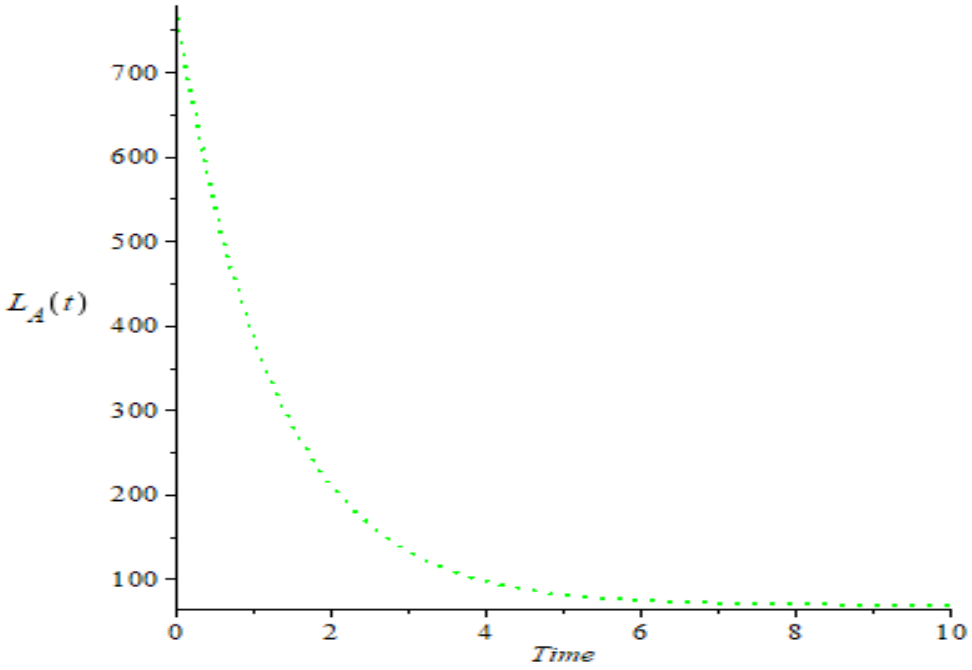


Fig 3: Profile of $L_A(t)$ over Time.

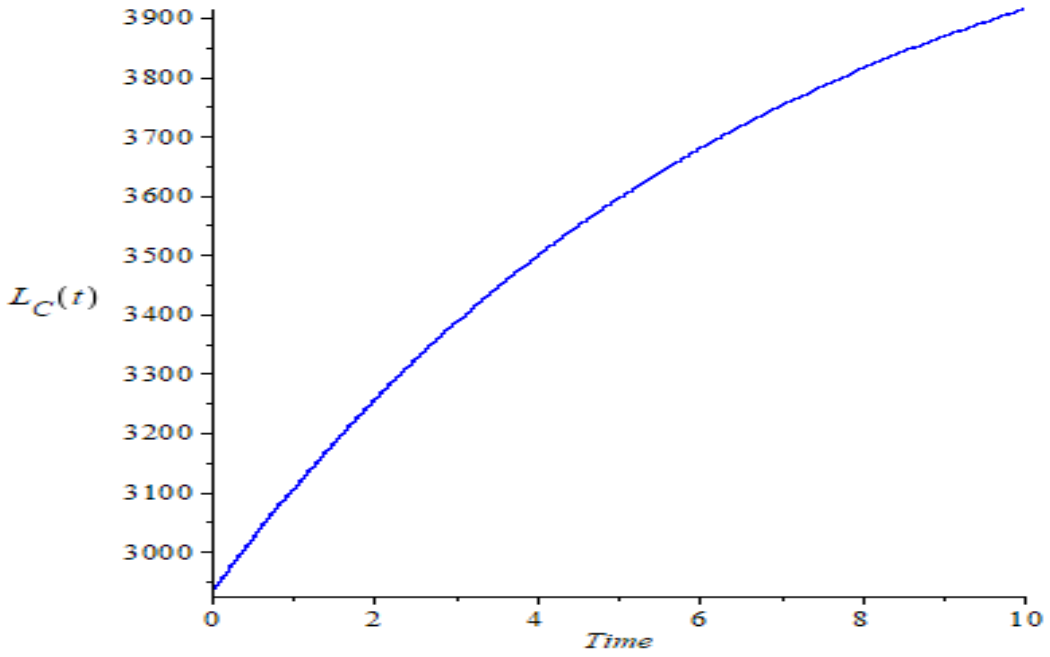


Fig 4: Profile of $L_C(t)$ over Time.

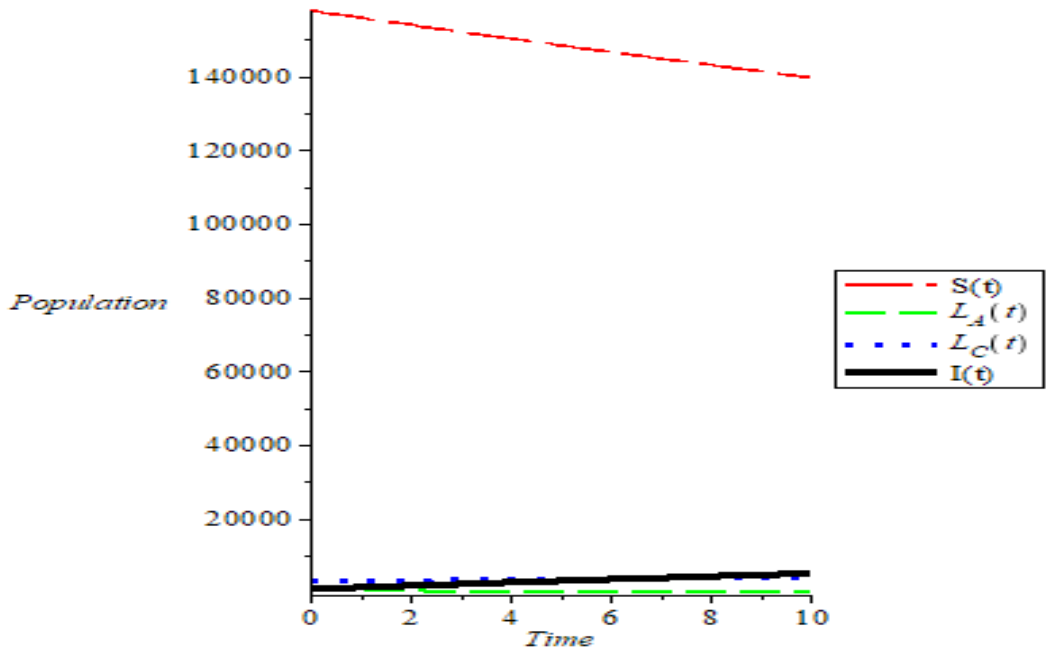


Fig 5: Profiles of the Four Compartments over Time.

Fig. 3 shows that more people are getting aware of their HIV/AIDS status showing sharp decline in $L_A(t)$ overtime and are place on ART, this is

eminent in fig. 4 with increase in $L_C(t)$. In fig. 5, the profile of the four compartments can easily be compared. It shows decline in the susceptible population and increase in $I(t)$ population resulting from lack of attainment of viral suppression.

$$R_0 = \frac{\begin{array}{l}0.0242365221\{(0.000863478)(0.0119)(0.0048046381)+(0.000863478)(0.0048046381)\\(0.1580022701)+(0.000863478)(0.0180713603)(0.7899703424)+(0.0119)^2\\(0.0048046381)+(0.0119)(0.0048046381)(0.1580022701)+(0.0119)(0.0180713603)\\(0.7899703424)+(0.004292719)(0.7899703424)(0.1580022701)\}\end{array}}{0.0119(0.0119+0.7899703424)(0.0119+0.1580022701)(0.000863478+0.0119)}$$

$$R_0 = \frac{0.0242365221\{0.0000000493+0.0000006555+0.0000123268+0.0000006803+0.0000090338+0.0001698824+0.0005358047\}}{0.0119(0.8018703424)(0.1699022701)(0.012763478)}$$

$$R_0 = \frac{0.0242365221(0.00069045)}{0.0119(0.001738891)}$$

$$R_0 = \frac{0.0000167341}{0.0000206928}$$

$$R_0 = 0.8086919122 < 1$$

Sensitivity Analyses

The sensitivity analyses was carried out to determine the impact of the parameters on the population using the sensitivity analyses software. The following values were generated:

$\tau_a = 0.02139172626$

$\tau_b = -0.05874596013$

$\phi_a = 0.01571204636$

$\phi_c = -0.21859715280$

$\phi_i = -0.72836166030$

It can be seen that τ_a is the most influential parameter that can determine the magnitude of $S(t)$ in the EE. Since the sign is positive and larger, it means that the transition rate from acutely infected into chronic infected will enlarge the total of susceptible humans. Using the same approach, the order of the most influential parameters in determining the magnitude of the variable $S(t)$ at the EE is τ_a, ϕ_a, τ_b, ϕ_c, ϕ_i.

Conclusion

The research work establishes the existence of the disease free equilibrium state and obtain the endemic equilibrium state as well as the basic reproduction number (R_0). Result revealed that $R_0 < 1$, which signifies that the disease can be controlled. The sensitivity analysis indicated that the parameter τ_a is the most sensitive which can significantly increase the susceptible population.

Recommendations

It is important that all people living with HIV have access to ART treatment and achieve viral suppression. The adoption of a test and treat policy of 2016 should be uphold and strengthen.

Viral suppression among people living with HIV should be scaled up to meet up with the UNAIDS 90-90-90 target in 2030.

There should be repository of related data for researchers.

Since there is no cure for HIV infection for now and it is known that effective anti-retroviral drugs (ARVs) can control the virus and help prevent onward transmission to other people. In line with the recommendation of WHO (2020), all people living with HIV should be provided with lifelong ART.

Acknowledgement

The research work was fully supported and sponsored by TETFund, Abuja, Nigeria (2015/2016, 4th Batch Institution-Based Research Intervention) in IBB University, Lapai, Niger State. The authors are extremely grateful.

References

Bawa, M., Abdulrahman, S., Jimoh, O. R. & Adabara, N. U. (2013). Stability Analysis of the Disease Free Equilibrium State for Lassa Fever Disease. Journal of Science, Technology, Mathematics and Education, Vol. 9(2), Pp 150-161.

David, G., Murray, D. and Fraser, L. (2001). "A Mathematical Treatment of AIDS and Condom Use," Math Medicine and Bio 18(3):225-262.

David, H.O., Avidan, U. and Meuman, P. (1995). "Rapid Turnover of Plasma Virus and CD4 lymphoctes in HIV-1 Infection, Nature, 373(6510):12-126.

Diekmann, O., & Heesterbeek, J.A.P. (2000). Mathematical Epidemiology of Infectious Diseases: Model Building, Analysis and Integration. New York: Wiley.

Diekmann, O., & Heesterbeek, J.A.P. & Roberts, M. G. (2010). The Construction of Next Generation Matrices for Compartmental Epidemic Models, J. R. Soc. Interface 7, Pp 873-885.

HIV Transmission question and answers. Accessed on 28th May 2015.
http://www.avert.org/Hiv-transmission-questionss-answers.htm#sthash.ifvu2w5u.dpuf

NAIIS (2019). HIV/AIDS in Nigeria-Pushing for the last miles. The Presidential announcement of the Nigeria HIV/AIDs indicator and impact survey results by his Excellency Muhammadu Buhari, President of the Federal Republic of Nigeria.

Ogata, K. (1990). Modern Control Engineering.

Palloni, A. and Glicklich, M. (1991). review of Approaches to Modeling the Demographic Impact of the AIDS Epidemic. In: the AIDs Epidemic and its Demographic consequences United Nations and WHO, New York, 1-37.

UNAIDS (2016). www. un.org/apps/new/story.app? News ID=54442#.v41umhlondg.

University of California Los-Angeles Researchers (UCLA) (2002). "Could Widespread use of Combination Anti-retroviral Therapy Eradicate HIV Pandemic?", Lancet Infection Disease, Vol.2; No.8, 15-30.

Van de Driessche, P., & Watmough, J. (2002). Reproduction Numbers and Sub-Threshold endemic equilibria for Compartmental Models of Disease Transmission. Mathematical Bioscience, 180, 29-48.

WHO (2020). WHO int/news-room/fact-sheets/detail/HIV-aids.

Yekeen, Taofeek A., Xu, X., Zhang, Y, Wu, Yousheng, K., Stephani, R, Tiina, H.,X. (2016). Assessment of health risk of trace metal pollution in surface soil and road dust from e-waste recycling area in China. Environment Science and Pollution Research, 23(17), 17511-17524