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SIMULATION OF DECREASING AND INCREASING EFFECT OF AN ENVIRONMENTAL VARIABLE ON THE POLLUTANT LEVEL AND THE RELATIVE HUMIDITY

[1]ABRAHAM I.M. [2]EKAKA-A .E.N. [3]EDE P.N, [4]UBONG .I.U.
[1]Institute of Geo-Sciences and Space Technology (IGST), Rivers State University, Port Harcourt, Rivers State, Nigeria. [2]Department of Mathematics, Rivers State University, Port Harcourt, Rivers State, Nigeria [3]Department of Geography, Rivers State University, Port Harcourt, Rivers State, Nigeria [4]Institute of Pollution Studies, Rivers State University, Port Harcourt, Rivers State, Nigeria.

ABSTRACT
It is an established fact that the pollutant level and the relative humidity in the atmosphere do interact. This process of interaction is defined by a continuous dynamical system of two non-linear differential equations of a first order with their specified initial condition values. One of the model parameters that define the dynamics is called the inter-competition coefficient which acts to inhibit either the growth of the relative humidity or the pollutant level. In this study, we have assumed that if the pollutant level contribution to inhibit the relative humidity is fixed, and the contributions of relative humidity to inhibit the pollutant level is being decreased, we have used the Ordinary Differential Equation of order 45 (ODE 45) to analyse and solve this important research question. The full novel results of this numerical simulation analysis that we have not seen elsewhere are presented and discussed.

Keywords: *Pollutant, Relative Humidity, Inter-competition coefficient, Simulation, modelling*

INTRODUCTION
Human activities release substances into the air, some of which cause problems for living things (AIHA, 2002). Pollution is the release of substances into the environment, which will result in deleterious effects.

The legal dictionary also define pollution as the discharge of toxic or contaminating substance that is likely to have an adverse effects on the natural environment or life. These toxic or contaminating substances are called pollutants.

Pollutants are classified as primary or secondary. Usually, primary pollutants are directly emitted from a process, such as ash from a volcanic eruption, the carbon monoxide gas from a motor vehicle exhaust or sulphur dioxide released from factories. Secondary pollutants are not emitted directly. Rather, they are formed in the air when primary pollutants react. An important example of secondary pollutant is ground level ozone which is one of the many secondary pollutants that makes up photochemical smog. Some pollutants may be both primary and secondary: that is, they are both emitted directly and formed from other primary pollutions (Tawari et al., 2012).

Other than gaseous compound, one of the most important pollutants produced by human activities is particulate matter (PM) (Dermirbas, 2004). Particulate matter can also cause serious problems in the environment. Studies have also confirmed the relationship between mortality and exposure to PM (Peters et al., 2001).

National Research Council (2002), stated that particulate pollution is the most important contaminant in our air and when particle levels go up, people die. WHO (2007), also has that small particles with a diameter of 10 microns (PM10) or less are able to penetrate deep into the lungs and appear to have the greatest health-damaging potential.

Air movements influence the fate of air pollutants. So any study of air pollution should include a study of meteorology. If the air is calm and pollutants cannot disperse, then the concentration of these pollutants will build up. Meteorology plays a crucial role in ambient air pollution. Meteorological factors such as wind speed and direction, relative humidity, temperature can significantly affect air quality. The most important role of meteorology is in the dispersion, transformation and removal of air pollution from the atmosphere (Radaideh, 2017).

Air pollutants increase with increase in relative humidity. Increase air pollutant concentrations do not typically result from sudden increases in emissions but rather from meteorological conditions that impede dispersion in the atmosphere or result in increased pollutant generation (Gorai et al., 2015).

Relative Humidity, which is amount of water vapour present in air expressed as a percentage of the amount needed for saturation at the same temperature, affects the natural deposition process of PH, whereby moisture particles adhere to PM, accumulating atmospheric PM concentration. With increasing humidity, moisture particles eventually grow in size to a point where dry deposition occurs, reducing PM10 concentrations in the atmosphere (Hermandez et. al., 2017).

In this study we have stimulated the decreasing effect of an environmental variable on the pollutant level and the relative humidity.

MATHEMATICAL FRAMEWORK

In this numerical simulation study, we have used the following non-linear ordinary differential equation, having the mathematical structure

$$\frac{d}{dt}\,p(t) = \alpha 1\, p(t) - \beta 1 p2(t) - r1\, p(t)Rh(t)$$

$$\frac{d}{dt}\,Rh(t) = \alpha 2\, Rh(t) - \beta 2\, Rh2(t) - r2\, p(t)Rh(t)$$

$$P(0) = P0 > 0$$

$$Rh(0) = Rh(0) > 0$$

$\alpha 1 =$ The growth rate of the Pollution Level

$\alpha 2 =$ The growth rate of Relative Humidity

$\beta 1 =$ The inhibiting self interaction factor between the molecules of the pollutant in the atmosphere.

$\beta 2 =$ The inhibiting self interaction factor between the molecules of the pollutant and the relative humidity in the atmosphere.

$r1 =$ The contribution of the relative humidity to inhibit the growth of the pollutant level over time due to its interaction with the pollutant level.

$r2 =$ The contribution of the pollutants to inhibit the growth of the relative humidity over time.

P(0) = The Initial pollution level
Rh(0) = The initial relative humidity value

For the purpose of this study, we have used the following parameter values:
$\alpha 1 = 0.14$
$\alpha 2 = 0.04725$
$\beta 1 = 0.0034$
$\beta 2 = 0.0063084$
$r1 = 0.001$
$r2 = 0.0001$
P(0) = 0.23
Rh(0) =0.48

METHOD OF ANALYSIS

We applied ODE 45 to simulate the above self interacting model to a varying single initial condition. From this numerical analysis, we have obtained predicted pollution level or pollutant level for the duration of Six months on the scenario of every 30days such as P(0), P(1), P(31), P(61), P(91), P(121), P(151). The interpretation of this result is presented in the body of this work.
We assumed that an inter-competition interaction (the effect that one species has on the other) exist between the environmental variable called Pollution Level (Particulate Level) and a meteorological variable such as Relative Humidity which has been recently derived by Ekaka-a and Akpode (2019). These two environmental variables are time dependent.
Without loss of generality, we have only applied the ODE 45 numerical method which is computationally more efficient than its counterparts numerical methods such as ODE 23, ODE 23tb and ODE 15s to quantify the effect of decreasing the inter-competition coefficient of the relative humidity on the particulate level when the inter-competition coefficient of the particulate level on the relative humidity is fixed.

RESULTS

On the application of the above methods which have applied to solve the proposed research problems, we have obtained the following results as displayed on Table 1- Table 5.

Table 1: Evaluating the decreasing effect of r1 by 50% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	11.0832	1.2936	1.9246	1.9245	0.0083
61	37.9448	38.7682	2.1702	6.7847	6.7746	0.1498
91	36.5342	38.8289	6.2809	19.7682	19.6688	0.5026
121	30.9417	36.0064	16.3684	41.5290	41.0702	1.1047
151	25.3757	33.1997	30.8327	59.5802	58.5850	1.6704
181	22.1688	31.6975	42.9824	67.7758	66.4059	2.0213

Authors numerically simulated results.

Table 2: Evaluating the decreasing effect of r1 by 70% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	11.0264	0.7745	1.9246	1.9245	0.0050
61	37.9448	38.4385	1.3012	6.7847	6.7786	0.0898
91	36.5342	37.9074	3.7587	19.7682	19.7086	0.3013
121	30.9417	33.9638	9.7671	41.5290	41.2544	0.6612
151	25.3757	30.0383	18.3741	59.5802	58.9856	0.9980
181	22.1688	27.8708	25.7208	67.7758	66.9570	1.2082

Authors numerically simulated results.

Table 3: Evaluating the decreasing effect of r1 by 90% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	10.9698	0.2576	1.9246	1.9246	0.0016
61	37.9448	38.1092	0.4334	6.7847	6.7827	0.0299
91	36.5342	36.9907	1.2494	19.7682	19.7483	0.1004
121	30.9417	31.9426	3.2350	41.5290	41.4377	0.2198
151	25.3757	26.9159	6.0697	59.5802	59.3832	0.3308
181	22.1688	24.0582	8.5228	67.7758	67.5046	0.4001

Authors numerically simulated results.

Table 4: Evaluating the decreasing effect of r1 by 95% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	10.9557	0.1287	1.9246	1.9246	0.0008
61	37.9448	38.0270	0.2167	6.7847	6.7837	0.0150
91	36.5342	36.7623	0.6243	19.7682	19.7583	0.0502
121	30.9417	31.4413	1.6147	41.5290	41.4834	0.1098
151	25.3757	26.1438	3.0268	59.5802	59.4819	0.1651
181	22.1688	23.1115	4.2524	67.7758	67.6405	0.1997

Authors numerically simulated results.

Table 5: Evaluating the decreasing effect of r1 by 99.99% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	10.9417	0.0003	1.9246	1.9246	0.0000
61	37.9448	37.9449	0.0004	6.7847	6.7847	0.0000
91	36.5342	36.5346	0.0012	19.7682	19.7682	0.0001
121	30.9417	30.9427	0.0032	41.5290	41.5289	0.0002
151	25.3757	25.3772	0.0060	59.5802	59.5800	0.0003
181	22.1688	22.1707	0.0085	67.7758	67.7756	0.0004

Authors numerically simulated results.

Table 6: Evaluating the Increasing effect of r1 by 101% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	10.9388	0.0257	1.9246	1.9246	0.0002
61	37.9448	37.9283	0.0433	6.7847	6.7849	0.0030
91	36.5342	36.4886	0.1247	19.7682	19.7702	0.0100
121	30.9417	30.8420	0.3222	41.5290	41.5381	0.0219
151	25.3757	25.2226	0.6033	59.5802	59.5999	0.0330
181	22.1688	21.9808	0.8480	67.7758	67.8028	0.0398

Authors numerically simulated results.

Table 7: Evaluating the Increasing effect of r1 by 102% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	10.9360	0.0515	1.9246	1.9246	0.0003
61	37.9448	37.9119	0.0866	6.7847	6.7851	0.0060
91	36.5342	36.4430	0.2495	19.7682	19.7721	0.0201
121	30.9417	30.7423	0.6442	41.5290	41.5472	0.0439
151	25.3757	25.0697	1.2059	59.5802	59.6195	0.0659
181	22.1688	21.7931	1.6951	67.7758	67.8298	0.0796

Authors numerically simulated results.

Table 8: Evaluating the Increasing effect of r1 by 103% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	10.9332	0.0772	1.9246	1.9246	0.0005
61	37.9448	37.8954	0.1300	6.7847	6.7853	0.0090
91	36.5342	36.3975	0.3741	19.7682	19.7741	0.0301
121	30.9417	30.6428	0.9660	41.5290	41.5563	0.0658
151	25.3757	24.9170	1.8078	59.5802	59.6391	0.0988
181	22.1688	21.6055	2.5412	67.7758	67.8567	0.1193

Authors numerically simulated results.

Table 9: Evaluating the Increasing effect of r1 by 104% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	10.9304	0.1029	1.9246	1.9246	0.0007
61	37.9448	37.8790	0.1733	6.7847	6.7855	0.0120
91	36.5342	36.3520	0.4988	19.7682	19.7761	0.0401
121	30.9417	30.5433	1.2875	41.5290	41.5654	0.0877
151	25.3757	24.7644	2.4089	59.5802	59.6587	0.1317
181	22.1688	21.4181	3.3864	67.7758	67.8836	0.1590

Authors numerically simulated results.

Table 10: Evaluating the Increasing effect of r1 by 105% for a fixed r2 on the pollutant level and the relative humidity using ODE 45 numerical method.

Days (time)	Pollution level P(t)	Pollution level modified Pm (t)	Estimated effect EE1	Relative Humidity concentration Rh(t)	Relative Humidity concentration modified Rhm(t)	Estimated effect EE2
1	0.2300	0.2300	0.0000	0.4800	0.4800	0.0000
31	10.9416	10.9276	0.1286	1.9246	1.9246	0.0008
61	37.9448	37.8626	0.2166	6.7847	6.7857	0.0149
91	36.5342	36.3064	0.6234	19.7682	19.7781	0.0501
121	30.9417	30.4439	1.6088	41.5290	41.5745	0.1096
151	25.3757	24.6121	3.0093	59.5802	59.6783	0.1645
181	22.1688	21.2310	4.2305	67.7758	67.9105	0.1987

Authors numerically simulated results.

DISCUSSION OF RESULTS

Table 1-5: Evaluate the decreasing effect of varying r1 (contribution of relative humidity to inhibit the pollution level) at 50%, 70%, 90%, 95% and 99.99% for a fixed r2 (contribution of pollutant level to inhibit relative humidity) on the pollution level and relative humidity. The particulate level indicates an increase while the relative humidity is vulnerable to depletion.

Table 6-10: Evaluate the increasing effect of varying r1 (contribution of relative humidity to inhibit the pollution level) at 101%, 102%, 103%, 104% and 105% for a fixed r2 (contribution of pollutant level to inhibit relative humidity) on the pollution level and relative humidity. The particulate level produces depletion while the relative humidity is vulnerable to increase. This is a good scenario in environmental science.

Looking at table 1, we have observed that when r1 is decreased by 50% for a fixed parameter value of r2, the relative humidity is vulnerable to depletion over every 30 days scenario predictions whereas the particulate level has shown an increase in its magnitude of predictions over the same 30 days scenario.

The mathematical model being simulated is called logistic interaction between two environmental variables called pollution level and relative humidity over time in the unit of days. The initial pollution level and the initial relative humidity also show that we are dealing with a self

interaction or intra-competition interaction between these environmental variables.
For every 30days, in which the first day of the month starts at day 1, we have constructed this scenario from a bigger time frame value.
The sample size of the independent variable called time is usually taken for granted; to circumvent this problem in numerical simulation; we have learnt that, the sample size of the independent variable must not be chosen to be smaller than the time point 181 of the sub data sets, it must be chosen so that the time set value is bigger than the time point 181. Therefore, what takes the value of 120, 130, 140, 160, 170, 179, the numerical simulation will attracts a simulation error because the numerical integration will fail to produce the expected result, but this problem will be overcome for t= 181, 182, 185, 190 and 200.
From the theory of logistical modelling and Locka Voltera formulation, the value of $\beta 1$ is popularly assumed to be smaller than the value of $\alpha 1$, the value of $\beta 2$ is always assumed to be smaller than $\alpha 2$,. It can be otherwise provided the expected result is not misleading or does not contradict reality.
P(o), α and Rh(o) were so chosen because they are both bigger than the expected limiting values. The initial condition should be chosen to obey the scientific law of carrying capacity which measures the maximum population size that can sustain the growth of both the pollution level and relative humidity.
In summary as r1 is being decreased by 50%, 70%, 90%, 95% and 99.99%, we have observed that the depletion of the relative humidity at day 181 value of 2.0213 has decreased to 1.2082 followed by a further decrease to 0.4001 then to 0.1997 and finally to 0.0004. this decreasing trend of depletion is an indicator that a transition will soon occur in which a depletion of the relative humidity will change to an increase of the relative humidity when the model parameter value is increased for a fixed r2 (table 6- table 10).

CONCLUSION AND FURTHER RESEARCH

This study simulated the decreasing effect of r1 for a fixed r2 on the pollutant level and the relative humidity. We have established that decreasing r1 and varying r2 is hostile to the environment but increasing r1 and fixing r2 bring about depletion in pollutant level and increases relative humidity. This is favourable to the environment. We are yet to look at the reverse condition of increasing and decreasing r2 for a fixed r1.

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