



PRODUCTION OF GEOIDAL MAP AND THREE DIMENSIONAL MODELLING FOR SURFACE MONITORING

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ABSTRACT

Dual Frequency Global Positioning System (DGPS) has emerged as a successful technology in providing precise positions of points on the surface of the earth over the reference ellipsoid with sub-metre level of accuracy. DGPS is one of the most frequently used positioning methods in geodesy. The end products of surveying with this receiver gives geodetic latitude (ϕ), geodetic longitude (λ) and ellipsoidal height (h) which are obtained with reference to the ellipsoid. This research involved the determination of Geoidal undulation for the production of Geoidal map of Bauchi metropolis, Bauchi state. DGPS receiver and precise Level instruments were used to obtain ellipsoidal and orthometric heights of the study area. Geoidal heights were derived from the differences between ellipsoidal heights and orthometric heights. The adjusted orthometric heights obtained using precise Level and the ellipsoidal heights which are part of the geodetic/Universal Transverse Mercator (UTM) coordinates obtained using DGPS were post processed using spectrum survey office Software (SSO). The Geoidal map, contour map and three dimensional surface Model (3D) were created using ArcGIS 10.2.1 Software version. The Microsoft Office Excel was used to deduce the ellipsoidal heights and orthometric heights in order to obtain geoidal heights for the production of Geoidal Map of the study Area. The statistical analysis of the result met the precision of second order geodetic control network and levelling specifications. The result of the spearman correlation coefficient computed using geodetic coordinates is 0.054 and the coefficient of determination

0.0029%. The result of the Z test computed (0.125) indicated that the measurements are precise at 95% confidence level. The reliability of the measurements of X and Y second order geodetic coordinates were computed at the scale of 1:5000 and found reliable at 95% confidence level. The mean value of the geoidal heights determined is 22.680 metre which can be used as the geoid of Bauchi State. The contour map, geoidal and 3D model were produced at the scale of 1:60,000 and the contour lines were interpolated at 0.2 meter contour interval which represented the terrain configuration. Therefore, the results obtained can be used for any work requiring the use of geoidal heights in Nigeria and any part of the world.

Keywords: Geoidal heights, Orthometric heights, Ellipsoidal heights, Correlation and Statistical analysis

Introduction

One of the basic goals of geodesy is the determination of the geoid which is the equipotential surface of the earth gravity field that coincides on the average with the mean sea level. The geoid is the surface which coincides with that surface to which the oceans would conform over the entire earth, if free to adjust to the combined effects of the earth's mass attraction (gravitation) and the centrifugal force of the Earth's rotation. Specifically, it is an equipotential surface, meaning that it is a surface on which the gravitational potential energy has the same value everywhere with respect to gravity (Aleem et al., 2013). The geoid surface is irregular, but considerably smoother than earth's physical surface. Sea level, if undisturbed by tides, currents and weather, would assume a surface equal to the geoid (Featherstone 2000). In geodesy, the different reference surfaces for heights determination are the earth surface, the telluroid, the geoid, the quasi-geoid and the ellipsoid. The heights determined with reference to the geoid is regarded as the orthometric height and the heights determined with reference to the ellipsoid is known as the ellipsoidal height. The ellipsoidal heights are geometric values while orthometric heights are physical values reflecting local variations in gravity as well as changes in topography, the conversion from ellipsoidal to orthometric height requires a geoid height model. Geoid comes from the word "geo"

which literarily means earth-shaped. Geoid is an empirical approximation of the figure of the earth (minus topographic relief). It is defined as the equipotential surface of the earth's gravity field which best fits, in the least square sense; the mean sea level. On the ocean, the geoid is on average at the same level as mean sea level, the surface obtained by removing from the instantaneous sea surface all periodic and quasi-periodic variations (tidal phenomena, air pressure, littoral seas, eddies and continual shifting of ocean currents).

Geodesy is concerned with the relative positioning of points and the gravity field of the earth. For geoidal mapping and three dimension (3D) surface modelling, a well defined coordinate system is needed on which measurements are tied to a set of reference points called a geodetic datum (geoid or ellipsoid). A control survey is a means of establishing precise positions of geodetic monuments. There are two types of survey controls. These include horizontal and vertical controls. Horizontal controls are defined with respect to an ellipsoid of revolution whilst vertical controls are defined with reference to a local geoid. Horizontal and vertical terrestrial geodetic control networks are important and valuable for the accurate positioning of construction and engineering projects, they serve as points of reference for correct positioning. Controls are established by classical and modern methods. The classical methods are traversing, triangulation and trilateration while the modern methods include the use of satellite techniques such as the Global Positioning System (GPS), and satellite altimeters. Satellite techniques could be used to establish and densify 3D networks more rapidly, with greater accuracy and less difficulty than terrestrial techniques (Poku and Gunter, 2006). Moreover, the use of classical methods is limited by such requirements as intervisibility between the instrument stations and target stations, favourable weather, atmospheric conditions and accessibility of stations. In addition, the accuracy levels are low. Hence classical geodetic networks established by terrestrial methods are insufficient to contemporary requirements (Heiskanen and Moritz, 1967).

Ellipsoids are reference surfaces usually determined on the physical surface of the earth. The difference in height between the physical surface and the ellipsoid is regarded as the ellipsoidal height while the difference

in height between the physical surface and the geoid is called the orthometric height. The difference in height between the geoid and the ellipsoid is regarded as the geoidal height or geoidal undulation. The height anomaly is derived from the difference between the quasi-geoid and the ellipsoid or the physical surface of the earth and the telluroid. The height anomaly is a quantity similar to the geoid height, however is located on the level of the topography not sea level. The surface formed by points which are above the reference ellipsoid (and thus a distance below the topography), is called the telluroid. The surface formed by points which are above the reference ellipsoid which coincide with the geoid at sea level, if free to adjust to the combined effects of the earth's mass attraction and the centrifugal force of the Earth's rotation is called the quasi-geoid. It lacks any physical meaning; it is not an equipotential surface, although out at sea it coincides with the geoid. Normal heights are very operational. They are always used together with so-called "quasi-geoid" heights (more correctly: height anomalies). Orthometric heights (more precisely: Helmert heights) on the other hand are always used together with geoid heights.

Presently, the most accurate positioning technology is the Global Positioning System (GPS). GPS gives accurately the 3D position of points (ellipsoidal latitudes, longitudes, and heights) and can measure under favourable weather conditions. In addition, it can measure when placed on any platform (static or dynamic). One major advantage of GPS technology over the traditional methods is that inter-visibility is not a requirement. In addition to providing highly accurate data, it is easy to use, portable, less labour intensive, and its surveys are relatively less costly. The coordinates of the GPS are referenced to the World Geodetic System 1984 (WGS 84), a global ellipsoid having its origin closed to the earth centre of the mass, which forms the origin of its coordinate system.

The mapping systems of various countries are based on their local coordinate systems. In the local coordinate systems, horizontal positions are referenced to the local ellipsoids that are defined differently by various countries to fit their topography, and heights are referenced to the local geoid (orthometric height, H). For accurate location and mapping of the natural resources which are sometimes transboundary, there is a need to integrate data obtained in one system into another. A modern height

system in a modern survey and mapping communities requires the ability to measure elevations relative to mean sea level easily, accurately, and at the lowest possible cost.

GPS applications range from cadastral surveys to monitoring sea level rise; from navigation and mapping to the use of remote sensing for resource management; from mineral exploration to assessment of potential flooding areas; from the construction and precise positioning of dams and pipelines to the interpretation of seismic disturbances. The height reference system is also implicated in many legal documents related to land management and safety such as easement, flood control, and boundary demarcation (Ayhan et al, 2009). Therefore, based on these information and the needs to support modern height system, the implementation of Accurate Height System (AMS) in Nigeria particularly in Bauchi metropolis has to be realized.

There are a lot of geodetic methods for determining of height or heights differences. These methods are classified as geometric levelling, trigonometric levelling, and GPS/Levelling. Generally, geoid undulation is required for many geodetic and surveying applications. The most notable application is for converting GPS-derived ellipsoidal height to orthometric height for engineering purpose. Classical methods of heights determination is the techniques usually adopted in the study area; hence there is high demand for digital dataset for the production of geoidal map and 3D surface model of the study area. There is a growing use of GPS surveys in Nigeria due to its numerous advantages over classical methods. Its applications include land and engineering surveying, GIS and navigation. If GPS data is properly processed and used, GPS can be an effective tool to promote national development as the data can be used for planning of communities, exploration and exploitation of natural resources, correct positioning of engineering and construction works, scientific investigation, enhancement of agricultural productivity and provision of services among other applications of which the study area is not an exception.

A digital terrain model (DTM) is a digital representation of ground surface topography. It represents a very important geospatial data type in the analysis and modelling of different hydrological and ecological phenomenon which are required in preserving our immediate

environment. DTMs are used in geographic information systems (GIS) and are the most common basis for digitally produced topographic maps and orthogonal projections of the earth (also called orthophotos). DTMs are particularly relevant for many applications such as lake and water volumes estimation, soil erosion volumes calculations, flood estimate, quantification of earth materials to be moved for channels, roads, dams, embankment etc. They are essential data for planning, decision making, and information gathering and measuring including volume changes. High accuracy DTMs are typical for industries involved in mining, land reclamation or construction activities or government agencies involved in urban and infrastructure planning.

The geoidal undulation varies globally between ± 110 m, when referred to the GRS 80 ellipsoid. The geoid model will give geoidal undulation at every point of observation and the fundamental relationship, to first approximation, that binds the ellipsoidal heights obtained from GPS measurements and heights with respect to a vertical (local) datum established from conventional spirit levelling (Heiskanen and Moritz, 1967; Featherstone et al., 1998; Olaleye et al., 2013). Ellipsoidal heights can't satisfy the aim in practical surveying, engineering or geophysical applications as they have no physical meaning and must be transformed to orthometric heights (H), which are referred to geoid, to serve the geodetic and surveying applications.

According to Raaed (2014), in this study, a proposed computational scheme is applied for the assessment of the orthometric correction for long line trigonometrically levelled height differences. This algorithm is based on the spherical harmonic coefficients of geopotential models and trigonometric elevation data. The applied algorithm does not demand any terrestrial gravity data and is route independent. In particular, two geopotential models with different resolutions were utilized. The results showed a reasonable applicability of the investigated algorithm to compute the orthometric correction for trigonometric levelling. Thus it is recommended to use this approach for computing the orthometric corrections in similar modern heighting applications, such as precise EDM trigonometric height traverses. Amal (2016) in his work, a large part of Baghdad University campus has been selected. The determination of

Geoidal height for the local area requires Ground Control Points which both Ellipsoidal and Orthometric heights are known to compute the difference between them.

This work investigates the use of ellipsoidal heights in place of orthometric heights for engineering surveys. DGPS observations were carried out to obtain the ellipsoidal heights for a number of points in the study area in Port Harcourt, Nigeria. Orthometric heights for the same set of points were determined using geodetic levelling. The results satisfied third order levelling which is good enough for engineering surveys (Badejo, 2016). Heister et al (2012); A multidisciplinary research project to examine hydrology, sedimentology and plant ecology of the wetlands of the Okavango Delta was initiated in 1993. One key research area is the determination of precise orthometric height differences along the Okavango/Jao/Boro river system. Traditional methods (geodetic and trigonometric levelling) to determine orthometric heights within the delta are prevented due to the wide swampy areas. Local geoid models do not have the required accuracy to transfer ellipsoidal heights derived by GPS to orthometric heights. Pre-emption analysis revealed that the determination of a local geoid, derived by a combination of GPS and levelling determined height differences, is the most suitable approach.

One of the major tasks of geodesy is the determination of geoid. This task is getting more crucial due to the development of global positioning systems (GPS). This is due to the fact that GPS provide ellipsoidal heights instead of orthometric heights. To convert ellipsoidal heights into orthometric heights, precise geoid heights are required. Nowadays, the most effective universal technique used for the determination of orthometric heights is the GPS and Levelling technique. This paper focuses on this technique and multiple regression analysis method was used to further determine the geoid undulations. ArcGIS 9.2 software version was used for generating the grid map of the area using the corrected orthometric heights obtained by the regression method (Edan et al., 2014). Orthometric is the height preferred by users because of its relationship with Mean Sea Level which approximate the geoid. If the geoid is known, it can be used to produce the geoidal map. Geoidal Maps are essential tool in all spheres of our day- to- day activities most especially in geophysical

studies, because they portrayed the geopotential configurations of any given place. However, as important as such maps have the same instrument for geospatial exploration purposes are lacking for many places including part of Mubi north. The aim of the research was to determine the Geoidal Undulation and produce the Geoidal Map of part of Mubi North Local Government Area of Adamawa State, Nigeria. Single Frequency Global Positioning System and Geodetic Level (Wild N3) instruments were used to obtain ellipsoidal and Orthometric heights of the areas. The adjusted Orthometric heights obtained from Geodetic Levelling and the Ellipsoidal heights which are part of the geodetic coordinates obtained from GNSS were post processed using Leica Geo-Office Software. The Geoidal lines and Digital Geoidal Model (DGM) were created using Surfer 7 Software. The Microsoft Office Excel was used to deduce the Ellipsoidal height, Orthometric height and Geoidal Undulations, for production of Geoidal Map of the study Area. The statistical analysis of the result met the Geodetic specifications and therefore can be used for any work required the use of geoidal undulation in the study area (Aleem et al., 2016)

Digital Elevation Model (DEM) represents a very important geospatial data type in the analysis and modelling of different hydrological and ecological phenomenon which are required in preserving our immediate environment. DEMs are typically used to represent terrain relief. DEMs are particularly relevant for many applications such as lake and water volumes estimation, soil erosion volumes calculations, flood estimate, quantification of earth materials to be moved for channels, roads, dams, embankment etc. In this study, three different sources of spatial data in the generation of DEMs (Shuttle Radar Topography Mission SRTM 30, Digitized Topographical map and Google Earth Pro.) were compared with field measured data from Total Station Instrument, the field data were used to generate a Digital Elevation Models DEMs from 495 radial points over the test site. The accuracy of generated DEMs were assessed statistically by comparing (1) estimates of some topographic attributes (slope and aspect), (2) overall spot height estimation performance and, (3) independence of spot estimation errors and the magnitude of field measured height. From the results obtained it was concluded that the DEMs from the satellite imagery (SRTM 30) does not perform well in

collecting data for topographic works. The digitized topographic map gives a good result but the variation from the reference in this study may be as a result of human activities and erosion that has occurred from when the topographic map was produced and also the quality of the topographic map. The Google Earth pro was also concluded to perform far better than the SRTM 30 data. Finally, it was recommended that Real Time Kinematic GPS combine with total station can be tested for speed and accuracy and also SRTM data and other global terrain data sources i.e., GTOPO, Microsoft Visual Earth and NASA World Wind can also be examined for suitability of their application over larger assessment area (Olalekan et al., 2011).

In order to obtain geoid heights of unknown points, geoid models are performed by using several techniques such as (Soycan and Soycan, 2003); Inverse Distance Weighting, Nearest Neighbor, Triangulation with Linear Interpolation, Natural Neighbor, Polynomial Regression, Local Polynomial, Radial Basis Function, Modified Shepard's Method, Minimum Curvature, Moving Average, Biharmonic Spline Interpolation, Kriging.

Spatial interpolation has been applied in many disciplines such as, geodesy, geophysics, civil engineering, water resources, meteorology, mathematics, marine science and agriculture etc. Specific applications under those disciplines are many such as mine exploration, climate change investigation, crustal deformation monitoring, classification of soil properties, population density modelling, digital terrain model (DTM) generation and use, chemical concentration modelling, soil Ph or moisture estimation and so on (Li and Heap, 2008).

Statement of Problem

In engineering works, engineers and surveyors are usually faced with the problem of determination of height differences between points. Some of the challenges are surveying of levelling networks, vertical applications, maintenance and control measurements of big structures like bridges, dams, very tall buildings and towers, determination of crustal movements of the earth and motorways, railways, sewers and pipelines measurement. Instruments used in surveying and measurement methods are determined in relation to topography of land, target precision, and the aim. In this study, accuracies of heights determination techniques were based on the

instrument and the measurement methods. Digital technology for the production of geoidal map and three dimensional surface modelling is lacking in the study area. The major concerned is lack of an existing geoidal map and digital 3D surfaces model for geodetic, surveying applications, exploration and exploitation. It is in light of the above that this research was carried out to produce a geoidal and three dimension surface modelling of the study area.

Aim and Objectives of the Study

The aim of this thesis is to produce geoidal map and three dimension surfaces modelling using dual frequency GPS and precise level. In order to achieve this aim, the following objectives were followed:

Production of three dimensional Surface model of the study area

Production of contour map of the study area

Production of geoidal map of the study area

Justification of the Study

Considering the numerous problems of flooding and the need of contour map, geoidal map and 3D surface model use for civil engineering work, environmental monitoring and control, with a comprehensive contour map, geoidal map and 3D surface model of the study area, the environmental monitory agency can easily identify the problematic areas most especially in the raining season in order to provide preventive measures for such occurrences. The products of this research will further be use as tools for developing the area and assist in controlling future developmental plans in the state and the nation in general.

The roots of under development of third world countries, such as Nigeria emanated from a number of factors which include poor quality of data collection, organization and management practices; and, lack of adequate knowledge to develop the area and manage the environment in a sustainable manner. The consequences of all these are obvious from air and water pollution, environmental degradation, diseases and death e.t.c. These are the challenges of surveyors, environmental managers and any other specializations that deal with the management of environment in Bauchi metropolis and Nigeria as a whole. The geoid that was determined

will be useful in geophysical exploration and for geodetic application. In general, the information provided will be used in all aspects of physical developments that concern land in the study area. For instance, for planning purposes, construction of civil engineering work, building engineering, design and construction of works, both on the surface and underground. The research can be apply in water/flood volumes estimation, soil erosion volumes estimation, for geophysical exploration of natural resources, geodetic application and scientific investigations.

Scope of the Study

The study area for the geoidal mapping and 3D surface modelling is restricted to Bauchi metropolis in the north east part of Nigeria. The research work involved data acquisition, field observations and reductions of measurement, data downloading, post processing, geo-processing, and compilation of contour map, geoidal map and 3D surface modelling of the study area.

Study Area

Bauchi state was founded by one Yakubu and it later became a state capital in the year 1976. Bauchi state is located between latitudes 090 30' and 090 50' north of the equator and longitudes 090 50' and 100 20' east of the Greenwich meridian. The total area of Bauchi state is 49,119 km² (18,965sq mi) with the population density 95/km² (250/sq mi) (National Population Commission of Nigeria, 2006). Bauchi state is bordered by seven states; Kano and Jigawa to the north, Plateau and Taraba to the south, Gombe and Yobe to the east and Kaduna to the west. The map of Nigeria is showed in Figure 1.2. Mean daily maximum temperatures range from 29.2°C in July and August to 37.6°C in March and April. The mean daily minimum ranges from about 11.7°C in December and January to about 24.7°C in April and May. The state is drained by several river systems. The dominant one is River Gongola which originates in the Jos Plateau area, southwest of Bauchi State. It traverses, in a southwest-northeast direction through the southern LGAs of the state including Dass, T/Balewa, Bogoro, Bauchi and Kirfi and, thence, to Gombe State. It has numerous headwaters and tributaries within the state. The state comprises several previously

independent powerful Emirates, including, for instance, Bauchi, Ningi, Katagum, Dass, and Duguri. The map of Bauchi state is showed in Figure 1.2. The study area is Bauchi metropolis the capital city of Bauchi state of Nigeria. It lies approximately between 090 40'E to 090 45'E and 090 55N to 100 15'N covering an area of 180km² with the population of 493,810 as at 2006 census and still growing, (National Population Commission of Nigeria, 2006).

Methodology

The methodology adopted for acquiring complete datasets for precise representation of complex surfaces is divided into two stages namely: field work and data processing. The former deals with the equipment setup and data collection, while the latter focused on data manipulation and processing. The basic data include Ellipsoidal heights and orthometric heights which were acquired using Dual Frequency Global Positioning System (DGPS) and precise level respectively. Geoidal heights were derived from the observed ellipsoidal heights and orthometric heights. Geoidal heights (Geoid undulation) were computed from the separation of the reference ellipsoid with the geoid surface measured along the ellipsoidal normal. The Geoidal heights were interpolated by gridding using spline method of interpolation in ArcGIS 10.2.1 to produce the three dimensional surface model, Geoidal map and contour map of the study area. Geoidal and contour maps were therefore produced and the results were finally presented and analyzed for further discussion and conclusion.

Equipment Used

The equipments needed for the research work are as follows:

Hardware Used

DGPS receiver and its accessories to acquired data for ellipsoidal height
Precise level and its accessories to acquired data for orthometric height
Computer and its accessories for computation, processing and analysis

Software Used

ArcGIS 10.2.1 software for interpolation

Microsoft Office excel 2007
Microsoft Office word 2007
Spectrum survey offices (SVO)

Flowchart of Methodology

The flowchart used for data acquisition, field observation, data downloading, post processing, geo-processing, and compilation of contour map, geoidal map and 3D surface model of the study area is presented in Figure 1.0.

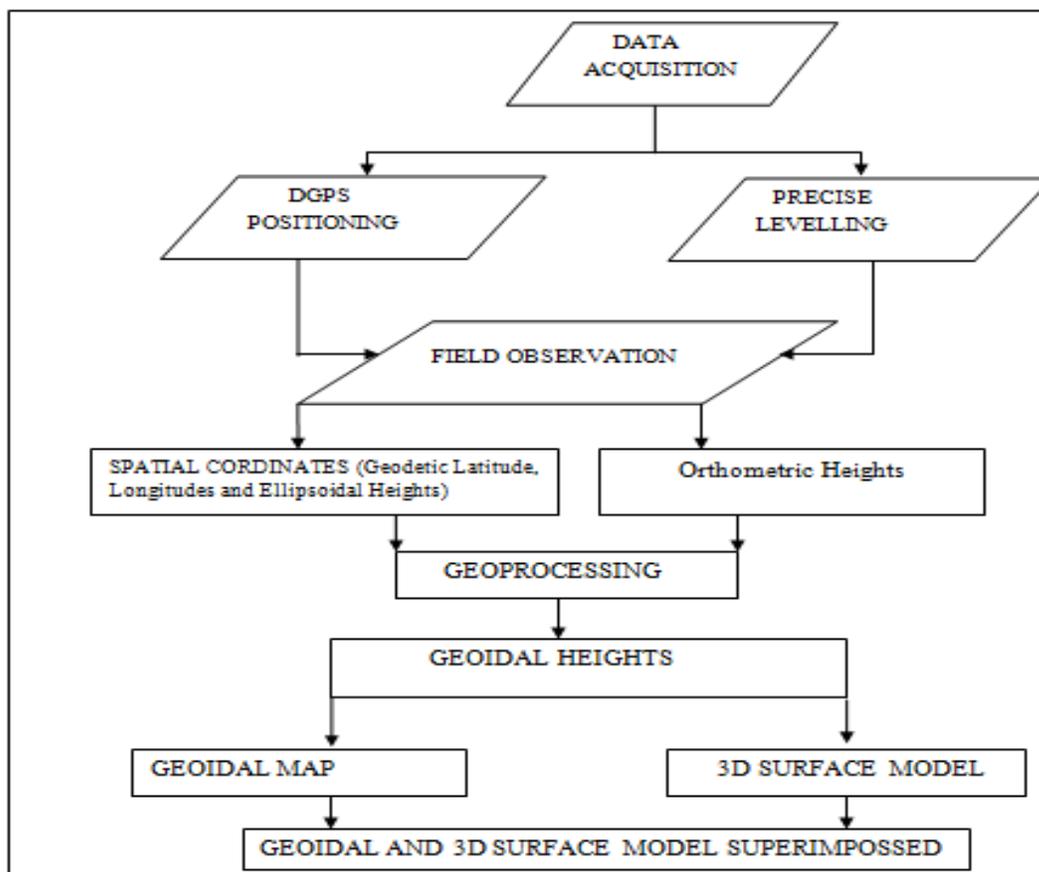


Figure 1.0: Flowchart of the Research Methodology (Zakari, 2020)

Reconnaissance

The factors considered in the reconnaissance include the design of the network and techniques adopted for effective execution of the work. In this research study, office and field reconnaissance were carried out. These are explained as follows;

Field Reconnaissance

Field reconnaissance was carried out to locate suitable positions for control establishment. In preliminary survey the existing control points adopted for connection were determined and in-situ was ascertained. The control points/benchmarks observed were established in the study area (good choice of stations marks). Site inspection played a vital role in facilitating the work and ascertaining the methods applied. The problems of intervisibility between the stations were completely avoided in case of precise levelling. The recce diagram of the study area was produced and control in-situ check was adopted on the existing control points used for connection of the DGPS observation to ascertain the quality of the original control used.

Office Reconnaissance

The equipments and the methods adopted were chosen at this stage. Both the DGPS receiver and the precise level were tested on a control points and reference datum respectively, in order to ascertain their accuracy. The coordinates of the control points used for connection, the orthometric heights of the benchmarks and Bauchi State street guide map was collected from Bauchi State Ministry of lands and Housing. Booking sheet was designed and adopted for all field observations and measurements.

Data Acquisition

The field operations were carried out for the purposes of acquiring the ellipsoidal heights and orthometric heights for a number of well distributed points in the research area. Precise levelling and DGPS field exercises were conducted in this work. The data captured includes latitude (ϕ), longitude (λ) and ellipsoidal heights (h) of all points of interest, using DGPS Receivers and Precise level for orthometric heights (H). These sets of data were obtained from the site by means of direct field observation. A total number of five hundred (500) points monumented by the Bauchi State Ministry of lands and Housing and the Department of Surveying and Geo-informatics, Federal Polytechnic Bauchi were observed and recorded. Additional two hundred (200) points was also monumented and observed. Apart from the pre-cast points, other two hundred and eighty four (284)

points were selected at random and their coordinates were determined and recorded. This becomes necessary due to the topographical nature of the study area.

DGPS Survey Observation

Planning was the first important step for DGPS surveys, so that almanac data can be analyzed to obtain optimal time sets when a geometrically strong array of operating satellites is available above 15° of elevation (above the horizon) and to identify topographic obstructions that may hinder signal reception. Planning software graphically display GDOP (geometric dilution of precision) at each time of the day (GDOP of 7 or below is usually considered suitable for positioning a value of 5 or lower is ideal).

Differential GPS observations were made at the most suitable locations along the levelling routes. The derived coordinates were comparable to GPS standard accuracy. The WGS-84 ellipsoid was adopted as the reference surface for the determination of ellipsoidal heights (h). In analogy the height of a point is defined as the distance from the ellipsoid measured along a normal to the reference ellipsoid. Ellipsoidal heights can be derived from geocentric Cartesian coordinates provided by GPS observations. DGPS receiver was used to determine both the Universal Traverse Mercator (UTM) coordinates and the geographical coordinates of all the required stations. The GPS Fast-static mode of observation was adopted throughout the measurement. The data obtained was recorded into the memory card for post processing. The GPS receiver and spectrum survey office were used, which are suitable for the survey operations.

Precise Levelling Observation

The observations of the precise levelling operation carried out were reduced and processed to obtain the orthometric heights. Orthometric heights (H) refer to an equipotential reference surface, the geoid. The orthometric height of a distinct point on the surface of the earth is the distance from that point to the geoid, measured along the plumb line normal to the geoid. The orthometric heights of points were determined with precise level through levelling.

Determination of Orthometric and Ellipsoidal heights

The reference surface used for the determination of orthometric heights is the geoid while the reference surface used for the determination of ellipsoidal heights regarded as the ellipsoid. Precise levelling operations were carried out on the control points in order to determine the geoid while ellipsoid heights were determined by DGPS observation. The vertical separation between the geoid and the ellipsoid were determined and it known as the geoidal heights. Geoidal heights were used in the production of geoidal map. The orthometric heights and the ellipsoidal were used in the compilation of contour map of the study. The three dimensional surface modelling were produced from derived orthometric and ellipsoidal heights observed with precise level and DGPS respectively.

Data Quality

The quality of data used in this thesis was determined by the validity and reliability of the data. Validity is measured by the precision while the reliability is determined by the accuracy of the data. The quality control test for the data used in this study was carried out by the researcher and the result indicated validity. Before performing a minimally constrained and fully constrained adjustment, the network was analyzed for possible outliers using loop closures analysis of repeated baselines and comparison of known and observed baselines. Detection of blunders was facilitated through the source of blunder (height of instrument, centering errors, etc.), display vectors in northing, easting, ellipsoidal height, and distance (geodetic latitude, longitude, and height). Baselines were processed on daily basis to allowing the user to identify problems that might exist. A list of the triple difference, float double difference, and fixed double difference vectors (dx-dy-dz) were normally listed.

GPS system validation was performed to verify that the complete system achieved accuracy fitting for the types of GPS control surveys used. The validation survey is similar to a production of GPS survey; except that it was carried out on permanent pillars with high accuracy of 3D coordinates. GPS receivers, GPS antennas, field support equipment, baseline processing software, network adjustment software, office staff (for planning, supervision, processing, adjustments, reporting) and field staff (for system

set-up and data collection): All of the above was verified during a GPS validation survey. The results indicated that the validity, reliability and hence quality of the data were satisfactory.

Data Processing

Two sets of data were involved in this research work: the first set of data was obtained using dual Frequency GPS receivers and the second set of the data was obtained using precise level. The GPS observations were post processed using spectrum survey offices software and the final coordinates and the heights of the points within the study area were determined and obtained the three dimensional coordinates (geodetic latitude (ϕ), geodetic longitude (λ) and ellipsoidal height (h)).

The precise levelling data was also processed and reduced to obtain the orthometric height (H) of all the required points. The difference between the ellipsoidal and orthometric heights determined with DGPS and precise level yields geoidal undulations.

Map compilation

The processed data (geoidal undulations) was used for the production of digital contour map, geoidal map and 3D surface model of the study area. There are different types of software in the market for production of contour, digital terrain and three dimensional surface models. In this research work ArcGIS 10.2.1 was used for generating contour, geoidal map and 3D surface model. ArcGIS 10.2.1 is a Contouring and 3D surface mapping software package. It transforms random surveying data, using interpolation, into continuous curved surface contour. ArcGIS 10.2.1's sophisticated interpolation engine will transform all XYZ data into publication quality maps. The methods were grouped into smoothing and exact interpolators. Smoothing interpolators are: Inverse Distance to a Power, Kriging, Polynomial Regression, Radial Basis Function, Spline, Modified Shepard's Method, Local Polynomial, Moving Average; while the exact interpolators are: Inverse Distance to a Power, Kriging, Nearest Neighbor, Radial Basis Function, Modified Sheppard's Method, Triangulation with Linear Interpolation, and Natural Neighbor. But Spline

methods of interpolation were used to approximate the geoid in the study area.

RESULTS AND DISCUSSION

Validity and Reliability Test

DGPS and precise levelling were carried out in line with, specifications for second order accuracy. The data were checked and the mean of the height differences were taken as the most probable value of the measurements. The statistical analysis of the result met the precise of second order geodetic control network and levelling specifications. The result of the pearman correlation coefficient computed using northing and easting coordinates is 0.054 and the coefficient of determination is 0.0029% which indicated that the control points were normally distributed. The result of the Z test computed (0.125) indicated that the measurements are precise at 95% confidence level. The reliability of the measurement of X and Y second order geodetic coordinates were computed at the scale of 1:5000 and found reliable at 95% confidence level. Therefore, the quality of the data was guaranteed as showed in Table 1.1.

Table 1.1: Validity and Reliability Test

Method	Software Used	Variables	Result	Remark
Correlation Pearson (Sig. 2-tailed), N= 916	Microsoft excel, 2010	Var00001 Var00002	0.054 0.103	Weak correlation
Reliability test	SPSS	Var00001 Var00002	Case: Scale: 1:5000 Valid:916 (100%) Excluded: 0 Total:(916) 100%	The reliability of the coordinates were computed at the scale of 1:5000 and found reliable at 95% confidence level.

Z-test (Sig. 2-tailed), N= 916	SPSS	Var00001 Var00002	Case: α : 0.05 Z: 0.125	The result of the Z-test computed (0.125) indicated that the measurements are precise at 95% confidence level.
Precision test (Sig. 2-tailed), N= 916	Spectrum survey office	Horizontal and Vertical control	Case: α : 0.05	The accuracy computed is 0.002m precision obtained on Hz plane and 0.001m precision obtained on vertical plane at 95% confidence level

Orthometric and Ellipsoidal Heights Determined and Compared

The transformation between the ellipsoidal heights and the orthometric heights was successfully done and geoidal undulation (N) were determined. The ellipsoidal height (h) and the orthometric height (H) were transformed by subtracting the geoid-ellipsoid separation, which is called geoid heights. Table 1.2 showed the sample of the results of the ellipsoidal and orthometric heights determined from the field. The ellipsoidal heights, orthometric heights and geoidal heights derived were used in the compilation of contour map, 3D surface model and geoidal map of the study area respectively. The geoidal undulation determined proved the theory that the vertical separation between geoid and reference ellipsoid is ± 100 m.

Table 1.2: Sample of Coordinates in UTM and Geoidal Heights from the Field

PILLER ID	EASTING (M)	NORTHING (M)	ELLIPSOIDAL HEIGHT (M)	ORTHOMETRIC HEIGHT (M)	GEIODAL HEIGHT (N) (M)
BA/SC A001	590272.076	1149941.615	631.2754	608.4822	22.7934
BA/SC A002	590347.560	1149587.566	627.4385	604.6680	22.7705
BA/SC A003	590630.023	1147949.555	617.9043	595.1340	22.7703
BA/SC A004	590823.989	1147613.867	623.7726	600.9910	22.7816
BA/SC A005	591265.472	1147062.809	617.3684	594.6100	22.7584
BA/SC A006	591395.051	1146527.667	619.1873	596.4290	22.7583
BA/SC A007	591555.206	1146140.217	632.0442	609.2860	22.7582
BA/SC A008	591590.524	1145830.944	632.2494	609.4910	22.7584
BA/SC A009	591731.936	1145413.177	632.9763	610.2180	22.7583
BA/SC A010	591809.382	1145160.880	636.0115	613.2530	22.7585
BA/SC A011	592396.177	1143857.553	650.6051	627.8260	22.7791
BA/SC A012	592774.826	1143966.743	646.2632	623.4840	22.7792
BA/SC A013	593129.503	1144077.509	646.5163	623.7610	22.7553

BA/SC A014	593902.825	1144124.718	654.3516	631.5980	22.7556
BA/SC A015	594590.563	1144112.597	653.8854	631.1100	22.7554
BA/SC A016	594947.966	1144082.290	650.2717	627.5490	22.7227
BA/SC A017	595287.237	1144062.160	649.3884	626.6660	22.7224
BA/SC A018	596266.843	1144118.378	636.9553	614.2330	22.7223
BA/SC A019	597197.115	1144278.835	625.1712	602.4720	22.6992
BA/SC A020	597628.428	1144528.347	620.3988	597.6990	22.6998
BA/SC A021	597984.162	1144842.818	619.0397	596.3400	22.6997
BA/SC A022	598643.209	1145444.136	616.6122	593.9360	22.6762
BA/SC A023	598982.890	1145771.887	609.7292	587.0530	22.6765
BA/SC A024	599499.223	1146225.203	606.1374	583.4610	22.6764
BA/SC A025	593928.789	1143964.028	655.0782	632.3230	22.7552
BA/SC A026	593927.641	1143739.841	655.9693	633.2140	22.7553
BA/SC A027	594034.588	1143566.951	656.3096	633.5540	22.7556

Sample Profile of the Orthometric and Ellipsoidal Heights

The vertical and horizontal axes represent the heights and the position of the beacons respectively. The vertical axis (heights) was plotted against horizontal axis (beacons ID) as indicated in Figure 1.1. The profile showed that both ellipsoidal and orthometric heights portrayed same terrain. The Pearson correlation computed is 0.993 and the coefficient of determination is 99% which indicated that the ellipsoidal and the orthometric heights are positively highly correlated. The profiles of both the orthometric and ellipsoidal heights are represented by dark green and blue colour respectively as showed below. It was observed that due to non-parallelism of equipotential surfaces, points with same Orthometric heights or same ellipsoidal heights are not on the same equipotential surface. That is the gravitation potential at those points is not the same even if the points has same Orthometric heights or same ellipsoidal heights.

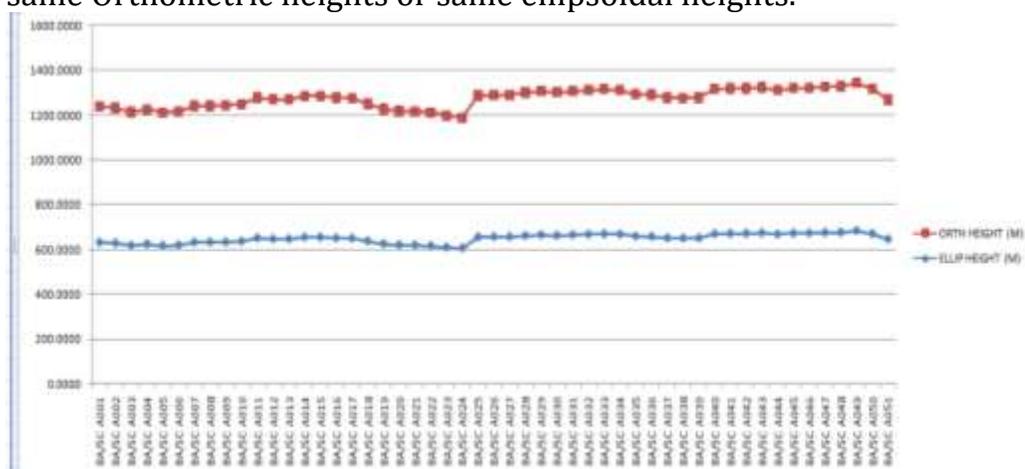


Figure 1.1: Profile of Orthometric and Ellipsoidal Heights (Author's Lab.)

Sample Profile of the Geoidal Heights

The sample of the X, Y coordinates determined and the three heights system tabulated on Table 1.2. The vertical axis (geoidal heights) was plotted against horizontal axis (beacons position) as indicated on Figure 1.2. The blue line represents geoidal undulation and the mean of the geoidal heights was 22.277m which could be used as a constant to model the entire geoid of the study area and Bauchi State in general. This research proved the theory that the geoid as an equipotential surface of the Earth's gravity field which best fits the mean sea level is irregular due to the uneven distribution of the Earth's masses (C.F. Gauss in 1828).

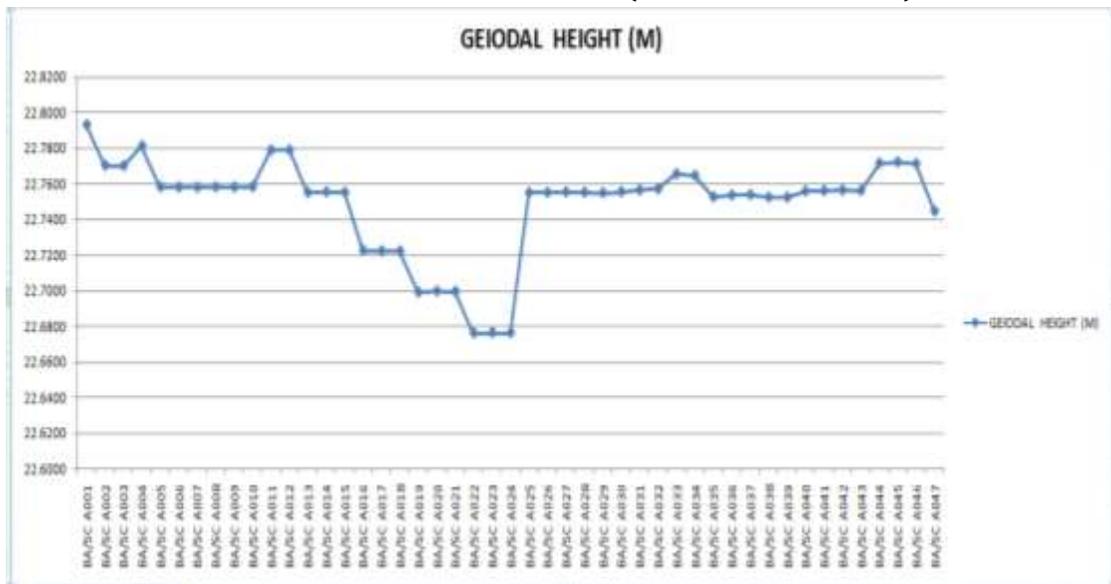


Figure 1.2: Profile of Geoidal Heights (Author's Lab.)

Compilation of Three Dimensional Surface Models

The ellipsoidal and orthometric heights were used in the production of three dimensional surface models as shown in Figure 1.3 and Figure 1.4 respectively. The three dimensional surface models were produced at the scale of 1:60,000 and they represent the terrain configuration. The 3D surface models produced from ellipsoidal and orthometric heights showed that they follow the same pattern, which is an indication that the two surfaces are true representation of the same terrain. The legend showed heights range on each of the 3D surface model which are represented by different colour ramp.

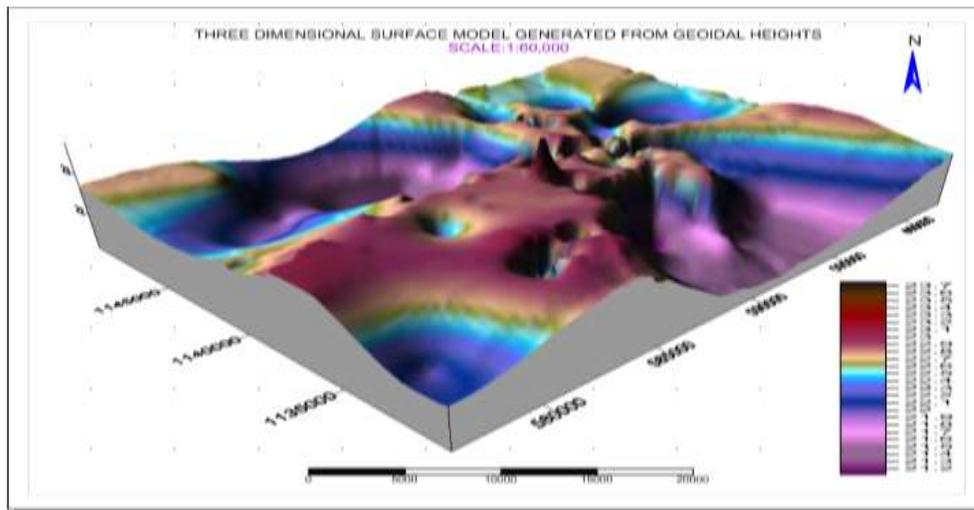


Figure 1.3: Three Dimensional Surface Model Generated from Geoidal Heights (Author’s Lab.)

Compilation of Contour Map

The spot heights were randomly observed in the study area and used for the determination of the geoidal heights. The geoidal heights were used in the production of contour map as showed in Figure 1.4. The contour map was produced at a scale of 1:60,000 and the contour lines were interpolated at 0.2 meter contour interval which represented the terrain configuration. The contour map produced from geoidal heights portrayed the geoid-ellipsoid separation.

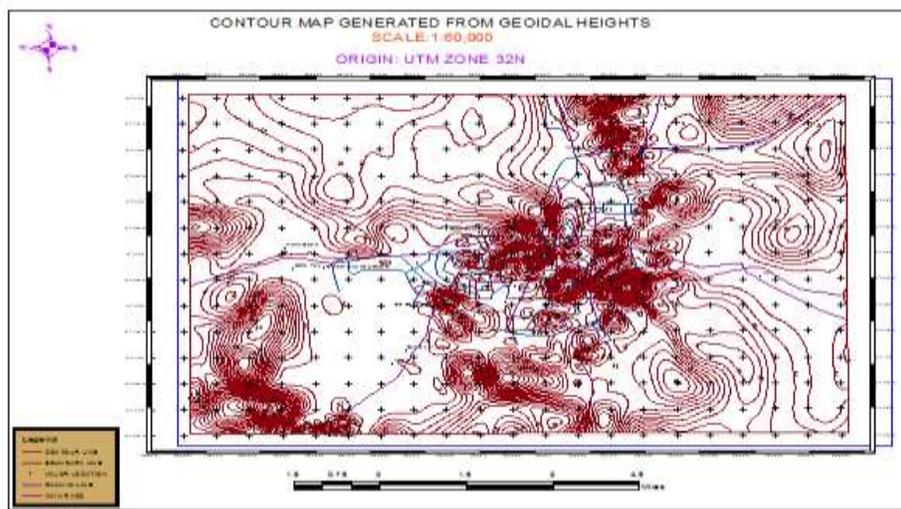


Fig. 1.4 Contour map Generated from Geoidal Heights (Author’s Lab.)

Production of Geoidal Map

The Geoidal heights were used in the production of Geoidal Map as indicated in Figure 1.5. The Geoidal Map was produced at the scale of 1:60,000 and it represents the geoidal configuration. The legend showed heights range on the geoidal map which are represented by different colour ramp.

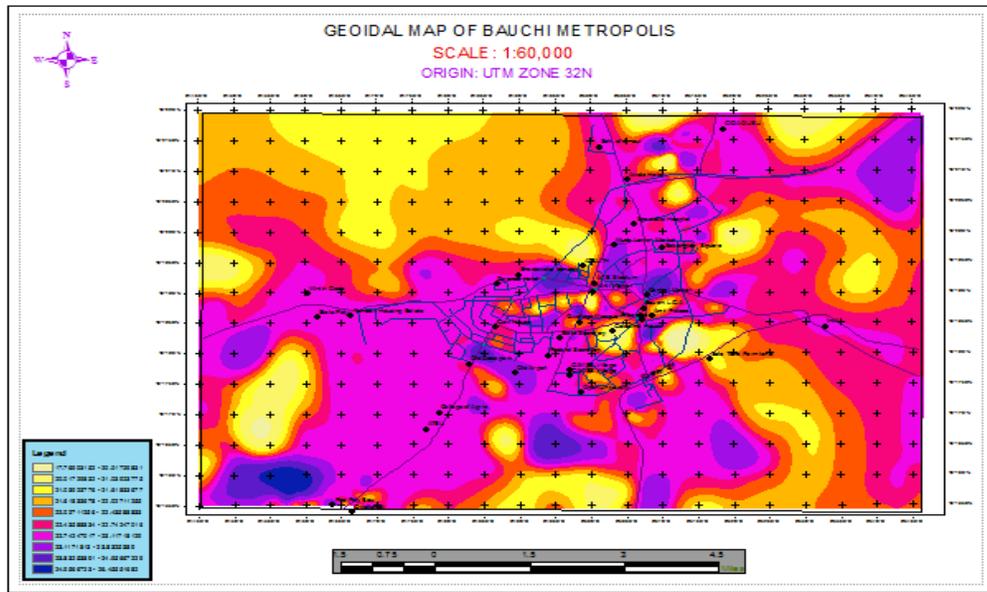


Figure 1.5: Geoidal Map Generated from Geoidal Heights (Author's Lab.)

Summary

In the determination of Geoidal undulation for production of geoidal map of Bauchi metropolis, the coordinates, ellipsoidal and orthometric heights were determined with the aid of Dual Frequency GPS receiver (DGPS) and precise Level Wild N3 instruments respectively. The heights determined with precise Levelling were reduced and earth surface-geoid separation were determined (orthometric heights). The ellipsoidal heights were determined with DGPS. The DGPS coordinates obtained and heights determined were post processed using the Spectrum survey offices software and the final adjusted coordinates and heights were determined. Geoidal Heights of the study area were obtained from the differences between the orthometric and the ellipsoidal heights. The heights determined were exported from Microsoft office excel 2007 to ArcGIS 10.2.1 version. Shape files were created for each layer and were used in the

production of the maps. The ellipsoidal and orthometric heights were also used for the production of contour map and digital geoidal model was created using ArcGIS 10.2.1 version. The geoidal undulations determined were used in the compilation of the Geoidal map of the study area.

Conclusion

In conclusion, levelled heights were established along with DGPS observations in Bauchi metropolis to unify the height system. The orthometric and ellipsoidal heights were determined using DGPS receiver and precise Level. The differences between orthometric and ellipsoidal heights were determined (Geoidal Undulation). The orthometric and ellipsoidal heights were used for the compilation of the contour map. Geoidal Undulations were used for the production of Geoidal map of the study area. The Geoidal Model and the 3D surface geoidal model were produced using ArcGIS 10.2.1 version.

Recommendations

In view of the foregoing results, it is therefore recommended that:

This research should be repeated using observations with Differential Global Positioning System in full static mode with more time spent on each station and geodetic level equipment to see if the accuracy of the result could be improved.

Other researchers should use the geoidal heights determined to produce a mathematical model of the study area and the Nigeria as a whole

Further research work should be under taking to simulate 3D model of the entire Bauchi State which could be utilize for densifying levelling networks of lower order for appropriate future planning.

The Nigeria government should make efforts towards the production of a National Geoid Model through the office of the Surveyor General of the Federation in order to keep in pace with other developing countries.

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