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MATHEMATICAL MODELING OF FILTRATION COMBUSTION WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY AND DIFFUSION COEFFICIENT IN A WET POROUS MEDIUM

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Abstract

This paper investigates the effect of temperature dependent thermal conductivity and diffusion coefficient on the filtration combustion in a wet porous medium. The model which relies on several assumptions and based on the conservation of total mass, chemical species and energy written in transient state mode of operation which governed the phenomenon is presented. The properties of solution were investigated. The coupled nonlinear governing equations were solved simultaneously for the temperature and concentration field analytically via parameter expanding method, direct integration and eigenfunction expansion technique. The influence of dimensionless parameter such as scaled thermal conductivity λ_1*, species diffusion coefficient* D_1*, Frank kamenetskii parameter* δ *peclet mass number* p_{em} *on the filtration combustion was investigated. It is established that the maximum temperature is attained when* δ *=0.5 for fixed time t. Simulation results also revealed that high temperature front created by combustion; the oxygen molar fraction, vapor molar fraction, passive gas molar fraction, molar concentration of the solid fuel and molar concentration of liquid depend appreciably on the values of the parameters involved.*

Keywords: *Analytical Solution, Filtration Combustion, Combustion, Porous medium, Eigenfunction expansion technique.*

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Introduction

Air injection leading to in situ combustion is generally considered applicable to recovery of heavy oils because it causes a significant reduction in oil viscosity. However, it can also be used to recover light oils by mechanisms such as combustion gas drive recovery, distillation and thermal expansion. The air injection process usually refers to high pressure air injection (HPAI), whereas the term in situ combustion traditionally has been used for heavy oil reservoirs (Negar *et al.,* 2014). The method of air injection has also been reported to increase recovery rates of light oils (Negar *et al.*, 2015) in this case; thermal expansion and gas drive promoted by the oxidation reaction are responsible for enhancing the recovery of oil. The reaction that takes place between light oil and injected oxygen occurs at lower temperatures, bounded by the boiling point; it is termed low temperature oxidation (LTO). Aldushin *et al.* (1997) described Filtration combustion as the propagation of exothermic reaction waves in a porous medium through which there is gas filtration. The porous solid is composed of both reactive and inert components. Filtration combustion covers a wide range of natural and technological combustion processes in porous media having a common mechanism of reaction front propagation. The principal feature of this mechanism is the delivery of gaseous reactants to the reaction front by filtration from the surrounding environment, where it reacts with the solid reactants. Filtration can be caused by two different mechanisms, referred to as forced and natural. In the former case an external force pushes the gas into the porous matrix, and is often used in technological processes while in the natural filtration combustion, the gas flow is induced by combustion process itself, which is due to consumption of gas in the reaction. Since last few decades, Filtration Combustion has been studied extensively; these include the work of Olayiwola (2015) who formulated a model for forward propagation of a combustion front through a porous medium with reaction involving oxygen and a solid fuel. Dependence of thermal conductivity and diffusion coefficient on temperature and gas composition was neglected. Existence and uniqueness of solution of the model was proved by actual solution method and he showed that temperature is a non-decreasing function of time. The

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system of partial differential equations, describing the problem under consideration was transform into a boundary value problem of coupled ordinary differential equation and the numerical technique was used to solve the reduced system. He observed that heat transfer and species consumption are significantly influence by the Frank-kamenetskii number. Grigori *et al*. (2012) studied the asymptotic approximation of long time solution for low temperature filtration combustion by considering a combustion process when air is injected into a porous medium containing immobile fuel and inert gas. They focus on the case when the reaction is active for all temperatures, but heat losses were neglected and developed a method for computing the traveling wave profile in the form of an asymptotic expansion and derived its zero-order approximation. Numerical simulations were performed in order to validate the asymptotic formulae. Chapiro and Marchesin (2015) studied the effect of thermal losses on traveling waves for in-situ combustion in porous medium. The purpose of their research was to identify waves that arise in one-dimensional models of combustion in porous media, and to understand how the waves fit together in solutions of Riemann problems. Diffusion effects and the dependence of gas density on temperature was disregard. They simplify the proof of uniqueness and existence of the travelling wave solution. Michael and Janet (1999) developed a model of filtration combustion in a packed bed by investigating the low velocity filtration combustion reaction of lean methane/air mixtures flowing through a packed bed and compare to experimental results. The reaction is represented with a complete methane/air kinetic mechanism. Their results for solid temperature agree with the experiments for a mixture with an equivalence ratio 0.15 which is consistent with the existing theory on filtration combustion and discovered that gas-phase transport is not important to wave propagation at this condition. They discovered that gas-phase dispersion is important only at higher equivalence ratios. Mailybaev *et al.* (2013) formulated a model for recovery of light oil by medium temperature oxidation. They considered two phase flow possessing a combustion front when a gaseous oxidizer (air) is injected into porous rock filled with light oil. The temperature of the medium is bounded by the

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boiling point of the liquid and, thus, relatively low. They disregarded the gas phase reactions. They observed that the initial period, the recovery curve is typical of gas displacement but after a critical amount of air has been injected the cumulative oil recovery increases linearly until all oil has been recovered, they conclude that oil recovery is independent of reaction rate parameters but recovery is much faster than for gas displacement and among their findings is that oil recovery is faster when the injected pressure is higher. Bruining *et al.* (2009) developed a model of filtration combustion in wet porous medium. By considering a porous rock cylinder thermally insulated on the side filled with inert gas, liquid and solid fuel. An oxidizer was injected. They assumed that the amount of liquid is small, so its mobility is negligible, and that only a small part of the available space is occupied by solid fuel and liquid, so that changes of rock porosity in the reaction, evaporation, and condensation processes can be neglected. They neglected the dependence of thermal conductivity and diffusion coefficients on the temperature and gas compositions. In our case, we incorporating temperature dependent thermal conductivity and diffusion coefficient. We made the same assumption with Bruining *et al.* (2009) and the aim is to provide an analytical solution describing the phenomena. We provided the criteria for the existence and uniqueness of solution of the equations, examine the properties of solution and provided the analytical solution of the model by parameter expanding, direct integration and eignfunction expansion methods.

Model Formulation

Following Bruining *et al. (*2009), we consider a porous rock cylinder thermally insulated on the side and filled with vaporizable liquid, inert gas, and combustible solid fuel. An oxidizer (air) is injected. The liquid can be water or light oil, and the combustible solid can be coke. We assume that the amount of liquid is small, so its mobility is negligible. We assume that only a small part of the available space is occupied by solid fuel and liquid, so that we can neglect changes of rock porosity in the reaction, evaporation, and condensation processes. We assume that the solid, gas, and liquid are in local thermal equilibrium, so they have the same

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temperature. Based on the above assumptions, a one-dimensional model with time t and space coordinate x is considered. The energy equation governing the system is giving by equation (1):

$$\left.\begin{array}{l}\rho c_g \frac{\partial}{\partial t}(T - T_{res}) + \rho c_g u \frac{\partial}{\partial x}(T - T_{res}) = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + Q_r K_r Y n_f\, e^{-\frac{E_r}{RT}} - \\ Q_e k n_l \left(\frac{P_{atm}}{\rho RT} e^{-\frac{Q_e}{R}\left(\frac{1}{T} - \frac{1}{T_b}\right)} - X\right)\end{array}\right\} \quad (1)$$

We consider a single component liquid (water), and denote by X its vapor molar fraction in the gas phase (mole of vapor/mole of gas). The gas has several components: oxygen, vapor, and passive (inert and combusted) gas. We denote the molar fractions of oxygen and passive gas in the gas-phase by Y and Z, respectively. Then, we write the mass balance equations for the components X, Y, Z as equation (2) to equation (4):

$$\phi\rho\left(\frac{\partial X}{\partial t} + u \frac{\partial X}{\partial x}\right) = \phi \frac{\partial}{\partial x}\left(\rho D_X \frac{\partial X}{\partial x}\right) + k n_l \left(\frac{p_{atm}}{\rho RT} e^{-\frac{Q_e}{R}\left(\frac{1}{T} - \frac{1}{T_b}\right)} - X\right) \quad (2)$$

$$\phi\rho\left(\frac{\partial Y}{\partial t} + u \frac{\partial Y}{\partial x}\right) = \phi \frac{\partial}{\partial x}\left(\rho D_Y \frac{\partial Y}{\partial x}\right) - \mu_o K_r Y n_f\, e^{-\frac{E_r}{RT}} \quad (3)$$

$$\phi\rho\left(\frac{\partial Z}{\partial t} + u \frac{\partial Z}{\partial x}\right) = \phi \frac{\partial}{\partial x}\left(\rho D_Z \frac{\partial Z}{\partial x}\right) + \mu_g K_r Y n_f\, e^{-\frac{E_r}{RT}} \quad (4)$$

As the solid fuel and the liquid do not move, their concentrations satisfy the equations for reaction and evaporation respectively giving by equation (5) to equation (6):

$$\frac{\partial n_f}{\partial t} = \mu_f K_r Y n_f\, e^{-\frac{E_r}{RT}} \quad (5)$$

$$\frac{\partial n_l}{\partial t} = - k n_l \left(\frac{p_{atm}}{\rho RT} e^{-\frac{Q_e}{R}\left(\frac{1}{T} - \frac{1}{T_b}\right)} - X\right) \quad (6)$$

Where; ρ [mole/m^3] is the molar density of gas, T [k] is the temperature of the reservoir when heated, T_{res} is the initial reservoir temperature, c_g is the heat capacity of rock, u [m/s] is the Darcy velocity of gas, λ [w/mk] is

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thermal conductivity of the porous medium, (Q_r and Q_e)[J/mole] are the heats enthalpies of combustion and evaporation of the solid and the liquid at reservoir temperature, K_r [1/s] is the pre exponential parameter, Y is the molar fraction of oxygen, X is the vapor molar fraction in the gas phase (mole of vapo/mole of gas), Z is the molar fraction of passive gas in the gas-phase, n_f Is the molar concentration of solid fuel, n_l Is the molar concentration of liquid, E_r [J/mole] is activation energe, μ_f is the moles of solid fuel , μ_o is the moles of oxygen , μ_g is the moles of gaseous product, $R = 8.314$[J/mole k] is the ideal gas constant, T_b is the boiling temperature of the liquid at atmospheric pressure p_{atm}, ϕ is the porosity, D_x [m²/s] is the diffusion coefficients for vapor of porous medium, D_y [m²/s] is the diffusion coefficients for oxygen of porous medium, D_z [m²/s] is the diffusion coefficients for passive gas in the gas-phase of porous medium,

Coordinate Transformation

The balance of mass can be eliminated by the means of streamline function (Olayiwola, 2015) giving by equation (7)

$$\eta(x.t) = \left(\rho^2\right)^{-\frac{1}{2}} \int_0^x \rho(x,t) ds \tag{7}$$

Then coordinate transformation is giving by equation (3.8) to (3.9)

$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial}{\partial \eta} \tag{8}$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial t} = -u\frac{\partial}{\partial \eta} + \frac{\partial}{\partial t} \tag{9}$$

We make the additional assumptions that ρc_g, ρD_X, ρD_Y, ρD_Z and λ are constant. Although these assumptions could be relaxed in the future, they considerably simplify the equations. Then equations (1) to equation (6) can be simplified as:

$$\rho c_g \frac{\partial}{\partial t}(T - T_{res}) = \frac{\partial}{\partial \eta}\left(\lambda \frac{\partial T}{\partial \eta}\right) + Q_r K_r Y n_f e^{-\frac{E_r}{RT}} - Q_e k n_l \left(\frac{P_{atm}}{\rho RT} e^{-\frac{Q_e}{R}\left(\frac{1}{T} - \frac{1}{Tb}\right)} - X\right) \tag{10}$$

$$\phi\rho\frac{\partial X}{\partial t} = \phi\frac{\partial}{\partial \eta}\left(\rho D_X \frac{\partial X}{\partial \eta}\right) + k n_l \left(\frac{p_{atm}}{\rho RT} e^{-\frac{Q_e}{R}\left(\frac{1}{T} - \frac{1}{T_b}\right)} - X\right) \tag{11}$$

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$$\phi\rho\frac{\partial Y}{\partial t}=\phi\frac{\partial}{\partial\eta}\left(\rho D_Y\frac{\partial Y}{\partial\eta}\right)-\mu_o K_r Y n_f\, e^{-\frac{E_r}{RT}} \qquad (12)$$

$$\phi\rho\frac{\partial Z}{\partial t}=\phi\frac{\partial}{\partial\eta}\left(\rho D_Z\frac{\partial Z}{\partial\eta}\right)+\mu_g K_r Y n_f\, e^{-\frac{E_r}{RT}} \qquad (13)$$

The initial and boundary conditions were formulated as follows:
Initial condition
At $t=0$ and $\forall\eta$

$$\left.\begin{array}{l} T=\dfrac{RT_0^{\,2}}{E}\left(1-\dfrac{\eta}{L}\right)+T_0,\\ X=X_0\left(1-\dfrac{\eta}{L}\right),\qquad Y=Y_0\left(1-\dfrac{\eta}{L}\right),\\ Z=Z_0\left(1-\dfrac{\eta}{L}\right),\qquad n_f=n_{fres,}\qquad n_l=n_{lres}\end{array}\right\} \qquad (14)$$

Boundary Condition

$$\left.\begin{array}{ll} T|_{\eta=0}=T_1, & T|_{\eta=l}=T_0\\ Y|_{\eta=0}=Y_{inj}, & Y|_{\eta=l}=0\\ X|_{\eta=0}=0, & X|_{\eta=l}=0\\ Z|_{\eta=0}=0, & Z|_{\eta=0}=0\end{array}\right\} \qquad (15)$$

3. Method of Solution

We let $D_x=D_y=D_z=\dfrac{\lambda}{\rho c_g}=T_{res}$ = constant, then the solutions of equations (10) to (15) are:

$$\left.\begin{array}{l} T(\eta,t)=\dfrac{1}{\mu_0}\left(A_1+\dfrac{\eta}{L}(B-A_1)+\displaystyle\sum_{n=1}^{\infty}\dfrac{2}{n\pi}\big((-1)^n-(A+B)((-1)^n-1)\big)e^{-D\left(\frac{n\pi}{L}\right)^2 t}\sin\dfrac{n\pi}{L}\eta\right.\\ \left.-\left(\dfrac{\phi Q_e\mu_0}{c_g}X+\dfrac{\phi(Q_r+\mu_g)}{c_g}Y+\dfrac{\phi\mu_0}{c_g}Z\right)\right)\end{array}\right\} \qquad (16)$$

$$\left.\begin{array}{l} X(\eta,t)=\\ \dfrac{c_g}{\phi Q_e\mu_0}\left(A_1+\dfrac{\eta}{L}(B-A_1)+\displaystyle\sum_{n=1}^{\infty}\dfrac{2}{n\pi}\big((-1)^n-(A+B)((-1)^n-1)\big)e^{-D\left(\frac{n\pi}{L}\right)^2 t}\sin\dfrac{n\pi}{L}\eta\right.\\ \left.-\left(\mu_0 T+\dfrac{\phi(Q_r+\mu_g)}{c_g}Y+\dfrac{\phi\mu_0}{c_g}Z\right)\right)\end{array}\right\} \qquad (17)$$

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$$Y(\eta,t)= \left\{ \begin{array}{l} \dfrac{c_g}{\phi(Q_r+\mu_g)}\left(A_1+\dfrac{\eta}{L}(B-A_1)+\sum_{n=1}^{\infty}\dfrac{2}{n\pi}\left((-1)^n-(A+B)\left((-1)^n-1\right)\right)e^{-D\left(\frac{n\pi}{L}\right)^2 t}\sin\dfrac{n\pi}{L}\eta \right. \\ \left. -\left(\mu_0 T+\dfrac{\phi Q_e\mu_0}{c_g}Y+\dfrac{\phi\mu_0}{c_g}Z\right)\right) \end{array} \right\} \quad (18)$$

$$Z(\eta,t)= \left\{ \begin{array}{l} \dfrac{c_g}{\phi\mu_0}\left(A_1+\dfrac{\eta}{L}(B-A_1)+\sum_{n=1}^{\infty}\dfrac{2}{n\pi}\left((-1)^n-(A+B)\left((-1)^n-1\right)\right)e^{-D\left(\frac{n\pi}{L}\right)^2 t}\sin\dfrac{n\pi}{L}\eta \right. \\ \left. -\left(\mu_0 T+\dfrac{\phi Q_e\mu_0}{c_g}X+\dfrac{\phi(Q_r+\mu_0)}{c_g}Y\right)\right) \end{array} \right\} \quad (19)$$

Where

A= $\left(\dfrac{\phi Q_e\mu_0}{c_g}X_0+\dfrac{\mu_0 RT_0^2}{}+\dfrac{\phi(Q_r+\mu_g)Y_0}{c_g}+\dfrac{\phi\mu_0}{c_g}Z_0\right)$, $B=\mu_0 T_0$,

$$A_1=\left(\mu_0 T_1+\frac{\phi(Q_r+\mu_g)}{c_g}Y_{inj}\right)$$

The dependence of thermal conductivity and diffusion coefficient on the temperature is taken into account by the mathematical expression giving by equation (20) and equation (21):

$$\lambda=\lambda_0\left(\frac{T}{T_0}\right) \quad (20)$$

$$D=D_0\left(\frac{T}{T_0}\right) \quad (21)$$

Where λ_0 is the initial thermal conductivity, D_0 is the initial diffusion coefficient, and T_0 is the initial temperature of the medium.

Substituting equation (16) to (21) into equations (5) and equation (6), (10) to (15), we have

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$$\rho c_g \frac{\partial}{\partial t}(T - T_{res}) = \lambda_0 \frac{\partial}{\partial \eta}\left(\frac{T}{T_0}\frac{\partial T}{\partial \eta}\right) + Q_r K_r n_f \left\{ \begin{array}{l} \left(\frac{c_g}{\phi(Q_r + \mu_g)}\left(A_1 + \frac{\eta}{L}(B - A_1) + \sum_{n=1}^{\infty}\frac{2}{n\pi}\left((-1)^n - (A+B)\left((-1)^n - 1\right)\right)e^{-D\left(\frac{n\pi}{L}\right)^2 t}\sin\frac{n\pi}{L}\eta - \left(\mu_0 T + \frac{\phi Q_e \mu_0}{c_g}X + \frac{\phi\mu_0}{c_g}Z\right)\right)\right)e^{-\frac{E_r}{RT}} - \\ k n_l \left(\frac{p_{atm}}{\rho RT}e^{-\frac{Q_e}{R}\left(\frac{1}{T} - \frac{1}{Tb}\right)} - X\right) \end{array} \right\} \quad (22)$$

$$\phi\rho\frac{\partial X}{\partial t} = \phi\rho D_0 \frac{\partial}{\partial \eta}\left(\frac{T}{T_0}\frac{\partial X}{\partial \eta}\right) + k n_l\left(\frac{p_{atm}}{\rho RT}e^{-\frac{Q_e}{R}\left(\frac{1}{T} - \frac{1}{Tb}\right)} - X\right) \quad (23)$$

$$\phi\rho\frac{\partial Y}{\partial t} = \phi\rho D_0 \frac{\partial}{\partial \eta}\left(\frac{T}{T_0}\frac{\partial Y}{\partial \eta}\right) - \mu_0 K_r n_f \left\{\left(\frac{c_g}{\phi(Q_r + \mu_g)}\left(A_1 + \frac{\eta}{L}(B - A_1) + \sum_{n=1}^{\infty}\frac{2}{n\pi}\left((-1)^n - (A+B)\left((-1)^n - 1\right)\right)e^{-D\left(\frac{n\pi}{L}\right)^2 t}\sin\frac{n\pi}{L}\eta - \left(\mu_0 T + \frac{\phi Q_e \mu_0}{c_g}X + \frac{\phi\mu_0}{c_g}Z\right)\right)\right)e^{-\frac{E_r}{RT}}\right\} \quad (24)$$

$$\phi\rho\frac{\partial Z}{\partial t} = \phi\rho D_0 \frac{\partial}{\partial \eta}\left(\frac{T}{T_0}\frac{\partial Z}{\partial \eta}\right) + \mu_g K_r n_f \left\{\left(\frac{c_g}{\phi(Q_r + \mu_g)}\left(A_1 + \frac{\eta}{L}(B - A_1) + \sum_{n=1}^{\infty}\frac{2}{n\pi}\left((-1)^n - (A+B)\left((-1)^n - 1\right)\right)e^{-D\left(\frac{n\pi}{L}\right)^2 t}\sin\frac{n\pi}{L}\eta - \left(\mu_0 T + \frac{\phi Q_e \mu_0}{c_g}X + \frac{\phi\mu_0}{c_g}Z\right)\right)\right)e^{-\frac{E_r}{RT}}\right\} \quad (25)$$

$$\phi\rho\frac{\partial n_f}{\partial t} = \mu_f K_r n_f \left\{\left(\frac{c_g}{\phi(Q_r + \mu_g)}\left(A_1 + \frac{\eta}{L}(B - A_1) + \sum_{n=1}^{\infty}\frac{2}{n\pi}\left((-1)^n - (A+B)\left((-1)^n - 1\right)\right)\times e^{-D\left(\frac{n\pi}{L}\right)^2 t}\sin\frac{n\pi}{L}\eta - \left(\mu_0 T + \frac{\phi Q_e \mu_0}{c_g}X + \frac{\phi\mu_0}{c_g}Z\right)\right)\right)e^{-\frac{E_r}{RT}}\right\} \quad (26)$$

Non – dimensionalization

Here we shall non-dimensionalized equation (22) to equation (26) using the following dimensionless variables as:

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$$\left.\begin{array}{l} X^1 = \dfrac{X}{X_0},\quad Y^1 = \dfrac{Y}{Y_0},\quad Z^1 = \dfrac{Z}{Z_0},\quad \theta = \dfrac{E}{RT_0}\left(T - T_0\right), \\ t^1 = \dfrac{t}{t_0},\quad \eta^1 = \dfrac{\eta}{L},\quad n_f{}^1 = \dfrac{n_f}{n_{fres}},\quad n_l{}^1 = \dfrac{n_l}{n_{lres}},\quad \varepsilon = \dfrac{RT_0}{E} \end{array}\right\} \tag{27}$$

an we obtain the dimensionless equations together with initial and boundary conditions as:

$$\left.\begin{array}{l} \dfrac{\partial\theta}{\partial t} = \lambda_1 \dfrac{\partial}{\partial\eta}\left((1+\varepsilon\theta)\dfrac{\partial\theta}{\partial\eta}\right) + \delta\left(a_1\left(A_1 + (B - A_1)\eta + \sum_{n=1}^{\infty} B_1\, e^{-P_{em}n^2\pi^2 t} \sin n\pi\eta - \right.\right. \\ \left.\left.(b_1(1+\varepsilon\theta) + a_2 X + b_2 Z)\right)\right) n_f\, e^{\frac{\theta}{1+\varepsilon\theta}} - \alpha\left(a_3 \dfrac{e^{-a\left(\frac{b-\theta}{1+\varepsilon\theta}\right)}}{1+\varepsilon\theta} - X\right) \end{array}\right\} \tag{28}$$

$$\frac{\partial X}{\partial t} = D_1 \frac{\partial}{\partial\eta}\left((1+\varepsilon\theta)\frac{\partial X}{\partial\eta}\right) + \alpha_1 n_l\left(a_3 \frac{e^{-a\left(\frac{b-\theta}{1+\varepsilon\theta}\right)}}{1+\varepsilon\theta} - X\right) \tag{29}$$

$$\left.\begin{array}{l} \dfrac{\partial Y}{\partial t} = D_1 \dfrac{\partial}{\partial\eta}\left((1+\varepsilon\theta)\dfrac{\partial Y}{\partial\eta}\right) - \gamma\left(a_1\left(A_1 + (B - A_1)\eta + \sum_{n=1}^{\infty} B_1\, e^{-P_{em}n^2\pi^2 t} \sin n\pi\eta - \right.\right. \\ \left.\left.(b_1(1+\varepsilon\theta) + a_2 X + b_2 Z)\right)\right) n_f\, e^{\frac{\theta}{1+\varepsilon\theta}} \end{array}\right\} \tag{30}$$

$$\left.\begin{array}{l} \dfrac{\partial Z}{\partial t} = D_1 \dfrac{\partial}{\partial\eta}\left((1+\varepsilon\theta)\dfrac{\partial Z}{\partial\eta}\right) + \gamma_1\left(a_1\left(A_1 + (B - A_1)\eta + \sum_{n=1}^{\infty} B_1\, e^{-P_{em}n^2\pi^2 t} \sin n\pi\eta - \right.\right. \\ \left.\left.(b_1(1+\varepsilon\theta) + a_2 X + b_2 Z)\right)\right) n_f\, e^{\frac{\theta}{1+\varepsilon\theta}} \end{array}\right\}$$

(31)

$$\left.\begin{array}{l} \dfrac{\partial n_f}{\partial t} = \gamma_2 n_f\left(a_1\left(A_1 + (B - A_1)\eta + \right.\right. \\ \left.\left.\sum_{n=1}^{\infty} B_1\, e^{-P_{em}n^2\pi^2 t} \sin n\pi\eta - (b_1(1+\varepsilon\theta) + a_2 X + b_2 Z)\right)\right) e^{\frac{\theta}{1+\varepsilon\theta}} \end{array}\right\} \tag{32}$$

$$\frac{\partial n_l}{\partial t} = -\gamma_3 n_l\left(a_3 \frac{e^{-a\left(\frac{b-\theta}{1+\varepsilon\theta}\right)}}{1+\varepsilon\theta} - X\right) \tag{33}$$

$$\left.\begin{array}{lll} \theta(\eta,0) = 1-\eta, & \theta(0,t) = b_3, & \theta(1,t) = 0 \\ X(\eta,0) = 1-\eta, & X(0,t) = 0, & X(1,t) = 0 \\ Y(\eta,0) = 1-\eta, & Y(0,t) = 1, & Y(1,t) = 0 \\ Z(\eta,0) = 1-\eta, & Z(0,t) = 0, & Z(1,t) = 0 \\ n_f = 1 & & \\ n_l = 1 & & \end{array}\right\} \tag{34}$$

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Where, $\lambda_1 = \dfrac{\lambda_0 t_0}{\rho c_g L^2}$, $\delta = \dfrac{t_0 Q_r k_r n_{fres}}{\rho c_g \varepsilon T_0} e^{-\frac{Er}{RT_0}}$, $P_{em} = \dfrac{Dt_0}{L^2}$, $a_1 = \dfrac{c_g}{\phi(Q_r + \mu_0)}$, $b_1 = \mu_0 T_0$,

$a_2 = \dfrac{\phi Q_e \mu_0 X_0}{c_g}$ $b_2 = \dfrac{\phi \mu_0 Z_0}{c_g}$, $\alpha = \dfrac{t_0 Q_e k n_{lres} X_0}{\rho c_g \varepsilon T_0}$, $a_3 = \dfrac{P_{atm}}{\rho R T_0 X_0}$, $D_1 = \dfrac{t_0 D_0}{L^2}$, $\alpha_1 = \dfrac{t_0 k n_{lres}}{\phi \rho}$,

$\gamma = \dfrac{t_0 \mu_0 k_r n_{fres} n_f^1}{\phi \rho Y_0} e^{-\frac{Er}{RT_0}}$ $\gamma_1 = \dfrac{t_0 \mu_g k_r n_{fres}}{\phi \rho Y_0} e^{-\frac{Er}{RT_0}}$, $\gamma_2 = t_0 \mu_f k_r e^{-\frac{Er}{RT_0}}$, $\gamma_3 = t_0 k X_0$, $a_3 = \dfrac{P_{atm}}{\rho R T_0 X_0}$,

$b_3 = \dfrac{T_1 - T_0}{\varepsilon T_0}$, $a = \dfrac{Q_e \varepsilon}{RT_0}$, $b = \dfrac{T_b - T_0}{\varepsilon T_0}$.

3.2 Analytical Solution

We solve equations (28) to (34) using parameter expanding method and eigenfunctions expansion technique and we obtain.

$$n_f(\eta,t) = 1 + \varepsilon \left(\begin{array}{l} p_9\left(t + p_1(1-\eta)t - \sum_{n=1}^{\infty} \frac{p_2}{q} e^{-qt} \sin n\pi\eta \right) + p_{10}\left((1 + p_1(1-\eta))t - \sum_{n=1}^{\infty} \frac{p_2}{q} e^{-qt} \sin n\pi\eta \right)\eta + \\ p_{11}\left(-\sum_{n=1}^{\infty} \frac{1}{q_2} e^{-q_2 t} \sin n\pi\eta - p_1(1-\eta)\sum_{n=1}^{\infty} \frac{1}{q_2} e^{-q_2 t} \sin n\pi\eta - \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} \frac{p_2}{q+q_1} e^{-(q+q_2)t} \sin^2 n\pi\eta \right) - \\ p_{12}\left(-\sum_{n=1}^{\infty} \frac{1}{q_3} e^{-q_3 t} \sin n\pi\eta - p_1(1-\eta)\sum_{n=1}^{\infty} \frac{1}{q_1} e^{-q_3 t} \sin n\pi\eta + \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} \frac{p_2}{q+q_1} e^{-(q+q_3)t} \sin^2 n\pi\eta \right) - \\ p_{13}\left(\begin{array}{l} \sum_{n=1}^{\infty}\left(p_5 t + \frac{p_6}{q_1} e^{-q_1 t} + \sum_{n=1}^{\infty} p_7\left(-\frac{1}{q} e^{-qt} + \frac{1}{q} e^{-q_1 t} \right) \right) \sin n\pi\eta + \\ p_1(1-\eta)\left(p_5 t - \frac{p_6}{q_1} e^{-q_1 t} + \sum_{n=1}^{\infty} p_7\left(-\frac{1}{q} e^{-qt} + \frac{1}{q} e^{-q_1 t} \right) \right) \sin n\pi\eta + \\ \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} p_2 \left(\begin{array}{l} -\frac{p_5}{q} e^{-qt} \sin n\pi\eta - \frac{p_6}{(q+q_1)} e^{-(q+q_1)t} \sin n\pi\eta + \\ \sum_{n=1}^{\infty} p_7\left(\frac{1}{(q+q_1)} e^{-(q+q_1)t} - \frac{1}{2q} e^{-2qt} \right) \sin^2 n\pi\eta \end{array} \right) \end{array} \right) \end{array} \right) \quad 35)$$

$$n_l(\eta,t) = 1 + \varepsilon \left(\begin{array}{l} m_3 \sum_{n=1}^{\infty}\left(p_5 t - \frac{p_6}{q_1} e^{-q_1 t} - \sum_{n=1}^{\infty} p_7\left(\frac{e^{-q_1 t}}{q_1} - \frac{e^{-qt}}{q} \right) \right) \sin n\pi\eta + \\ p_8\left((1 + p_1(1-\eta))t - \sum_{1}^{\infty} \frac{p_2}{q} e^{-qt} \sin n\pi\eta \right) \end{array} \right) \quad (36)$$

$$\theta(\eta,t) = b_3(1-\eta) + \sum_{n=1}^{\infty} \frac{2}{n\pi} e^{-\lambda_1 (n\pi)^2 t} \sin n\pi\eta + \varepsilon\left(\sum_{n=1}^{\infty} \theta_{1n}(t) \sin n\pi\eta \right) \quad (37)$$

$$X(\eta,t) = \sum_{n=1}^{\infty}\left(p_5 + p_6 e^{-q_1 t} + \sum_{n=1}^{\infty} p_7\left(e^{-qt} - e^{-q_1 t} \right) \right) \sin n\pi\eta + \varepsilon\left(\sum_{n=1}^{\infty} X_{1n}(t) \sin n\pi\eta \right) \quad (38)$$

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$$Y(\eta,t)=(1-\eta)+\sum_{n=1}^{\infty}\frac{2}{n\pi}e^{-D_1(n\pi)^2 t}\sin n\pi\eta+\varepsilon\left(\sum_{n=1}^{\infty}Y_{1n}(t)\sin n\pi\eta\right) \quad (39)$$

$$Z(\eta,t)=\sum_{n=1}^{\infty}\frac{2}{n\pi}e^{-D_1(n\pi)^2 t}\sin n\pi\eta+\varepsilon\left(\sum_{n=1}^{\infty}Z_{1n}(t)\sin n\pi\eta\right) \quad (40)$$

Where, $p=\alpha_1 a_3 a^2 b,\ \ p_1=b_3(e-2),\ \ p_2=\frac{2}{n\pi}(e-2),\ \ q=\lambda_1 n^2\pi^2$, $p_4=pp_2$,

$p_{26}=\alpha_1 m_3 a_3 a^2 b,$

$p_8=m_3 a_3 a^2 b$ $p_3=\frac{2p}{n\pi}\left((-1)^n-1\right)-\frac{2pp_1}{n\pi}$,

$p_5=\frac{p_3}{q_1},\ \ p_6=\left(\frac{2}{n\pi}-\frac{p_3}{q_1}\right),\ \ p_7=\frac{p_4}{(q_1-q)}e^{-q_1 t},$

$q_1=\alpha_1+D_1 n^2\pi^2$ $p_{12}=\frac{2m_2 n_{f0} a_1 b_2}{n\pi}$, $p_{13}=m_2 n_{f0} a_1 a_2$, $q_2=p_{em}n^2\pi^2$, $q_3=D_1 n^2\pi^2$, $p_{27}=\alpha_1 a_3 a^2 b,$

$p_{14}=ma_1 A_1,\ \ p_{15}=ma_1(B-A_1),\ \ p_{16}=ma_1 B_1,\ \ p_{17}=ma_1 b_1,\ \ p_{18}=ma_1 a^2$,

$p_{20}=m_1 a_1 A_1,\ \ \ p_{21}=m_1 a_1(B-A_1), p_{22}=m_1 a_1 B_1,\ p_{23}=m_1 a_1 b_1, p_{24}=m_1 a_1 a^2,\ \ \ p_{25}=m_1 a_1 b_2$

Results and Discussion

The systems of equations describing filtration combustion with temperature dependent thermal conductivity and diffusion coefficients in wet porous medium is solved analytically using parameter expanding method, direct integration and eigenfunctions expansion technique. Analytical solution given by equations (35) to (40) are computed for the following parameters values of λ_1=0.4, D_1=0.3, δ=0.4, p_{em}=1 using computer symbolic algebraic package MAPLE 17.

Figure 4.1: shows the graph of temperature $\theta(\eta,t)$ against distance η and time t for different values of scaled thermal conduct λ_1. It is observed that the temperature increases and later decreases along the distance with increase in time, but decreases with increase in scaled thermal conduct λ_1.

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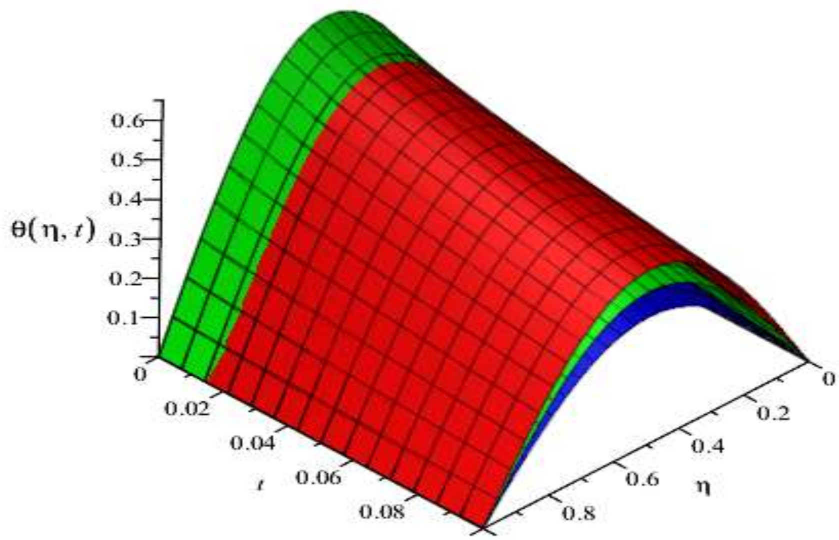

Figure 4.1: Relation among temperature $\theta(\eta,t)$, time t and distance η at various values of scaled thermal conduct λ_1.

Figure 4.2: shows the graph of vapour molar fraction in the gas phase $X(\eta,t)$ against distance η and time t for different values of scaled thermal conduct λ_1. It is observed that the vapour molar fraction in the gas phase increases and later decreases along the distance with increase in time, but increases with increase in scaled thermal conduct λ_1.

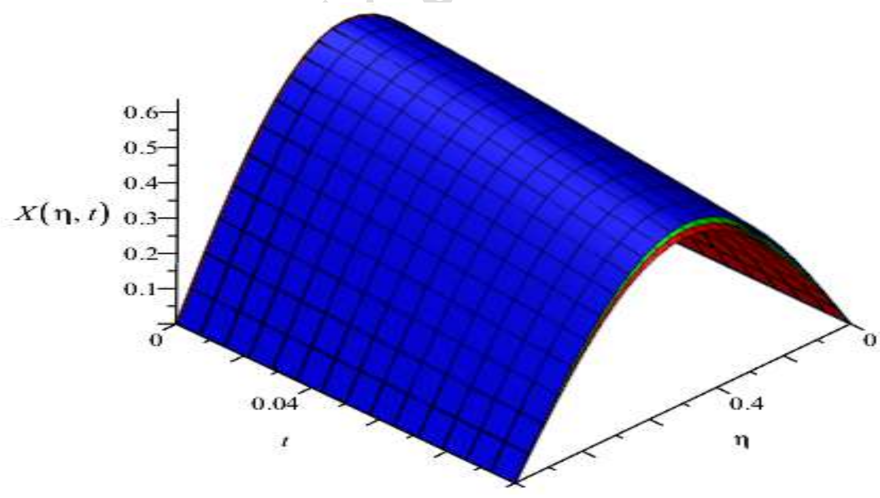

Figure 4.2: Relation among vapour molar fraction in the gas phase $X(\eta,t)$, time t and distance η at various values of scaled thermal conduct λ_1.

Figure 4.3: shows the effect of scaled thermal conductivity λ_1 on the molar concentration of solid fuel. It is observed that the molar concentration of solid fuel increases with time t, but increases with increase in scaled thermal conductivity λ_1.

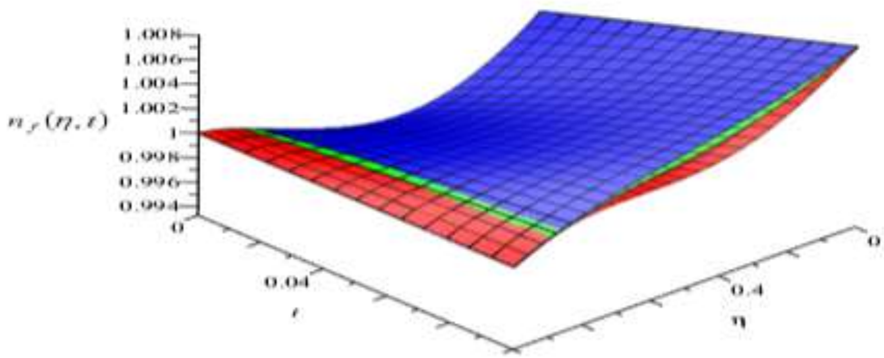

Figure 4.3: molar concentration of solid fuel $n_f(\eta,t)$ – time t relationships at various values of scaled thermal conductivity λ_1.

Figure 4.4: shows the graph of vapour molar fraction in the gas phase $X(\eta,t)$ against distance η and time t for different values of species diffusion coefficient D_1. It is observed that the vapour molar fraction in the gas phase increases and later decreases along the distance with increase in time, but decreases with increase in species diffusion coefficient D_1.

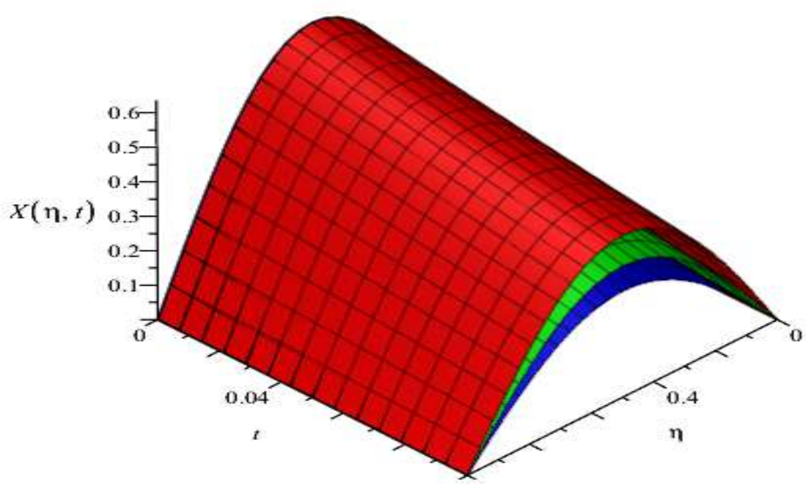

Figure 4.4: Relation among vapour molar fraction in the gas phase $X(\eta,t)$, time and distance η at various values of species diffusion coefficient D_1.

Figure 4.5: shows the graph of molar fraction of oxygen $Y(\eta,t)$ against distance η and time t for different values of species diffusion coefficient D_1. It is observed that the molar fraction of oxygen increases and later decreases along the distance with increase in time, but decreases with increase in species diffusion coefficient D_1.

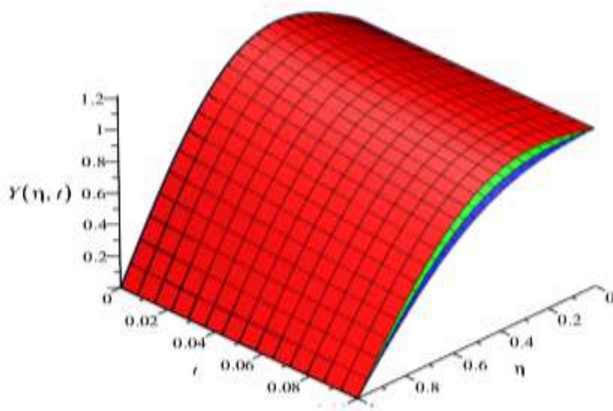

Figure 4.5: Relation among molar fraction of oxygen $Y(\eta,t)$, time t and distance η at various values of species diffusion coefficient D_1.

Figure 4.6: shows the graph of molar fraction of passive gas in the gas phase $Z(\eta,t)$ against distance η and time t for different values of species diffusion coefficient D_1. It is observed that the molar fraction of passive gas in the gas phase increases and later decreases along the distance with increase in time, but decreases with increase in species diffusion coefficient D_1.

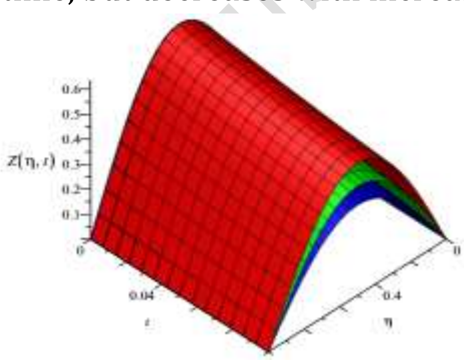

Figure 4.6: Relation among molar fraction of passive gas in the gas phase $Z(\eta,t)$, time t and distance η at various values of species diffusion coefficient D_1.

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Figure 4.7: shows the graph of molar concentration of solid fuel $n_f(\eta,t)$ against distance η and time t for different values of species diffusion coefficient D_1. It is observed that the molar concentration of solid fuel decreases and later increases along the distance with increase in time, but decreases with increase in species diffusion coefficient D_1.

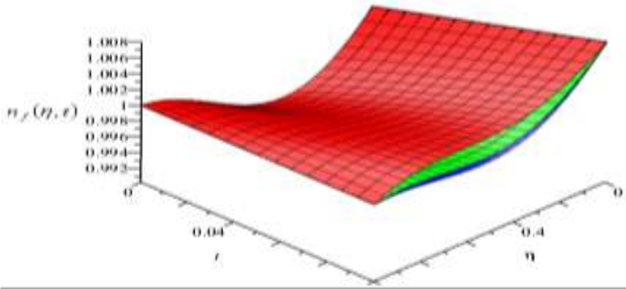

Figure 4.7: Relation among molar concentration of solid fuel $n_f(\eta,t)$, time t and distance η at various values of species diffusion coefficient D_1.

Figure 4.8: shows the graph of temperature $\theta(\eta,t)$ against distance η and time t for different values of Frank-kamenesskii parameter δ. It is observed that the temperature increases and later decreases along distance with increase in time, but increases with increase in Frank-kamenesskii parameter δ.

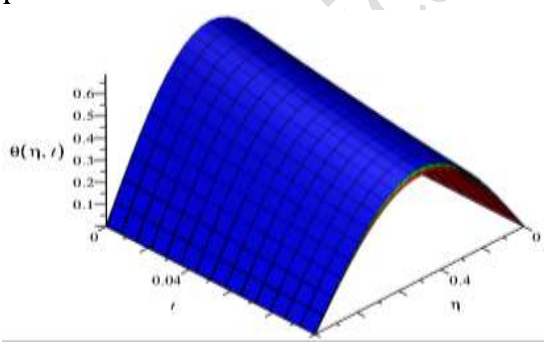

Figure 4.8: Relation among temperature $\theta(\eta,t)$, time t and distance η at various values of Frank-kamenesskii parameter δ.

Figure 4.9: shows the graph of molar concentration of solid fuel $n_f(\eta,t)$ against distance η and time t for different values of peclet mass p_{em}.It is observed that the molar concentration of solid fuel increases oscillate

along the distance with increase in time, but increases with increase in peclet mass p_{em}.

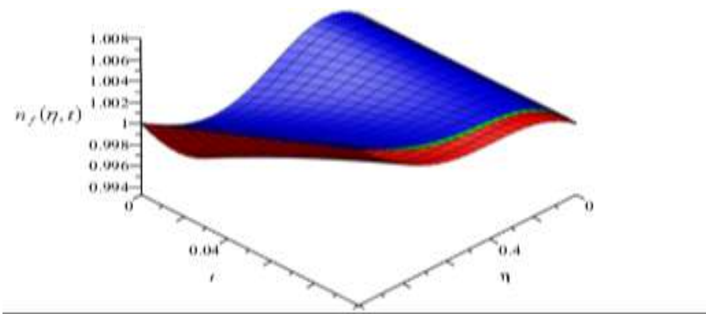

Figure 4.9: Relation among molar concentration of solid fuel $n_f(\eta,t)$, time t and distance η at various values of peclet mass p_{em}.

Conclusion

We have studied filtration combustion of gases in a wet porous medium with temperature dependent thermal conductivity and diffusion coefficient analytically. From the studies made on this paper, we discovered that the maximum temperature is attained when $\delta = 0.5$ for fixed time t. Simulation results also revealed that high temperature front created by combustion; the oxygen molar fraction, vapor molar fraction, passive gas molar fraction, molar concentration of the solid fuel and molar concentration of liquid depend appreciably on the values of the parameters involved. The influence of dimensionless parameter such as scaled thermal conductivity, species diffusion coefficient, Frank kamenetskii parameter, peclet mass number on the filtration combustion are of great importance. Future researchers may wish to study two-dimensional problems.

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