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MATHEMATICAL MODELING FOR PROGNOSIS OF NASARAWA STATE POPULATION GROWTH

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ABSTRACT

This study developed some specific population growth models and subjected them to accuracy test in order to determine the most appropriate model for the prognosis of Nasarawa State population. The study employed method of differential equation, exponential, geometric and hyperbolic functions as governing equations. Available census data of 1991 and 2006, Mean Absolute Error (MAPE) and Root Mean Square Error (RMSE) were used to obtain the growth rate and compared the three developed models for the population prognosis accuracy. The result of the analysis shows that exponential growth model with MAPE and RMSE values of 0.51% and 13.75 respectively is the most accurate and closely followed by geometric growth model with values 0.56% and 14.87 respectively. The study concludes that exponential growth model is the best for the prognosis of Nasarawa State population growth with approximate growth rate of 2.9% per annum, the projected population of the state will be 3,757,536 million by the year 2030. Hence, it is recommended that Nasarawa state government should institute developmental programmes, policies and work towards industrialization of the state to effectively improve its absorptive capacity development through the population size increase.

Keywords: *Population prognosis, Exponential, Geometric, Hyperbolic, Specific model, Nasarawa state.*

INTRODUCTION

Population is the most important element of the world, but to know the actual population size and growth rate have become the most serious problem all over the world; especially, the developing countries. A country policy, education, culture, politics, social activities, economy and exploration of her natural resources are influenced by the size and growth rate of her population. It implies that population size has major effect on social, economic, political and environmental development (Ekakitie and Ekereke, 2019). Besides, prognosis of any country's population plays a significant role in the planning as well as decision making for the demographic and socio-economic development plans (Ofori et al, 2013, Venkatesha, et al., 2017). In order to provide better living conditions to the people in terms of basic needs like good water, health, education, food and housing etc. a country requires to plan and execute appropriate schemes. To achieve this, the population size and growth rate must be obtained.

Population growth is the change in the size resulting from the differences in birth and death which are measured both in absolute and relative terms (Gee, 1999 in Ashirize. 2015), while growth rate is the average annual percentage change in the population size, which is affected by birth, death and the balance of the immigrants and emigrants within a specified period of time. (Mundi index (2012).

Population's size can change through three processes: fertility, mortality, and migration (Burea, 2013). Nigeria, as at today is at the verge of been termed over populated as the indicators signals in our states and cities. Although, the actual population or perfect estimation is yet to be ascertained in each state that make up Nigeria. Among the overpopulation indicators existing in Nigeria are unemployment, increase poverty and crime rate, high cost of living and environmental degradation in some states.

Nasarawa state is one of the states with the closest cities bordering the Nigeria Federal Capital city with attendant population flowing into some of its cities. For Nasarawa state to have a meaningful development, it must put in place a proper plan and scheme. Primary step is to have a good estimate of her population size, growth and growth rate. Hence, there is need to adequately model the population of Nasarawa state, this will assist the government in planning and balance allocation of resources to match up the growth.

Nasarawa state was created in October 1996 out of the western half of Plateau state with its capital in Lafia. It is centrally located in the Middle Belt region of Nigeria. The state lies between latitude 7^0 $45^/$ and 9^0 $25^/$ N of the equator and between longitude 7^0 and 9^0 37' E of the Greenwich meridian. It shares boundary with Kaduna state in the North, Plateau State in the East, Taraba and

Benue states in the south whereas Kogi state and the Federal Capital Territory verges it in the West. The state has a total land area of 26,875.59 square kilometers. It has thirteen Local Government Areas, namely, Akwanga, Awe, Doma, Karu, Keana, Kokona, Lafia, Nasarawa, Toto, Wamba, Keffi, Nasarawa Eggon and Obi. The people of Nasarawa state includes among others; the Gwandara, Alago, Eggon, Gbagi, Egbira, Migili, Kantana, Fulani, Hausa, Kanuri, Tiv, Afo, Gade, Nyankpa, Koro, Jukun, Mada, Ninzam, Buh, Basa, Agatu, Arum, Kulere, and also settler groups like the Igbo and Yoruba.

In order to obtain a good and reliable population estimate, there is need to employ mathematical models for the purpose of obtaining precise and accurate estimation of parameters and predictions (Vankatesha, 2017, Mussa and Jung, 2019). Therefore, this study attempt to develop specific population growth models for the population prognosis of Nasarawa state.

MATERIALS AND METHODS

A research or study is best understood as a process of arriving at dependent solutions to the problems through the systematic collection, analysis and interpretation of data (Dean, 2017).

In this study, secondary population data of Nasarawa state were collected from the Nigerian population censor, National Bureau of Statistics. Exponential, geometric and hyperbolic functions were employed as the governing equations to develop specific mathematical model equations to compute and estimate the future population size of Nasarawa state

Exponential Growth Model

The population size of a country, city or a community within a time period depends on birth, death, immigration rate situation of the country. It is possible to develop a general exponential growth model of population without taking these vital parameters into consideration, which begins with the assumption that the population grows at a rate proportional to the size of the current population as proposed by Thomas R. Malthius in 1978. According to this assumption, the model equation is expressed in simple differential equation thus;

$$\frac{dp(t)}{dt} = \alpha P(t) \tag{1}$$

Where P(t) is the total population size at certain time t and α is the constant growth rate which is the difference between the birth and death rate for a certain population size and period.

applying separation of variable to equation (1), thus;

$$\int_{p(t_0)}^{p(t)} \frac{dp(t)}{P(t)} = \int_{t_0}^{t} \alpha dt$$

$$\ln P(t) - \ln P(t_0) = \alpha(t - t_0)$$

$$\ln \frac{P(t)}{P(t_0)} = \alpha(t - t_0)$$

$$P(t) = P(t_0)e^{\alpha(t-t_0)}$$

Note, as $t \to \infty$, $P(t) \to \infty$

This result is referred to as exponential growth model, simply written as

$$P(t) = P_0 e^{\alpha n} \qquad (2)$$

Where, $P(t)$ is the population at any other year (time), P_0 is the initial population or base year population and $n = (t\text{-}t_0)$ is the projection period in year.

The parameter α (growth rate) can be obtained from equation (2) thus;

$$\alpha = \frac{1}{(t - t_0)} \ln \frac{P(t)}{P(t_0)} = \frac{1}{n} \ln \frac{P(t)}{P(t_0)} \qquad (3)$$

The growth rate as it is obtained in (3) is constant. However, human population is believed to be dynamics, therefore, the growth rate cannot be constant.

Geometric Growth Model

The geometric model assumes that the percentage increase in population is constant from decade to decade. This model method is useful for cities or populations which have unlimited scope for expansion and where a constant rate of growth is anticipated. The percentage increase in growth drops gradually as the cities reach the saturation points. The geometric growth model for projecting population depends on the availability of accurate data of the age and gender composition of the city population. The existing population structure mostly determines the future population composition of the city, but it may change depending on, birth, death and age-specific migration rates. The governing equation is given as

$$P(t) = P(t_0)(1+\alpha)^{(t-t_0)}$$

Written as

$$P(n) = P(t_0)(1+\alpha)^{n} \quad (4)$$

Where P(n) = The population at some time in the future.

$P(t_0)$ = base (initial) population

α = geometric percentage increase

n = (t-t_0) = projection period in year.

We obtained α from (4) as,

$$\alpha = \sqrt[n]{\frac{P(n)}{P(t_0)}} - 1 \quad (5)$$

Where ,

$$\alpha\% = \left(\sqrt[n]{\frac{P(t)}{P(t_0)}} - 1\right)100 \quad (6)$$

2.3 **Hyperbolic Growth Model**

The differential equation governing the exponential growth model assumes that change in population is proportional to the previous population with the constant relative growth rate α as given in (1);

$$\frac{dp(t)}{dt} = \alpha P(t)$$

For the hyperbolic growth model, the growth rate α is assumed to be function of the population P, that is, α is proportional to P (Dean, 2013). Thus, to maintain the dimension, it can be written as

$$\alpha(p) = \frac{\alpha P(t)}{P_0}$$

Hence,

$$\frac{dp(t)}{dt} = \alpha(p)P(t) = \frac{\alpha P(t)}{P_0} P(t) = \frac{\alpha P(t)^2}{P_0} \quad (7)$$

Using separation of variable

$$\int \frac{dp(t)}{P(t)^2} = \int \frac{\alpha}{P_0} dt$$

To obtain,

$\frac{1}{-P(t)} = \frac{\alpha}{P_0} t + k$, if P_0 is a constant population at t = 0, hence, $k = -\frac{1}{P_0}$ so ,

$$\frac{1}{-P(t)} = \frac{\alpha}{P_0} t - \frac{1}{P_0} = \frac{1}{P_0}(\alpha t - 1)$$

We have

$$P(t) = \frac{p_0}{(1-\alpha t)} = \frac{p_0}{[1-\alpha(t-t_0)]} = \frac{p_0}{(1-\alpha n)}$$

$$P(t) = p_0 (1-\alpha n)^{-1} \quad (8)$$

Equation (8) gives the hyperbolic growth model equation. The growth rate is obtaining as

$$\alpha = \frac{P(t) - p_0}{nP(t)} \quad (9)$$

Where P_0= Initial population, n = Period, α = growth rate and P(t) =population at time t.

Mean Absolute Percentage Error (MAPE)

The Mean Absolute Percentage Errors (MAPE) of each model is computed to compare their prognosis accuracy using the formula.

$$MAPE = \frac{1}{N} \sum_{n}^{N} \left| \frac{A_n - F_n}{A_n} \right| 100 \quad (10)$$

Where A_n = actual value, F_n = forecast value and N=number of observations of the population respectively.

According to Lewis 1982; MAPE < 10% is highly accurate, 10%-20% is good and 21%-50% is reasonable forecasting, while >50% is inaccurate forecasting. However, the lower MAPE values are better because they signify smaller percentage errors.

Root Mean Square Error (RMSE)

The RMSE for the specific models of Nasarawa population was computed also to obtain the most accurate model by using the following formula;

$$RMSE = \left[\frac{1}{N} \sum_{n=1}^{N} (A_n - F_n)^2 \right]^{\frac{1}{2}} \quad (11)$$

Where, A_n, F_n and N have the same meaning as in equation (10).

RESULTS AND DISCUSSION

Modeling

In other to predict the population size of Nasarawa state, three specific population growth models were obtained by determining the growth rate of the population using equations (3), (5), and (9) the governing equations; the exponential, geometric and hyperbolic growth models in equations (20), (4) and (8) respectively, together with the two available actual population censors data of 1,207,876 million and 1,869,377 million in 1999 and 2006 respectively (NBS, 2011).

Exponential Growth Model for Nasarawa State Population

Let (t_0, P_0) = (1991, 1207876) and (t, P_t) = (2006, 1869377) using equation (3), the growth rate is

$$\alpha = \frac{1}{15}\ln 1.547656 = 0.0291 \qquad (12)$$

The growth rate percent = $\alpha\% = 2.91\%$

The required specific exponential growth model for Nasarawa state population is obtained from (2) as;

$$P(n) = 1,207,876e^{0.0291n} \qquad (13)$$

Where, n =0, 1, 2, 3, …30. Starting from base year 1991

Hyperbolic Growth Model for Nasarawa State Population

The specific hyperbolic growth model for Nasarawa state population is obtained using equations (9) and (8) to get growth rate and model equation respectively, thus

$$\alpha = \frac{661510}{28,040,655} = 0.02359 \qquad (14)$$

$$P(n) = 1,207,876(1 - 0.02359n)^{-1} \qquad (15)$$

Equations (14) and (15) give the specific growth rate and hyperbolic growth model equation for Nasarawa State population.

3.1.3 Geometric Growth Model for Nasarawa State Population

To project the future population of Nasarawa state using the geometric growth model when the vital population data (birth, death and migration) are not available. The growth rate α is obtained using equation (5), thus;

$$\alpha = \left\{ \left(\frac{P(n)}{P(t_0)} \right)^{\frac{1}{n}} - 1 \right\} = [(1.5476563819)1/15 - 1] = 0.0295 \tag{16}$$

The estimated percentage growth rate of Nasarawa state population is

$$\alpha\% \approx 2.95\%$$

Substituting for α and P_0 in equation (4) gives specific geometric growth model for Nasarawa state population as

$$P(n) = 1{,}207{,}876(1.0295)^{n} \tag{17}$$

Equations (13), (15) and (17) give the specific simple model equations for Nasarawa state population growth. These equations are utilized to obtain the prognosis of Nasarawa state population.

Results

Table 1: Available data of Nasarawa State Population projected by Nigeria National Populat Commission

Year	Projected population (million)
2008	1,984,974
2009	2,045,424
2010	2,107,717
2011	2,171,900
2012	2,238,051
2013	2,306,209
2014	2,376444
2015	2,448817
2016	2,523,395

Source: National Bureau of Statistics (NBS,2011and NBS,2017), Abuja

Table 2: Prognosis of Nasarawa State Population from the three developed specific Models,
their MAPEs and RMSEs. (1991 – 2019)

Year	n	Population of Nasarawa state (Million)		
		Exponential growth(million)	**Geometric growth Model**	**Hyperbolic Growth model(million**
1991	0	1,207,876	1,207,876	1,207,876
1992	1	1,243,541	1,243,508	1,237,058
1993	2	1,280,260	1,280,191	1,267,685
1994	3	1,318,063	1,317,957	1,299,876
1995	4	1,356,982	1,356,837	1,333,726
1996	5	1,397,050	1,396,863	1,369,396
1997	6	1,438,302	1,438,071	1,407,026
1998	7	1,480,771	1,480,494	1,446,783
1999	8	1,524,495	1,524,169	1,488,852
2000	9	1,569,509	1,569,132	1,533,440
2001	10	1,615,853	1,615,421	1,580,782
2002	11	1,663,565	1,663,076	1,631,140
2003	12	1,712,686	1,712,137	1,684,812
2004	13	1,763,258	1,762,645	1,742,139
2005	14	1,815,323	1,814,643	1,803,499
2006	15	1,868,925	1,868,175	1,869,343
2007	16	1,924,109	1,923,286	1,940,176
2008	17	1,980,924	1,980,023	2,016,588
2009	18	2,039,415	2,038,434	2,099,266
2010	19	2,099,634	2,098,567	2,189,013
2011	20	2,161,631	2,160,475	2,286,777
2012	21	2,225,459	2,224,209	2,393,682
2013	22	2,291,171	2,289,823	2,511,072
2014	23	2,358,824	2,357,373	2,640,570
2015	24	2,428,474	2,426,916	2,784,150
2016	25	2,500,181	2,498,510	2,944,243
2017	26	2,574,006	2,572,216	3,123,871
2018	27	2,650,010	2,648,096	3,326,840

2019	28	2,728,258	2,726,215	3,558,018
MAPE		0.51%	0.56%	25.26%
RMSE		13,746.97	14,872.14	211,001.83

Source: Computed by the Authors.

Table 3: Prognosis of Nasarawa State Population using the two accurate specific Models, (2020 – 2030)

Year	Population of Nasarawa state (Million) (2020 -2030)	
	Exponential growth(million)	**Geometric growth Model(million)**
2020	2,808,817	2,806,638
2021	2,891,754	2,889,434
2022	2,977,141	2,974,673
2023	3,,065,048	3,062,425
2024	3,155,552	3,152,767
2025	3,248,727	2,245,774
2026	3,344,654	3,341,524
2027	3,443,414	3,440,099
2028	3,545,089	3,541,582
2029	3,649,767	3,646,058
2030	3,757,536	3,753,617

Source: Computed by the Authors

Discussion

Comparing the three models based on the results of their computed MAPE and RMSE, it is found that exponential growth model with MAPE and RMSE of 0.51% and 13.746 respectively has the lowest error values followed by Geometric growth model with MAPE (0.56%) and RMSE (14,872) as against that of Hyperbolic growth model with MAPE (25.26%) and RMSE (211,001) the highest values among the three models. This implies that exponential growth model is the most accurate because of its least prognosis error. According to the three models, the population growth rate of Nasarawa state are 2.9%, 2.951% and 3% respectively. However, 2.9% growth rate of the most accurate model is adopted for the state.

Table 2 shows the results of the population based on the developed specific models for Nasarawa state population with their MAPEs and RMSEs. Table 3 gives the result of the prognosis of Nasarawa state population for 11 years from 2020 to 2030. The prognosis values points to the fact that Nasarawa state population is on the increase. This may be attributed to the fact that it is close to Federal Capital with the influence of workers from the capital city seeking lower cost of living and accommodation. Besides, it has fast agrarian land of rural setting with few develop cities. Hence, exponential growth model is the best model to project Nasarawa state population.

CONCLUSION

Mathematical modeling of population is an essential tool both in predicting the future population and growth rate of a certain community or country.

This paper explored the exponential, geometric and hyperbolic growth equation as governing equations to developed specific Exponential, hyperbolic and geometric population growth models for Nasarawa state. The three developed models were compared using MAPE and RMSE for prognosis accuracy. The MAPE and RMSE of the Exponential, Geometric and Hyperbolic population growth model were obtained as 0.51% and 17.75, 0.56% and 14.87, 25.26% and 211,011 respectively. Based on the results, exponential growth model is the most accurate of the three developed models and closely followed by geometric growth model. Hence, the study concludes that, exponential model is the best population growth model for the prognosis of Nasarawa state population with the population growth rate of 2.9% per annum. Therefore, the anticipated population of the state using Exponential growth model is 3,757,536 million by the year 2030.

However, it is essential to note that, more accurate prediction of population growth is much more complicated than the simple deterministic models presented in this paper. Besides, accurate prediction must take into consideration factors such as the age distribution, birth rates, death rates of the population and scarcity of available resources as the population grows. Therefore, in the absence of these vital data, as in the case of Nasarawa state, it is concluded that an exponential growth model is the best most accurate deterministic trend model leading to a precise accurate prognosis for Nasarawa state population.

RECOMMENDATIONS

Population projection of a community, state or country is important in planning, making sensitive and critical decision in the area of economy, politics, social and the demographic development of the nation.

Based on the analysis results of the developed models and conclusion of this paper that, population of Nasarawa state is on the increase and by 2030, it will be 3,757,536 million. This paper therefore, recommends that Nasarawa state government should institute developmental programmes, policies and work towards industrialization of the state, in the areas of agriculture, with the aim of mechanized production, processing and packaging, mineral exploration, exploitation and processing, education and social facilities. These will definitely have effect on improving the state absorptive capacity for development through the population progression.

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