



## EFFECT OF INTRA-COMPETITION COEFFICIENTS ON THE RESOURCE BIOMASS OF A RESOURCE- DEPENDENT INTERACTING BIOLOGICAL SPECIES

**\*DIKE, ADEDOTUN OLASUMBO \*\*GEORGE,  
ISOBEYE**

*\*Department of Mathematics, Federal University, Oye-Ekiti, Ekiti State.*

*\*\*Department of Mathematics/Statistics, Ignatius Ajuru University of  
Education, Port Harcourt, Rivers State.*

### Abstract

The effect of intra-competition coefficient on the resource biomass of a resourcedependent interacting biological species was investigated. To facilitate the investigation, a continuous system of nonlinear first order ordinary differential equation indexed by the appropriate initial conditions, was considered. A MATLAB ODE45 numerical scheme was used to generate the data needed for the analysis. The key result of the investigation shows that increase in intra-competition coefficient of the interacting biological species increases the resource biomass while the population density of the species diminishes.

**Keywords:** Intra-competition coefficient, resource biomass, resource-dependent interacting biological species, system of differential equation

### INTRODUCTION

This study deals essentially with the interaction between two biological species in an environment. In our immediate environment, living and non-living organisms share one thing or the other in common. In ecology, biological interaction is the effect organisms living together in an environment has on one another. The relationship could be the one involving same species inhabiting the same ecological area termed intra-specific interactions or the one involving different species inhabiting the same ecological area termed inter-specific interaction. The interaction could be long term or short term.

If the resource made available is insufficient for both populations, then there will be lowered productivity or growth rate or one species might survive while the other goes into extinction. Limited supply of resources could be responsible for competition. Competition will occur between organisms in an ecosystem when their niches overlap, both of them will try to use the same resource and there is every possibility that the resource will be in short supply.

Competition coefficient is a number giving the degree to which an individual of one species affects the growth or equilibrium level of a second species population relative to the effect of an individual of the second species (Volterra, 1996). Competition coefficients have two subscripts, one for each species involved in the competition. The species affected is listed first and the species causing the effect is listed second. Competition co-efficient is the key component of the Lotka-Volterra model because it was designed to create the situation whereby two species could co-exist indefinitely. If two species must co-exist, then intra-competition coefficient must be greater than inter specific competition [Case (2000), Gotelli, (2008), Mittelberg (2012), Vandermeer and Goldberg, (2013)].

The mathematical structure under consideration has the following form:

$$\frac{dx_1(t)}{dt} = F_1(x_1, x_2, R), \quad x_1(0) = x_{10} \geq 0 \quad (1)$$

$$\frac{dx_2(t)}{dt} = F_2(x_1, x_2, R), \quad x_2(0) = x_{20} \geq 0 \quad (2)$$

$$\frac{dR(t)}{dt} = F_3(x_1, x_2, R), \quad (0) = R_0 \geq 0 \quad (3)$$

Increase in the population size gradually stabilizes the dynamical system, as soon as the population size reaches a reduced number, productivity picks up and survival is increased thereby putting the population back in a particular growth pattern. These rising and falling helps keep the population from getting too high or too low. The regulating effect is the major consequence of intraspecific completion.

Fluctuation in environmental condition can also enable the co-existence of species that differ in their growth strategies. In a situation where resource is abundant and the resource competition is weak, the niche expansion might not be selectively advantageous even though free niche were available. Charles Darwin realized how important the struggle for life essential resource competition is in shaping the evolution of all organisms when resources are limited.

## MATERIAL AND METHODS

The physical model consists of two competing resources-dependent biological species (Goat and Sheep) in an environment. To achieve the objective of this work, the following system of non-linear first order ordinary differential equations, as given by George (2019), has been considered.

$$\frac{dx_1(t)}{dt} = a_1x_1 - a_2x_1^2 - \alpha x_1x_2 + \alpha_1x_1R, \quad x_1(0) = x_{10} \geq 0 \quad (4)$$

$$\frac{dx_2(t)}{dt} = b_1x_2 - b_2x_2^2 - \beta x_1x_2 + \beta_1x_2R, \quad x_2(0) = x_{20} \geq 0 \quad (5)$$

$$\frac{dR(t)}{dt} = c_1R - c_2R^2 - \alpha x_1R - \beta_1x_2R, \quad (0) = R_0 \geq 0 \quad (6)$$

$x_1$  and  $x_2$  denote the population sizes of species 1 and 2, respectively.  $a_1$  and  $b_1$  represent the intrinsic growth rates of the first and second species, respectively.  $a_2$  and  $b_2$  are the intra-competition coefficients of species 1 and 2, respectively.  $\alpha_1$  and

$\beta_1$  denote the growth rate coefficients of species 1 and 2, while  $\alpha$  and  $\beta$  are the inter competition coefficients of species 1 and 2, respectively.  $R$  is the resource biomass,  $c_1$  is the intrinsic growth rate of the resource biomass and  $c_2$  is the intra-competition coefficient of the resource biomass.

It is expected that the interacting biological species grow exponentially with unlimited supply of resources in the ecosystem by constant  $c_1$  and in the event of limited supply of resources the less powerful species reduce in weight and go into extinction.

In the absence of any interaction between the species, that is, if

$$\alpha = \alpha_1 = \alpha_2 = b_2 = \beta = \beta_1 = c_2 = 0,$$

then equations (4) – (6) become linear equations as follows:

$$\frac{dx_1}{dt} = a_1x_1, \quad x_1(0) = x_{10} \geq 0 \quad (7)$$

$$\frac{dx_2}{dt} = b_1x_2, \quad x_2(0) = x_{20} \geq 0 \quad (8)$$

$$\frac{dR}{dt} = c_1R, \quad (0) = R_0 \geq 0 \quad (9)$$

Solving equations (7) – (9) by the technique of separation of variables give

$$x_1 = x_{10}e^{a_1t} \quad (10)$$

$$x_2 = x_{20}e^{b_1t} \quad (11)$$

$$R = R_0e^{c_1t} \quad (12)$$

Equations (10) – (12) clearly show that as  $t \rightarrow \infty$ ,  $x_1(t)$ ,  $x_2(t)$  and  $R(t)$  will grow exponentially. This is mathematically true, but not scientifically correct. This is

because; no population grows exponentially, as space and limiting resources can inhibit such growth.

To facilitate the interpretation of the mathematical analysis, the following parameter values given by George (2019) were used in the simulation for the system (4) - (6)

$$a_1 = 5, \quad a_2 = 0.22, \quad \alpha = 0.007, \quad \alpha_1 = 0.02, \quad b_1 = 3,$$

$$b_2 = 0.26, \quad \beta = 0.008, \quad \beta_1 = 0.04, \quad c_1 = 10, \quad c_2 = 0.3$$

## RESULTS AND DISCUSSION

This section presents the numerical illustrations of the results. A MATLAB ODE45 numerical scheme was used to generate data and these are presented in Tables (1) – (6) below.

Table 1: Effect of variation of intra-competition coefficient,  $a_2$ , of the first biological species,  $x_1$ , on the resource biomass,  $R$ , of the dynamical system, using a MATLAB ODE45 numerical scheme

| S/N | $a_2$  | $R$     |    |        |         |    |        |         |
|-----|--------|---------|----|--------|---------|----|--------|---------|
| 1   | 0.2200 | 29.6205 | 10 | 0.0990 | 27.7753 |    |        |         |
| 2   | 0.0110 | 2.7017  | 11 | 0.1100 | 28.1112 |    |        |         |
| 3   | 0.0220 | 16.4358 | 12 | 0.1210 | 28.3823 |    |        |         |
| 4   | 0.0330 | 21.2069 | 13 | 0.1320 | 28.6144 |    |        |         |
| 5   | 0.0440 | 23.6405 | 14 | 0.1430 | 28.8080 |    |        |         |
| 6   | 0.0550 | 25.1196 | 15 | 0.1540 | 28.9646 |    |        |         |
| 7   | 0.0660 | 26.1060 | 16 | 0.1650 | 29.1197 |    |        |         |
| 8   | 0.0770 | 26.8221 | 17 | 0.1760 | 29.2452 | 19 | 0.1980 | 29.4364 |
| 9   | 0.0880 | 27.3550 | 18 | 0.1870 | 29.3514 | 20 | 0.2090 | 29.5360 |

Table 2: Effect of variation of intra-competition coefficient,  $a_2$ , of the first biological species,  $x_1$ , on the population of competing species,  $x_1$  and  $x_2$ , of the dynamical system, using a MATLAB ODE45 numerical scheme

| S/N | $a_2$  | $x_1$    | $x_2$                   | $R$     |
|-----|--------|----------|-------------------------|---------|
| 1   | 0.2200 | 24.9340  | 15.3302                 | 29.6205 |
| 2   | 0.0110 | 459.3840 | $1.2664 \times 10^{-7}$ | 2.7017  |
| 3   | 0.0220 | 240.0914 | 6.6799                  | 16.4358 |
| 4   | 0.0330 | 162.2846 | 9.8074                  | 21.2069 |
| 5   | 0.0440 | 122.5693 | 11.4044                 | 23.6405 |
| 6   | 0.0550 | 98.4680  | 12.3730                 | 25.1196 |
| 7   | 0.0660 | 82.2936  | 13.0244                 | 26.1060 |
| 8   | 0.0770 | 70.6762  | 13.4905                 | 26.8221 |
| 9   | 0.0880 | 61.9371  | 13.8423                 | 27.3550 |
| 10  | 0.0990 | 55.1190  | 14.1160                 | 27.7753 |
| 11  | 0.1100 | 49.6537  | 14.3356                 | 28.1112 |
| 12  | 0.1210 | 45.1759  | 14.5162                 | 28.3823 |
| 13  | 0.1320 | 41.4371  | 14.6660                 | 28.6144 |
| 14  | 0.1430 | 38.2707  | 14.7934                 | 28.8080 |
| 15  | 0.1540 | 35.5562  | 14.9040                 | 28.9646 |
| 16  | 0.1650 | 33.1968  | 14.9972                 | 29.1197 |
| 17  | 0.1760 | 31.1333  | 15.0804                 | 29.2452 |
| 18  | 0.1870 | 29.3122  | 15.1543                 | 29.3514 |
| 19  | 0.1980 | 27.6941  | 15.2214                 | 29.4364 |
| 20  | 0.2090 | 26.2412  | 15.2783                 | 29.5360 |

Table 3: Effect of variation of intra-competition coefficient,  $b_2$ , of the second biological species,  $x_2$ , on the resource biomass,  $R$ , of the dynamical system, using a MATLAB ODE45 numerical scheme

| S/N | $b_2$  | $R$     |
|-----|--------|---------|
| 1   | 0.2600 | 29.6205 |
| 2   | 0.0130 | 1.9893  |
| 3   | 0.0260 | 14.4709 |
| 4   | 0.0390 | 19.5549 |
| 5   | 0.0520 | 22.3139 |
| 6   | 0.0650 | 24.0455 |
| 7   | 0.0780 | 25.2350 |
| 8   | 0.0910 | 26.1016 |
| 9   | 0.1040 | 26.7608 |
| 10  | 0.1170 | 27.2654 |
| 11  | 0.1300 | 27.6958 |
| 12  | 0.1430 | 28.0445 |
| 13  | 0.1560 | 28.3239 |
| 14  | 0.1690 | 28.5725 |
| 15  | 0.1820 | 28.7886 |
| 16  | 0.1950 | 28.9541 |
| 17  | 0.2080 | 29.1370 |
| 18  | 0.2210 | 29.2805 |
| 19  | 0.2340 | 29.4030 |
| 20  | 0.2470 | 29.5035 |

Table 4: Effect of variation of intra-competition coefficient,  $b_2$ , of the second biological species,  $x_2$ , on the population of competing species,  $x_1$  and  $x_2$ , of the dynamical system, using a MATLAB ODE45 numerical scheme

| S/N | $b_2$  | $x_1$   | $x_2$    | $R$     |
|-----|--------|---------|----------|---------|
| 1   | 0.2600 | 24.9340 | 15.3302  | 29.6205 |
| 2   | 0.0130 | 15.6797 | 227.2459 | 1.9893  |
| 3   | 0.0260 | 19.8589 | 131.5374 | 14.4709 |
| 4   | 0.0390 | 21.5600 | 92.5572  | 19.5549 |
| 5   | 0.0520 | 22.4687 | 71.3762  | 22.3139 |
| 6   | 0.0650 | 23.0651 | 58.1153  | 24.0455 |
| 7   | 0.0780 | 23.4630 | 48.9978  | 25.2350 |
| 8   | 0.0910 | 23.7530 | 42.3533  | 26.1016 |
| 9   | 0.1040 | 23.9740 | 37.2960  | 26.7608 |
| 10  | 0.1170 | 24.1509 | 33.3226  | 27.2654 |
| 11  | 0.1300 | 24.2885 | 30.1068  | 27.6958 |
| 12  | 0.1430 | 24.4033 | 27.4589  | 28.0445 |
| 13  | 0.1560 | 24.5022 | 25.2424  | 28.3239 |
| 14  | 0.1690 | 24.5840 | 23.3543  | 28.5725 |
| 15  | 0.1820 | 24.6542 | 21.7288  | 28.7886 |
| 16  | 0.1950 | 24.7192 | 20.3190  | 28.9541 |
| 17  | 0.2080 | 24.7697 | 19.0743  | 29.1370 |
| 18  | 0.2210 | 24.8176 | 17.9765  | 29.2805 |
| 19  | 0.2340 | 24.8611 | 16.9989  | 29.4030 |
| 20  | 0.2470 | 24.9017 | 16.1235  | 29.5035 |

Table 5: Effect of simultaneous variation of intra competition coefficients,  $a_2$  and  $b_2$ , of the first and second biological species,  $x_1$  and  $x_2$ , on the resource biomass,  $R$ , of the dynamical system, using MATLAB ODE45 numerical scheme

| S/N | $a_2$  | $b_2$  | $R$     |
|-----|--------|--------|---------|
| 1   | 0.2200 | 0.2600 | 29.6205 |
| 2   | 0.0110 | 0.0130 | 0.0270  |
| 3   | 0.0220 | 0.0260 | 10.2977 |
| 4   | 0.0330 | 0.0390 | 15.2009 |
| 5   | 0.0440 | 0.0520 | 18.5094 |
| 6   | 0.0550 | 0.0650 | 20.8278 |
| 7   | 0.0660 | 0.0780 | 22.5245 |
| 8   | 0.0770 | 0.0910 | 23.8221 |
| 9   | 0.880  | 0.1040 | 24.8425 |
| 10  | 0.0990 | 0.1170 | 25.6642 |
| 11  | 0.1100 | 0.1300 | 26.3397 |
| 12  | 0.1210 | 0.1430 | 26.9108 |
| 13  | 0.1320 | 0.1560 | 27.3937 |
| 14  | 0.1430 | 0.1690 | 27.8057 |
| 15  | 0.1540 | 0.1820 | 28.1780 |
| 16  | 0.1650 | 0.1950 | 28.4641 |
| 17  | 0.1760 | 0.2080 | 28.7648 |
| 18  | 0.1870 | 0.2210 | 28.9938 |
| 19  | 0.1980 | 0.2340 | 29.2376 |
| 20  | 0.2090 | 0.2470 | 29.4367 |

Table 6: Effect of simultaneous variation of intra-competition coefficients,  $a_2$  and  $b_2$ , of the first and second biological species,  $x_1$  and  $x_2$ , on the competing species,  $x_1$  and  $x_2$ , of the dynamical system, using MATLAB ODE45 numerical scheme

| S/N | $a_2$  | $b_2$  | $x_1$    | $x_2$                   | $R$     |
|-----|--------|--------|----------|-------------------------|---------|
| 1   | 0.2200 | 0.2600 | 24.9340  | 15.3302                 | 29.6205 |
| 2   | 0.0110 | 0.0130 | 459.3842 | $1.4846 \times 10^{-7}$ | 0.0270  |
| 3   | 0.0220 | 0.0260 | 244.2650 | 64.7554                 | 10.2977 |
| 4   | 0.0330 | 0.0390 | 216.0301 | 62.2480                 | 15.2009 |
| 5   | 0.0440 | 0.0520 | 147.5201 | 54.4875                 | 18.5094 |
| 6   | 0.0550 | 0.0650 | 113.3820 | 47.5927                 | 20.8278 |
| 7   | 0.0660 | 0.0780 | 92.4222  | 41.9991                 | 22.5245 |
| 8   | 0.0770 | 0.0910 | 78.1270  | 37.4837                 | 23.8221 |
| 9   | 0.880  | 0.1040 | 67.7113  | 33.8019                 | 24.8425 |
| 10  | 0.0990 | 0.1170 | 59.7743  | 30.7565                 | 25.6642 |
| 11  | 0.1100 | 0.1300 | 53.5157  | 28.2023                 | 26.3397 |
| 12  | 0.1210 | 0.1430 | 48.4516  | 26.0307                 | 26.9108 |
| 13  | 0.1320 | 0.1560 | 40.7484  | 24.1657                 | 27.3937 |
| 14  | 0.1430 | 0.1690 | 37.7524  | 22.5479                 | 27.8057 |
| 15  | 0.1540 | 0.1820 | 35.1669  | 21.1295                 | 28.1780 |
| 16  | 0.1650 | 0.1950 | 32.9190  | 19.8828                 | 28.4641 |
| 17  | 0.1760 | 0.2080 | 30.9329  | 18.7666                 | 28.7648 |
| 18  | 0.1870 | 0.2210 | 29.1815  | 17.7748                 | 28.9938 |
| 19  | 0.1980 | 0.2340 | 27.6107  | 16.8761                 | 29.2376 |
| 20  | 0.2090 | 0.2470 | 26.2044  | 16.0665                 | 29.4367 |

It is observed in Table 1 that, increase in the intra-competition coefficient,  $a_2$ , of the first biological species,  $x_1$ , while other model parameters remain fixed, results in an increase in the resource biomass,  $R$ . It is further shown that,  $R$  becomes small when

$a_2$  is small. Table 2 shows that increase in the intra-competition coefficient,  $a_2$ , of the first biological species,  $x_1$ , decreases the population of the first species,  $x_1$ , as the population of the second biological species,  $x_2$ , increases gradually.

In Table 3, it is shown that increase in the intra-competition coefficient,  $b_2$ , of the second species,  $x_2$ , while other model parameters remain fixed, results in an increase in the resource biomass,  $R$ . Furthermore, Table 4 reveals that, increase in the intracompetition coefficient,  $b_2$ , of the second species,  $x_2$ , results in a decrease in  $x_2$ , but increases the population of the first species,  $x_1$ .

Table 5 shows that, simultaneous increase in the intra-competition coefficients,  $a_2$  and  $b_2$ , of the first and second biological species,  $x_1$  and  $x_2$ , while other model parameters remain fixed, results in an increase in the resource biomass,  $R$ , of the system. Furthermore, in Table 6 it is revealed that, simultaneous increase in  $a_2$  and  $b_2$  results in the simultaneous decrease of the population of the two species,  $x_1$  and  $x_2$ , respectively. It also shows that when the intra-competition coefficients,  $a_2$  and  $b_2$ , of both species are very small and the population of the first species,  $x_2$ , is large, the population of the second species,  $x_2$ , diminishes.

## CONCLUSION AND RECOMMENDATION

The effect of intra-competition coefficients on the resource biomass of a resourcedependent biological species was considered. To achieve the purpose of the study, a system of nonlinear first order ordinary differential equations was considered to analyses the variations of the competition coefficients. The key result of the investigation shows that increase in intra-competitions of the interacting biological species increases the resource biomass, while the population density of the species diminishes.

A further extension of this analysis should be considered using a second order nonlinear dynamical system. Also worthy of consideration is the interaction among three biological species.

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