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EXCEL SOLUTION TEMPLATES FOR COST MINIMISATION OF GENERATING CAPACITY OF CERTAIN ELECTRIC POWER

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ABSTRACT

This research work investigated the problem of generating and maintaining electric power from certain power plant and obtained the optimal power generating strategies using backward dynamic programming. In the sequel, the work exploited several works of Ukwu to design solution templates and the corresponding algorithm for the optimal generating policy. The work went further to provide illustrative examples which demonstrate the practicality and practicability of the solution templates. The solution templates are imperative for the undertaking of sensitivity analyses and certainly obviate the need for cumbersome manual computations associated with dynamic programming recursions. The research recommends among others that appropriate measures need to be taken in boosting generating capacity of certain electric power supply.

Introduction:

Operations Research is the science of rational decision making and the study, design and integration of complex situations and systems with the goal of predicting system behavior and improving or optimizing system performance. The formal activities of operation research were initiated in England during World War II to make decisions regarding the best utilization of war material (Sharma, 1989). After the war the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector. Operations research has developed to today's dominant and indispensable decision-making tool. It encompasses managerial

KEY WORDS: *Dynamic Programming, Backward Recursion, Excel Solution, Electric Power and Cost Minimization.*

decision making, mathematical and computer modeling and the use of information technology for informed decision making (Weigel and Cao, 1999).

In a sense, every effort to apply science to management of organized systems, and to their understanding, was a predecessor of operations research. It began as a separate discipline, however, in 1937 in Britain as a result of the initiative of A.P. Rowe, superintendent of the Bawdsey Research Station, who led British scientists to teach military leaders how to use the then newly developed radar to locate enemy aircraft. By 1939 the Royal Air Force had formally commenced efforts to extend the range of radar equipment so as to increase the time between the first warning provided by radar and the attack by enemy aircraft. At first they analyzed physical equipment and communication networks, but later they examined behavior of the operating personnel and relevant executives. Results of the studies revealed ways of improving the operators' techniques and also revealed unappreciated limitations in the network (Taha, 1999).

However, many operations research practitioners have made vital contributions on how to improve solutions of optimization problems using electronic methods for different systems and situations. Electronic solutions may be implemented on the platform of Programming languages such as C, C++, FOTRAN and applications such as Microsoft excel etc.

Consider the following base problem:

"An electric power utility forecasts that r_t kilowatt-hours (kWh) of generating capacity will be needed during year t (the current year is year 1). Each year, the utility must decide by how much generating capacity should be expanded. It costs $c_t(x)$ dollars to increase generating capacity by x kWh during year t. It may be desirable to reduce capacity, so x need not be nonnegative. During each year, 10 % of the old generating capacity becomes obsolete and unusable (capacity does not become obsolete during its first year of operation). It costs the utility $m_t(i)$ dollars to maintain i

units of capacity during year *t*. At the beginning of year 1, 100,000 kWh of generating capacity are available. Formulate a dynamic programming recursion that will enable the utility to minimize the total cost of meeting power requirements for the next T years" (Winston, 2004).

The problems to be investigated are as follows:

1. The extension of the above problem to arbitrary initial generating capacity and percentage (%) obsolescence of old generating capacity.
2. The incorporation of time value of money concept into (1) above.
3. Electronic implementation of the solutions of (1) and (2) on excel platform.

Dynamic programming and the Development of Operations Research

The concept of dynamic programming is largely based on mathematical recursions and the following Richard Bellman's principle of optimality as variously confirmed by Winston (2004), given the current state, the optimal decision must not depend on previously reached states or previously chosen decision. Future decisions for the remaining stages will constitute an optimal policy regardless of the policy adopted in previous stages, Taha (2006). Verma (2010): says an optimal policy (set of decisions) has the property that whatever be the initial stage and initial decisions, the remaining decisions must constitute an optimal policy for the state resulting from the first decisions. Gupta and Hira (2000) find in their research that an optimal policy (a sequence of decisions) has the property that whatever the initial stage and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. Inventory control is one of the most important aspects of today's complex supply chain management environment. One of the inventory models that have recently received renewed attention is that of the Newsboy Problem. Hadley and Whitin (1963) are credited for their seminal work on the classical version of this problem. Their models have been the foundation for many subsequent

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works extending the original models to other diverse scenarios and applications.

Nevertheless, despite its importance and the numerous publications related to the Newsboy Problem, wider dissemination, specifically in popular texts and subsequently in class rooms of operations management, operations research, and management science and like subjects for the multi-product Newsboy model and its variations remain limited. This could be attributed to the difficulties in the existing methodologies for such models which mostly require the utilization of numerical, iterative, and/or dynamic programming tools to solve, Zhang et. al (2009). Developing a solution method that is portable to the classroom environment in related fields is one of our main motivations for this work. Most importantly, we extend the developed models to cover what we designate as the Gardener Problem where both supply and demand are random in nature. As will be noted, a review of the literature reveals that research in this arena of random yielding suppliers for the multi-constrained, multi-product Newsboy Problem is still in its infancy. In this section, we briefly mention relevant research efforts in the Newsboy area. Following Hadley and Whitin's work (1963) where they present a dynamic programming solution approach to the multi constrained Newsboy Problem; several papers have been published advancing other solution methods that avoid the known complexities of dynamic programming particularly when the number of products and constraints is large.

Development of operations research paralleling that in Britain took place in Australia, Canada, France, and, most significantly for future developments, in the United States, which was the beneficiary of a number of contacts with British researchers. Sir Robert Watson-Watt, who with A.P. Rowe launched the first two operational studies of radar in 1937 and who claims to have given the discipline its name, visited the United States in 1942 and urged that operations research be introduced into the War and Navy departments (Russell and Keith, 2000). Reports of the British work had already been sent from London by American observers, and James B. Conant, then chairman of the National Defense Research Committee, had become aware of operations research during a visit to England in the latter half of 1940. Another stimulant was Blackett's memorandum, "Scientists at the Operational Level," of December 1941, which was widely circulated in the U.S. service departments (Kafka, 2000).

The first organized operations research activity in the United States began in 1942 in the Naval Ordnance Laboratory (Horner, 2000). This group, which dealt with mine warfare problems, was later transferred to the Navy Department, from which it designed the aircraft mining blockade of the Inland Sea of Japan. As in Britain, radar stimulated developments in the U.S. Air Force. In October 1942, all Air Force commands were urged to include operations research groups in their staffs. By the end of World War II there were 26 such groups in the Air Force. In 1943 Gen. George Marshall suggested to all theatre commanders that they form teams to study amphibious and ground operations (Sanders, 2000).

The early development of industrial operations research was cautious, and for some years most industrial groups were quite small. In the late 1950s, largely stimulated by developments in the United States, the development of industrial operations research in Britain was greatly accelerated. Although in the United States military research increased at the end of the war, and groups were expanded, it was not until the early 1950s that American industry began to take operations research seriously. The advent of the computer brought an awareness of a host of broad system problems and the potentiality for solving them and within the decade about half the large corporations in the United States began to use operations research. Elsewhere the technique also spread through industry (Hadley and Whitin, 1963). Societies were organized, beginning with the Operational Research Club of Britain, formed in 1948, which in 1954 became the Operational Research Society.

Similar developments took place in the British Army and the Royal Navy, and in both cases radar again was the instigator. In the army, use of operations research had grown out of the initial inability to use radar effectively in controlling the fire of anti-aircraft weapons. Since the traditional way of testing equipment did not seem to apply to radar gun sights, scientists found it necessary to test in the field under operating conditions, and the distinguished British physicist and future Nobel Laureate P.M.S. Blackett organized a team to solve the anti-aircraft problem (Wiston, 2004). Blackett's Anti-aircraft Command Research Group included two physiologists, two mathematical physicists, an astrophysicist, an army officer, a former surveyor, and subsequently a third physiologist, a general physicist, and two mathematicians. By 1942 formal operations research groups had been established in all three of Britain's military services (King, 2000).

Many other national societies appeared; the first international conference on operations research was held at Oxford University in 1957. In 1959 an International Federation of Operational Research Societies was formed. The first appearance of operations research as an academic discipline came in 1948 when a course in nonmilitary techniques was introduced at the Massachusetts Institute of Technology in Cambridge. In 1952 a curriculum leading to a master's and doctoral degree was established at the Case Institute of Technology (now Case Western Reserve University) in Cleveland. Since then many major academic institutions in the United States have introduced programs. In the United Kingdom courses were initiated at the University of Birmingham in the early 1950s (Galbraith, 1973). The first chair in operations research was created at the newly formed University of Lancaster in 1964. Similar developments have taken place in most countries in which a national operations research society exists. The first scholarly journal, the *Operational Research Quarterly*, published in the United Kingdom, was initiated in 1950; in 1978 its name was changed to the *Journal of the Operational Research Society*. It was followed in 1952 by the *Journal of the Operations Research Society of America*, which was renamed *Operations Research* in 1955. The International Federation of Operational Research Societies initiated the *International Abstracts in Operations Research* in 1961. Despite its rapid growth, operations research is still a relatively young scientific activity.

The concepts and methods of Operations Research (O.R.) are pervasive. Operations research consultants advise the public and private sectors on energy policy, design and operation of urban emergency systems, defense, health care, water resource planning, the criminal justice system and transportation issues. They also address a wide variety of design and operational issues in communication and data networks, computer operations, marketing, finance, inventory planning, manufacturing, and many areas designed to improve business productivity and efficiency (Ukwu, 2015 & 2016).

Methodology

The research methods that are appropriate for finding solutions to this problem in dynamic programming are analytical and computerized methods. But due to the modern world and advancement of technology, computer based solution is the best for the purpose of time utilization and sensitivity analyses. Elements of the dynamic programming (DP) model and the dynamic programming (DP) models and recursions are laid out as follows.

Problem 1:

Consider the problem (1) as stated in chapter 3: The extensions to arbitrary initial generating capacity and percentage (%) obsolescence of old generating capacity

"An electric power utility forecasts that r_t kilowatt-hours (KWh) of generating capacity will be needed during year t (the current year is year 1). Each year, the utility must decide by how much generating capacity should be expanded. It costs $c_t(x)$ dollars to increase generating capacity by x kWh during year t. It may be desirable to reduce capacity, so x need not be nonnegative. During each year, g % of the old generating capacity becomes obsolete and unusable (capacity does not become obsolete during its first year of operation). It costs the utility $d_t(i)$ dollars to maintain i units of capacity during year t. At the beginning of year 1, K kWh of generating capacity is available. Formulate a dynamic programming recursion that

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will enable the utility to minimize the total cost of meeting its power requirements for the next T years. ”

Elements of the relevant dynamic programming recursions:

The stage, represented by time, t

The alternatives at each stage (year) t, represented by x_t: the amount of generating capacity to add.

The state at stage t, represented by i_t: the amount of available generating capacity at the beginning of year t.

Let S_t denote the set of the alternatives at the beginning of year t. Note that $x_t \varepsilon S_t$. The recursive relations for cost minimization are obtained as follows:

Let $f_t(i_t)$ be the minimum cost incurred by the utility during years t, $t+1$, …, T, given that i_t kwh of generating capacity are available at the beginning of year t. Then by the Bellman's optimality principle, $f_t(i)$ = min{(cost during stage t) + f_{t+1}(new state at stage t +1)}.

The above equation translates as follows:

$f_t(i_t) = \min\{K_t + c_t(x_t) + d_t(i_t + x_t) + \beta_{t+1} f_{t+1}(\, i_{t+1})\}$

$(1) f_{T+1}(.) = 0$

where the minimum is over the set S_{t+1}.

Terms in the recursion:

K_t: Deterioration cost in period t

r_t : Power demand (kWh) during period t

i_t : Available electric power in kWh at the beginning of period t

$d_t(i_t)$: Maintenance cost for i_t kWh of power during period t

$d_t\ (x_t)$: Cost of increasing the generating capacity by x_t kWh during period t.

$C_t\ (x_t)$: Total cost incurred in period t.

Also the available capacity at the beginning of period $t+1$ is given as $i_{t+1} = (1 - 0.01 g_t) i_t + \ x_t - \ r_t \geq 0$ and i_t is the available electric power in kWh at the beginning of period t. i_{t+1} is the state at the beginning of year $t+1$,

$(1-0.01g_t)i_t$ is the undepreciated i_t at the end of year t, x_t added generated capacity in year t and r_t power demand in year t.

The above relation (1) is equivalent to

$$f_t(i_t) = \min\{c_t(x_t) + d_t(i_t + x_t) + f_{t+1}(\,i_{t+1})\}$$

$f_t(i_t) = \min\{K_t + c_t(x_t) + d_t(i_t + x_t) + f_{t+1}\big((1-0.01g_t)i_t + \ x_t - r_t\ \big)\}$,
(2) $\quad$ $\mathrm{t} \in \{\mathrm{T}, \mathrm{T}-1, \dots, 2,1\}$
where $f_{T+1}(.) = -s(.)$,
Penultimate stage is given as $T-1$, where the minimum is over all feasible x_t.
The terminal condition $f_{T+1}(.) = 0$ simply states the obvious fact that there are no costs incurred at the end of the horizon (from the beginning of year T+1)
Constraints on $\boldsymbol{x_t}$:
$i_t + x_t \ \geq\ r_t$ or $x_t \ \geq\ r_t$- i_t so the feasible of x_t are those values of x_t satisfying

$$x_t \ \geq\ r_t -\ i_t.$$

Cost incurred in year t:
If x_t kWh is added during a year that begins with i_t kWh of available capacity, then during year t, the sum of the generating cost and the utility maintenance cost is given by
$c_t(x_t) + d_t(i_t + x_t)$,
where
$c_t(x_t) =\ \mathrm{k}_0 +\ \mathrm{k}_1 x +\ \mathrm{k}_2\mathrm{x}^2 + \mathrm{k}_3\mathrm{x}^3 + \mathrm{k}_4\mathrm{x}^4$,
$d_t(y_t) =\ \mathrm{d}_0 +\ \mathrm{d}_1 y +\ \mathrm{d}_2\mathrm{y}^2 + \mathrm{d}_3\mathrm{y}^3 + \mathrm{d}_4\mathrm{y}^4$.
for some real constants $\mathrm{k}_0, \mathrm{k}_1, \mathrm{k}_{2,} \dots \mathrm{k}_4$ and d_0, d_1..., d_4 .
The state at the beginning of year t:
The state at the beginning of year t+1 will be $i_{t+1} = i_t + x_t - r_t$.

Attribute:

c_t, d_t, and r_t must be continuous functions due to practical considerations.

$$c_t(x_t) \gg\ d_t(i_t)\ \forall\, x_t \geq\ i_t$$

$c_t(x_t)\ >\ d_t(i_t)$ if $0 <\ i_t -\ x_t$ is small enough.
During year 1 of operation, generating capacity = installed capacity i.e. all things being equal (optimal supply in thermal condition). System is generating optimally. This means
$c_t(x_t) = 0$ iff $x_t = 0$.

$i_{t+1} = (1 - 0.01g_t)i_t,\ \ t \in \{T, T-1, \dots, 2,1\}$
Every year some turn around maintenance is carried out on the electric power facility to restore the generating power to its initial capacity (capacity during year t).
Maintenance cost for restoration K_t (restoration maintenance cost): this impacts on the generating power only.

Problem 2:
The incorporation of time value of money concept modifies the recursion in problem 1 to
$f_t(i_t) = \min\{K_t + c_t(x_t) + d_t(i_t + x_t) + \beta f_{t+1}(\, i_{t+1})\}\ , K_1 = 0, f_{T+1}(.) = 0$
(3)
The definitions and notations in problem 1 are all preserved.
In (3) and (4) β is a discount factor in fractional or decimalized format. Note that

$$\beta = \frac{1}{1+r} \tag{4}$$

3 where r is the positive discount rate per period in fractional or decimalized format. Therefore

$$0 < \beta < 1 \tag{5}$$

Problem 3:
Electronic implementation of problem 1 and 2 on excel platform; Excel solution template for cost minimization of generating capacity of certain electric power, interface and codes involved;

Excel Implementation Template Design and interface for generating Optimal Production Schedule

Terminal Stage, T							
	A	B	C	D	E	F	G
1	Excel Solution…						
2	Recursions						
3							
4	No. of periods:	T	Max Dem =	5			

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5	$c(x) = k_0 + k_1x + k_2x^2 + k_3x^3 + k_4x^4$						
6							
7	coefficients of c:	k_0	k_1	k_2	k_3	k_4	
8	coefficients of d:	d_0	d_1	d_2	d_3	d_4	
9	Demand:	r_T					
10		Stage	= [1]				
11	I	$x(i)$	$f(i) = c(x)$				
12	0	= [3]	= [5v]				
13	= [2v]	=[4v]					
14							
15							
16							
17							

Penultimate Stage, T – 1							
19	Demand:	r_{T-1}					
20		Stage	= [6]	Computations			
21	I	$x(i)$	$C(x) + d(i + x)$	$f(i + x - r)$	C+D	x^*	$f(i)$
22	0	= [8]	=[10v]	=[11v]	= [12v]	=[13v]	= [14v]
23	= [7v]	= [9v]					
24							
25							
26							
27							
28							
29							
30							
31							

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33	1	= [8]	=[10v]	=[11v]	= [12v]	=[13v]	= [14v]
34	= [7v]	= [9v]					
35							
36							
37							
38							
39							
40							
41							
42							

Stage (T-2)							
91	Demand :	r_{T-2}					
92		Stage	= [15]	Computations			
93	I	$x(i)$	$C(x) + (i + x)$	$f(i + x - r)$	C+D		$f(i)$
.	0	= [17]	=[19v]	=[20v]	= [21v]	=[22v]	= [23v]
.	= [16v]	= [18v]					
.							

100							

The tabular process continues down to the last stage, stage 1. Any stage number less than one indicates infeasibility. Blank cells also indicate forbidden choices.

Exposition on the Solution Template

1. "= [m] "indicates code segment m to be typed in the resident cell location.

"= [mv] "with letter v fixed indicates code segment m to be typed in the resident cell location followed by vertical crosshair-dragging activity.
Type the titles of the template in excel rows 1 and 2 as indicated above. Type the production cost function c(x) and their parameters in the indicated cell locations. An identifier succeeded by a colon indicates desired user input subject to the imposed restrictions.

2. Initialization of Stage numbering:
3. = [1]: Type the following code segment =$B$4, in cell location C10, <Enter> to initialize the terminal stage number at integral input value m.

Stage T Implementations

Incoming inventory, *i*:

Step 1: Input the number 0 in cell location A12.
Step 2: = [2v]: Type the following code segment =IF($A12<MAX($B$9,$D$4),1+$A12,""), in cell location A12,<Enter> to secure the next feasible higher contiguous integer. Click the cursor back on A12, position the cursor at the bottom right edge of the cell until a crosshair appears. Then drag the crosshair vertically down to cell location A21, to secure all feasible values of *i*.
Henceforth the act of clicking the cursor back on an indicated cell location, positioning the cursor at the bottom right edge of the cell until a crosshair appears and then dragging the crosshair vertically down to an indicated cell location will be described as clerical routine/duty.

Stage T Production Quantity, x_T:
Step 1: =[3]: “=$B$9-$A12”, in B12,<Enter>, to initialize the production quantity
Step 2. =[4v]: “=IF($A13 ="","",MAX($B$9-$A13,0))”, in B12, followed by the clerical duty, down to B21.

Minimum Costs, $f_T(i)$ for Entering Inventory i
[5v]:
“=IF($A13="","",($B$7+$C$7*$B13+$D$7*$B13^2+$E$7*$B13^3+$F$7*$B13^4+$B$8+$C$8*($A13+$B13)^1+$D$8*($A13+$B13)^2+$E$8*($A13+$B13)^3+$F$8*($A13+$B13)^4+$D$9))”, in C12, followed by the clerical duty, down to C21.

Stage (T-1) Computations.
Incoming inventory, $i = 0$:
Step 1: Input the number 0 in cell location A26.
Step 2: = [7v]: Type the following code segment “=IF(COUNT($A$26:$A26)<MAX($B$9+$B$23,$D$4),0,"")”, in cell location A26,followed by the clerical duty down to A35.

Production Quantity, $x_{T-1}(i)$:
Step 1: =[8]: “=MAX($B$23-$A26,0)<Enter>
Step 2. =[9v]:
“=IF(OR($A27="",$B26>=MAX($B$9+$B$23,$D$4)),"",1+$B26)”,
in B23, followed by the clerical duty, down to B35.
Implementation of

$$C_t(x_t) + (i_t + x_t - r_t) + K_t$$

=[10v]:
“=IF(OR($A26="",$B26=""),"",($B$7+$C$7*$B26+$D$7*$B26^2+$E$7*$B26^3+$F$7*$B26^4+$B$8+$C$8*($A26+$B26)^1+$D$8*($A26+$B26)^2+$E$8*($A26+$B26)^3+$F$8*($A26+$B26)^4+$D$23)), in C22, followed by the clerical duty, down to C35.
Implementation of *f (i+x-r)*

=[11v]: “=IF(OR($A26="",$B26="",MAX($A26,0)+MAX($B26,0)-MAX($B$23,0)>=6),"",$H$9*(IF($A26+MAX($B26,0)-MAX($B$23,0)=0,MAX($C$12,0),IF($A26+MAX($B26,0)-MAX($B$23,0)=1,MAX($C$13,0),IF($A26+MAX($B26,0)-MAX($B$23,0)=2,MAX($C$14,0),IF($A26+MAX($B26,0)-MAX($B$23,0)=3,MAX($C$15,0),IF($A26+MAX($B26,0)-MAX($B$23,0)=4,MAX($C$16,0),IF($A26+MAX($B26,0)-MAX($B$23,0)=5,MAX($C$17,0),""))))))))”,
in D22, followed by the clerical duty, down to D35.

Implementation of C+D

=[12v]: “=IF(OR($C26="",$D26=""),"",$C26+$D26)”,
in E22, followed by the clerical duty, down to E35.
Implementation of x*
=[13v]: “=IF($E26=MIN($E$26:$E$35),$B26,"")”,
in F22, followed by the clerical duty, down to F35.
Minimum Costs, $f_{T-1}(i)$, for Entering Inventory
= [14v]: “=IF($F26="","",$E26)”,
inG22, followed by the clerical duty, down to G35.
Incoming inventory, i = 1:
Step 1: Input the number 1 in cell location A37
Step 2: = [7v]: Type the following code segment “=IF(COUNT($A$37:$A37)<MAX($B$9+$B$23,$D$4),1,"")”, in cell location A38,followed by the clerical duty down to A47.
Production Quantity, $\boldsymbol{x_{T-1}(i)}$:
Step 1: =[8]: “=MAX($B$23-$A37,0)<Enter>
Step 2: = [9v]: “=IF(OR($A38="",$B37>=MAX($B$9+$B$23,$D$4)),"",1+$B37)”,
in B38, followed by the clerical duty, down to B47.

Implementation of $(i+x-r) + c(x)$

=[10v]:“=IF(OR($A37="",$B37=""),"",($B$7+$C$7*$B37+$D$7*$B37^2+$E$7*$B37^3+$F$7*$B37^4+$B$8+$C$8*($A37+$B37)^1+$D$8*($

A37+$B37)^2+$E$8*($A37+$B37)^3+$F$8*($A37+$B37)^4+$D$23))
, in C37, followed by the clerical duty, down to C47.

Implementation of *f (i+x-r)*

=IF(OR($A37="",$B37="",MAX($A37,0)+MAX($B37,0)-MAX($B$23,0)>=6),"",$H$9*(IF($A37+MAX($B37,0)-MAX($B$23,0)=0,MAX($C$12,0),IF($A37+MAX($B37,0)-MAX($B$23,0)=1,MAX($C$13,0),IF($A37+MAX($B37,0)-MAX($B$23,0)=2,MAX($C$14,0),IF($A37+MAX($B37,0)-MAX($B$23,0)=3,MAX($C$15,0),IF($A37+MAX($B37,0)-MAX($B$23,0)=4,MAX($C$16,0),IF($A37+MAX($B37,0)-MAX($B$23,0)=5,MAX($C$17,0),"")))))))),

in D37, followed by the clerical duty, down to D47.

Implementation of C+D

=[12v]: "=IF(OR($C37="",$D37=""),"",$C37+$D37)",

in E37, followed by the clerical duty, down to E47.

Implementation of x*

=[13v]: "=IF($E37=MIN($E$37:$E$46),$B37,"")",

in F37, followed by the clerical duty, down to F47.

Minimum Costs,$f_{T-1}(i)$, for Entering Inventory

= [14v]: "=IF($F37="","",$E37)", in G37, followed by the clerical duty, down to G47.

Incoming inventory i

Step 1: Input the number *i* in cell location A(37+(*i* -1)*11)

Step 2: Replace the " 37" in the code segment " = [7v] :" with(37+11(*i*-1)) 1n cell location A(1+(37+11(*i*-1))),followed by the clerical duty down to A(9+(1+(37+11(*i*-1)))).

Production Quantity, $x_{T-1}(i)$

Add 11(i-1) to all relative row references in steps 1 and 2 corresponding to i =1,incorporating "=[8]:" and " =[9v]:".

Implementation of *(i+x-r)+c(x)*:

Add 11(i-1) to all relative row references in " =[10v]:" and the ensuing clerical duty.

Implementation of *f (i+x-r)*:

Add 11(i-1) to all relative row references in " =[11v]:" and the ensuing clerical duty.

Implementation of *C+D*:

Add 11(i-1) to all relative row references in " =[12v]:" and the ensuing clerical duty.

Implementation of *x**:

Add 11(i-1) to all relative row references in " =[13v]:" and the ensuing clerical duty.

Minimum Costs, $f_{T-1}(i)$:

Add 11(i-1) to all relative row references in " =[14v]:" and the ensuing clerical duty.

Stage (T-2).

Implementations of *i, xi, (i+x-r)+c(x)*:

Add 69 to all row references of cells in stage T-1, for corresponding *i* values, excluding the $B$8:$F$8, with global scope.

Implementations of *f (i+x-r)*:

Replace the code segment in $D26 (for i = 0, in stage T-1) with the code segment below:

"=IF(OR($A95="",$B95="",MAX($A95,0)+MAX($B95,0)-MAX($B$92,0)>=6),"",$H$23*(IF($A95+MAX($B95,0)-MAX($B$92,0)=0,MAX($G$26:$G$35,0),IF($A95+MAX($B95,0)-MAX($B$92,0)=1,MAX($G$37:$G$46,0),IF($A95+MAX($B95,0)-MAX($B$92,0)=2,MAX($G$48:$G$57,0),IF($A95+MAX($B95,0)-MAX($B$92,0)=3,MAX($G$59:$G$68,0),IF($A95+MAX($B95,0)-MAX($B$92,0)=4,MAX($G$70:G$79,0),IF($A95+MAX($B95,0)-MAX($B$92,0)=5,MAX($G$81:$G$90,0),""))))))))", without the quotes.

Then apply the clerical duty from the input cell $D95down to $D159.

For $i \in \{1,2,3,4,5\}$, add 11i to all relative row references corresponding to *i* = 0.

Implementations C+D, x* and f (i):

The code segments for **C+D, x* and f (i)** are invariant and hence may be copy from stage T-1 down to stage 1.

Stage t, $t \in \{T-3, \cdots, 1\}$.

Implementations of *i, x(i),(i+x)+c(x)*:

Add $69(T-2-t)$ to all row references of cells in stage T-2, for corresponding i values, excluding the $B$8:$F$8, with global scope, bringing the exposition to an end.

Optimal Generating Policy

Table 4.1.1(a,b,c & d) furnishes in particular, the alternative generated power for each entering inventory, the optimal generated power and the optimal (minimum) cost.

Desired optimal results can be read bottom up starting from stage 1. For example if the demand is 4,000kWh and the entering inventory is 1,000kWh in stage 1and it is optimal to generated 3,000kWh. Then the associated minimum cost for stage 1, to the terminal stage is N73.81m. The generating inventory is 0 kWh. Then taking in to stage 2 with entering inventory 0kWh, where the demand is optimal to generate 6,000kWh. The minimum cost for stage 2 to the terminal stage is N59.78m. The entering inventory is 2,000kWh. This takes in to stage 3 where the demand is 4,000kWh and the entering inventory is 2,000kWh. It is optimal to generate 2,000kWh. The corresponding cost for year 3 to the leading stage is N37.67m. the entering inventory is 0. This takes into the leading stage, stage 4. Here the demand is 5,000kWh and the entering inventory is 0. It is optimal to produce 5,000kWh at the cost of N22m.

4.1 Summary of Results

Table 1:

Stage **t**	Demand r_t in 1,000kWh	Vector of entering inventory levels $\{i_t\}$ in 1,000kWh	Corresponding vector of optimal generating qty $\{x_t(i_t)\}$ in 1,000kWh	Corresponding vector of minimum cost from stage t to stage $T = 4$ in million of naira	Vector of ending inventory levels i_{t+1} in 1,000kWh
1	4	(0,1,2,3,4,5)	(4,3,2,1,0,0)	(74.81,73.81,72.81,71.81,70.81,70.82)	(0,0,0,0,0,1)

2	6	(0,0,0,0,0,1)	(6,6,6,6,6,5)	(57.78,59.78,59.78,59.78,59.78,58.78)	(2,0,0,0,0,0)
3	4	(2,0,0,0,0,0)	(2,4,4,4,4,4)	(37.67,39.67,39.67,39.67,39.67,39.67)	(0,0,0,0,0,1)
4	5	(0,0,0,0,0,1)	(0,0,0,0,0,0)	(22,22,22,22,22,21)	(0,0,0,0,0,0)

Table 2:

Stage t	Demand r_t in 1,000kWh	Vector of entering inventory levels $\{i_t\}$ in 1,000kWh	Corresponding optimal generating qty $\{x_t(i_t)\}$ in 1,000kWh	Corresponding vector of minimum cost from stage t to stage $T = 4$ in million of naira $f_t(i_t)$	Vector of ending inventory levels i_{t+1} in 1,000kWh
1	4	0	4	(74.81,73.81,72.81,71.81,70.81,70.82)	0
2	6	0	6	(57.78,59.78,59.78,59.78,59.78,58.78)	2
3	4	2	2	(39.67,37.67,37.67,37.67,37.67,39.67)	0
4	5	0	0	(22,21,20,19,18,17)	0

Discussion

This work furnishes carefully, the inventory i, the generated electric power X^* and the optimal value of generated electric current $f(i)$ as obtained at each stage in the table above. The results in the table are read from the first stage to the last stage (bottom-up). The restoration cost K_t is increasing function of T due to system deterioration.

Thus, the study has unveiled the tremendous computational power of the electronic solution templates for optimal generating cost for certain electric power. The templates circumvent the need for error-prone cumbersome manual computations. The templates may be optimally deployed for sensitive analyses. Note that it is virtually impossible to perform the computations manually, especially for large-sized problems. Electronic computations are the only way forward for solving practical problems of reasonable size and the undertaking sensitivity analyses. The computations can be easily demonstrated in just a matter of seconds.

Conclusion and Recommendation

The solution templates can be modified to solve other classes of practical problems with similar structure, subject to restrictions on demands or their equivalents. The maximum demand of 5,000 kWh may be relaxed at the cost of generating more template rows at each stage and consequently more tables. The result of this study is of tremendous interest to stake holders in the Nigerian power sector. This is because, the study contributes to the body of knowledge by electronically implementing the optimal generating power of a power plant and providing the corresponding algorithm, with incorporation of time value of money concepts, subject to the prevailing constraints. The research recommends among others that appropriate measures need to be taken in boosting generating capacity of certain electric power supply.

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