BERKELEY PUBLICATION AND RESEARCH INTERNATIONAL

03.31.2020

Journal of Pure and Applied Science Vol. 12 No. 9

ON THE DETERMINISTIC DIFFERENTIAL EFFECT OF THE RATE OF FORMATION OF VAPOUR PHASE ON THE DISTRIBUTION OF CARBON DIOXIDE IN THE ATMOSPHERE.

UDOH, N. A.[1], EKAKA-A, E. N.[2], OKOKO, E. E.[3], EZEORAH, J. N.[4]

[1]Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt, Nigeria. [2]Rivers State University of Science and Technology, Nkpolu, Port Harcourt, Nigeria. [3]Benue State Polytechnic, Ugbokolo, Nigeria. [4]University of Calabar, Nigeria.

ABSTRACT

The deterministic modeling of the effect of the rate of formation of vapour phase on the distribution of carbon dioxide in the atmosphere is one of the challenging problems in the discipline of the environmental science informatics. To tackle this problem numerically, we have utilized the simulation method of ODE 45. From the novel results that we have obtained that we have not seen elsewhere, we have observed that an increase in the rate of formation of vapour phase will lead to a high concentration of carbon dioxide in the atmosphere.

Keywords *– ODE 45, Vapour Density, Mathematical Quantification, Interpolation.*

Introduction:

Carbon dioxide in the atmosphere is important because it helps to absorb heat in the atmosphere without which the planet would be unfavourably cold [1]. However, a gradual increase in carbon dioxide concentrations in the atmosphere causes global warming as a result of high temperature which in turn leads to several adverse effects on human and its environment [2-7]. There is a link between the growing pattern of carbon dioxide in the atmosphere and the environment, and this bothers on the rate of formation of vapour phase in the atmosphere. In the modeling of the interpolation of the density of vapour molecules in the atmosphere, the popular method usually used had been the nonlinear ordinary differential equations of first order [8-17]. For the purpose of effective environmental management, in this present work, an efficient mathematical technique which is called the ordinary differential equation of order 45 will be utilized for the quantification of the effect of the rate of formation of vapour phase on the spread of carbon dioxide in the atmosphere.

MODEL EQUATIONS

Following Sundar et al 2009, the following continuous dynamical system of nonlinear differential equations of first order will be considered:

$$\frac{d}{dt}C_v = Q_v - \mu_0 C_v + \mu_1 \lambda_1 C_d C + \mu_2 r_1 C_r C \quad (1)$$

$$\frac{d}{dt}C_d = \lambda C_v - \lambda_0 C_d - \lambda_1 C_d C, \quad (2)$$

$$\frac{d}{dt}C_r = rC_d - r_0 C_r - r_1 C_r C \quad (3)$$

$$\frac{d}{dt}C = Q - \delta C - \alpha C C_r + \theta k C_a + \pi v C_a C_r \quad (4)$$

$$\frac{d}{dt}C_a = \alpha C C_r - kC_a - vC_a C_r. \quad (5)$$

with the following initial boundary conditions: $C_v(0) \geq 0, C_d(0) \geq 0, C_r(0) \geq 0, C(0) \geq 0$ **and** $C_a(0) \geq 0$ where:

$C_v(t)$ is the density of the vapour molecules at time t in unit of months,

$Q_v(t)$ is the rate of formation of vapour phase in the atmosphere,

$C_d\ (t)$ is the density of cloud droplets,

$C_r\ (t)$ is the density of raindrops in the atmosphere,

$C_a\ (t)$ is the cumulative concentration of gaseous pollutants in the absorbed phase,

(t) is the cumulative concentration of hot gases in the atmosphere,

Q is the cumulative emission rate of gaseous pollutants,

μ_0 is the natural depletion rate coefficient of vapour phase,

μ_1 is the interaction rate coefficient of cloud droplets and hot gases,

$\boldsymbol{\mu_2}$ is the interaction rate coefficient of raindrops and hot gases,
r is the growth rate coefficient of raindrops due to cloud droplets,
$\boldsymbol{r_0}$ is the natural depletion rate of raindrops,
$\boldsymbol{r_1}$ is the depletion rate coefficient of raindrops due to interaction with hot gases,
λ is the growth of cloud droplets formed from the vapour phase,
$\boldsymbol{\lambda_0}$ is the natural depletion rate of density of cloud droplets,
$\boldsymbol{\lambda_1}$ is the depletion rate coefficient of cloud droplets due to interaction with hot gases, b
δ is the natural removal rate coefficient of hot gases in the atmosphere,
k is the natural removal rate coefficient of gaseous pollutants in the atmosphere,
α is the removal rate coefficient of hot gases due to interactions with raindrops,
ν is the removal rate coefficient of gaseous pollutants due to interactions with raindrops,
θ and π are reversible rate coefficients.
$\theta \leq 0$ and $\pi \leq 1$.
The parameter values used in this work are all taken from published [16]. These values are: $\boldsymbol{\mu_0} = 2.8$, $\boldsymbol{\mu_1} = 0.001$, $\boldsymbol{\mu_2} = 0.001$, $\lambda = 2.5$, $\boldsymbol{\lambda_0} = 0.1$, $\boldsymbol{\lambda_1} = 0.8$, $r = 0.8$, $\boldsymbol{r_0} = 0.7$, $\boldsymbol{r_1} = 0.01$, $\delta = 0.35$, $\alpha = 0.65$, $k = 0.40$, $\theta = 0.001$, $\pi = 0.00001$, $\nu = 0.50$, $\boldsymbol{Q} = 1$ and $\boldsymbol{Q_v} = 15$.

METHOD OF ANALYSIS

In this study, the model equations consist of seventeen (17) deterministic parameter values each of which is varied at 100%. However, considering the fact that the atmosphere cannot be static over a long period of time [16], so, in the event of environmental perturbation, what will be its attending effect on the concentration of carbon dioxide in the atmosphere? It is against this background that we are proposing an efficient numerical method called ordinary differential equation of order 45 (ODE 45) numerical simulations to quantify the effect of the variation of a model parameter $\boldsymbol{Q_v}$ on the concentration of carbon dioxide in the atmosphere.

RESULTS

The full results which we have obtained by applying the method explained above are presented as shown in Tables 1-7 as follows:

Table 1: Quantifying the effect of a 10% variation of the rate of formation of vapour phase value of 15 on the density of vapour molecules $\boldsymbol{C_v}$ using ODE45 numerical simulation: $\boldsymbol{Q_v}$ =1.50

Empirical Example	Time (days)	$C_v(old)$	$C_v(new)$	EPD (%)
1	1	0.800000000000000	0.800000000000000	0.000000000000000
2	31	5.357170616447103	0.535985727016081	89.994984939054461
3	61	5.357159013310056	0.535981854939583	89.995035547611778
4	91	5.357155358430993	0.535984840920682	89.994972983616023
5	121	5.357153369623351	0.535965262794487	89.995335100461389
6	151	5.357152508610882	0.536006335415905	89.994566431432574
7	181	5.357151806375762	0.535950834046728	89.995601143710886
8	211	5.357151307285308	0.535900182066694	89.996545713802945
9	241	5.357150934339046	0.536016834623474	89.994367506287062
10	271	5.357150645085467	0.535938843128204	89.995822805172338
11	301	5.357150414195882	0.535831274869963	89.997830311987002
12	331	5.357150225628127	0.536063498540664	89.993495124027234
13	361	5.357150068724607	0.536066096212524	89.993446341141109
14	391	5.357149936126265	0.535943224523864	89.995739695286517
15	421	5.357149822593103	0.536030514065079	89.994110080617148
16	451	5.357149724292417	0.536029256900302	89.994133364060460
17	481	5.357149638339301	0.535997949351112	89.994717610365839
18	511	5.357149562566436	0.535983117770514	89.994994324673257
19	541	5.357149495246379	0.535977412552642	89.995100696214720
20	571	5.357149435046462	0.536006008224736	89.994566798562943

Table 2: Quantifying the effect of a 50% variation of the rate of formation of vapour phase value of 15 on the density of vapour molecules $\boldsymbol{C_v}$ using ODE45 numerical simulation: $\boldsymbol{Q_v}$= 7.50

Empirical Example	Time (days)	$C_v(old)$	$C_v(new)$	EPD (%)
1	1	0.800000000000000	0.800000000000000	0.000000000000000
2	31	5.357170616447103	2.678602713816083	49.999675097289639

3	61	5.357159013310056	2.678589119732624	49.999820556426030
4	91	5.357155358430993	2.678584900936724	49.999865194851665
5	121	5.357153569623351	2.678582849129042	49.999886799635526
6	151	5.357152508610882	2.678581636536837	49.999899531861622
7	181	5.357151806375762	2.678580835897414	49.999907922909202
8	211	5.357151307285308	2.678580267833972	49.999913868564597
9	241	5.357150934339046	2.678579843893738	49.999918301271173
10	271	5.357150645085467	2.678579515419371	49.999921733083205
11	301	5.357150414195882	2.678579253433253	49.999924468514060
12	331	5.357150225628127	2.678579039612610	49.999926699861291
13	361	5.357150068724607	2.678578861794013	49.999928554704262
14	391	5.357149936126265	2.678578711591663	49.999930120893104
15	421	5.357149822593103	2.678578583041633	49.999931460847598
16	451	5.357149724292417	2.678578471775404	49.999932620341383
17	481	5.357149638339301	2.678578374527280	49.999933633408261
18	511	5.357149562566436	2.678578288803516	49.999934526369728
19	541	5.357149495246379	2.678578212674078	49.999935319130230
20	571	5.357149435046462	2.678578144610122	49.999936027789914

Table 3: Quantifying the effect of a 90% variation of the rate of formation of vapour phase value of 15 on the density of vapour molecules $\boldsymbol{C_v}$ using ODE45 numerical simulation:$\boldsymbol{Q_v}$= 13.50

Empirical Example	Time (days)	$\boldsymbol{C_v(old)}$	$\boldsymbol{C_v(new)}$	EPD (%)
1	1	0.800000000000000	0.800000000000000	0.000000000000000
2	31	5.357170616447103	4.821456677568016	9.999941708677078
3	61	5.357159013310056	4.821444879304216	9.999967010029732
4	91	5.357155358430993	4.821441168942653	9.999974868103143
5	121	5.357153569623351	4.821439354167424	9.999978691922982
6	151	5.357152508610882	4.821438278172799	9.999980952138225
7	181	5.357151806375762	4.821437566193838	9.999982444857148
8	211	5.357151307285308	4.821437060272843	9.999983503992795
9	241	5.357150934339046	4.821436682262765	9.999984294681376
10	271	5.357150645085467	4.821436389124861	9.999984907127050
11	301	5.357150414195882	4.821436155147279	9.999985395762204
12	331	5.357150225628127	4.821435964069418	9.999985794609593

13	361	5.357150068724607	4.821435805086639	9.999986126308144
14	391	5.357149936126265	4.821435670738510	9.999986406487025
15	421	5.357149822593103	4.821435555711917	9.999986646292378
16	451	5.357149724292417	4.821435456116427	9.999986853956159
17	481	5.357149638339301	4.821435369051685	9.999987035151877
18	511	5.357149562566436	4.821435292282674	9.999987195188906
19	541	5.357149495246379	4.821435224087269	9.999987337192501
20	571	5.357149435046462	4.821435163107111	9.999987464129875

Table 4: Quantifying the effect of a 99.99% variation of the rate of formation of vapour phase value of 15 on the density of vapour molecules $\boldsymbol{C_v}$ using ODE45 numerical simulation: $\boldsymbol{Q_v}$=14.9985

Empirical Example	Time (days)	$C_v(old)$	$C_v(new)$	EPD (%)
1	1	0.800000000000000	0.800000000000000	0.000000000000000
2	31	5.357170616447103	5.356634902444021	0.009999942907120
3	61	5.357159013310056	5.356623299159944	0.009999967310671
4	91	5.357155358430993	5.356619644228195	0.009999975116557
5	121	5.357153569623351	5.356617855390781	0.009999979011371
6	151	5.357152508610882	5.356616794375676	0.009999981041144
7	181	5.357151806375762	5.356616092136533	0.009999982427066
8	211	5.357151307285308	5.356615593040035	0.009999983471521
9	241	5.357150934339046	5.356615220083317	0.009999984362873
10	271	5.357150645085467	5.356614930825566	0.009999984980684
11	301	5.357150414195882	5.356614699939392	0.009999985347996
12	331	5.357150225628127	5.356614511363572	0.009999985850528
13	361	5.357150068724607	5.356614354452824	0.009999986278353
14	391	5.357149936126265	5.356614221857379	0.009999986471787
15	421	5.357149822593103	5.356614108322683	0.009999986712348
16	451	5.357149724292417	5.356614010020510	0.009999986923596
17	481	5.357149638339301	5.356613924071091	0.009999987015038
18	511	5.357149562566436	5.356613848291991	0.009999987272867
19	541	5.357149495246379	5.356613780972557	0.009999987386892
20	571	5.357149435046462	5.356613720773011	0.009999987492335

Table 5: Quantifying the effect of a 101% variation of the rate of formation of vapour phase value of 15 on the density of vapour molecules $\boldsymbol{C_v}$ using ODE45 numerical simulation:$\boldsymbol{Q_v}$=15.15

Empirical Example	Time (days)	$C_v(old)$	$C_v(new)$	EPI (%)
1	1	0.800000000000000	0.800000000000000	0.000000000000000
2	31	5.357170616447103	5.410742014557371	0.999994249684676
3	61	5.357159013310056	5.410730428560112	0.999996735526354
4	91	5.357155358430993	5.410726778541104	0.999997508487433
5	121	5.357153569623351	5.410724992030169	0.999997885268485
6	151	5.357152508610882	5.410723932329523	0.999998107810666
7	181	5.357151806375762	5.410723230949793	0.999998254861349
8	211	5.357151307285308	5.410722732455848	0.999998359159401
9	241	5.357150934339046	5.410722359953150	0.999998437055672
10	271	5.357150645085467	5.410722071042264	0.999998497446453
11	301	5.357150414195882	5.410721840423892	0.999998545608341
12	331	5.357150225628127	5.410721652078160	0.999998584951985
13	361	5.357150068724607	5.410721495355165	0.999998617610355
14	391	5.357149936126265	5.410721362911249	0.999998645244590
15	421	5.357149822593103	5.410721249507484	0.999998668852797
16	451	5.357149724292417	5.410721151318196	0.999998689281637
17	481	5.357149638339301	5.410721065468625	0.999998707259010
18	511	5.357149562566436	5.410720989778024	0.999998722938834
19	541	5.357149495246379	5.410720922538055	0.999998737000203
20	571	5.357149435046462	5.410720862409632	0.999998749572044

Table 6: Quantifying the effect of a 105% variation of the rate of formation of vapour phase value of 15 on the density of vapour molecules $\boldsymbol{C_v}$ using ODE45 numerical simulation:$\boldsymbol{Q_v}$=15.75

Empirical Example	Time (days)	$C_v(old)$	$C_v(new)$	EPI (%)
1	1	0.800000000000000	0.800000000000000	0.000000000000000
2	31	5.357170616447103	5.625027613516266	4.999971370088755
3	61	5.357159013310056	5.625016092377736	4.999983730223054
4	91	5.357155358430993	5.625012460824312	4.999987576835350
5	121	5.357153569623351	5.625010682927926	4.999989450058050

6	151	5.357152508610882	5.625009628214420	4.999990557912897
7	181	5.357151806375762	5.625008930076169	4.999991289804780
8	211	5.357151307285308	5.625008433860224	4.999991809278400
9	241	5.357150934339046	5.625008063040117	4.999992197048638
10	271	5.357150645085467	5.625007775422754	4.999992497560495
11	301	5.357150414195882	5.625007545831611	4.999992737294368
12	331	5.357150225628127	5.625007358319889	4.999992933003022
13	361	5.357150068724607	5.625007202292939	4.999993095808520
14	391	5.357149936126265	5.625007070427659	4.999993233250454
15	421	5.357149822593103	5.625006957524381	4.999993350972272
16	451	5.357149724292417	5.625006859761396	4.999993452756419
17	481	5.357149638339301	5.625006774289343	4.999993541958939
18	511	5.357149562566436	5.625006698922642	4.999993620261818
19	541	5.357149495246379	5.625006631972767	4.999993690003771
20	571	5.357149435046462	5.625006572111341	4.999993752508710

Table 7: Quantifying the effect of a 110% variation of the rate of formation of vapour phase value of 15 on the density of vapour molecules C_v using ODE45 numerical simulation: Q_v=16.50

Empirical Example	Time (days)	$C_v(old)$	$C_v(new)$	EPI (%)
1	1	0.800000000000000	0.800000000000000	0.000000000000000
2	31	5.357170616447103	5.892884625280804	9.999943014489766
3	61	5.357159013310056	5.892873177863662	9.999967580252994
4	91	5.357155358430993	5.892869567281929	9.999975229537425
5	121	5.357153569623351	5.892867799205299	9.999978955608201
6	151	5.357152508610882	5.892866750150006	9.999981159357265
7	181	5.357151806375762	5.892866055697758	9.999982615471552
8	211	5.357151307285308	5.892865562068649	9.999983649048909
9	241	5.357150934339046	5.892865193161474	9.999984420609259
10	271	5.357150645085467	5.892864907017948	9.999985018601896
11	301	5.357150414195882	5.892864678594746	9.999985495633613
12	331	5.357150225628127	5.892864492031135	9.999985885036404
13	361	5.357150068724607	5.892864336788533	9.999986208925920
14	391	5.357149936126265	5.892864205588070	9.999986482535867
15	421	5.357149822593103	5.892864093250838	9.999986716787864

Vol. 12, No.9
International Journal of Pure and Applied Science

16	451	5.357149724292417	5.892863995979040	9.999986919487885
17	481	5.357149638339301	5.892863910927858	9.999987096769361
18	511	5.357149562566436	5.892863835937016	9.999987252809435
19	541	5.357149495246379	5.892863769322915	9.999987391651057
20	571	5.357149435046462	5.892863709750238	9.999987515732430

DISCUSSION OF RESULTS

The differential effect of the variation of the rate of formation of vapour phase $\boldsymbol{Q_v}$ on the distribution of carbon dioxide in the atmosphere is given in Tables 1-7. The number of months under consideration in this work is stated in the first column while the exact day of the month in which the simulation is performed is given in the second column. The $\boldsymbol{C_v(old)}$ data is the density of vapour phase $\boldsymbol{C_v}$ when all the model parameter values are varied at 100% while the $\boldsymbol{C_v(new)}$ data are the values obtained when the rate of formation of vapour phase $\boldsymbol{Q_v}$ is varied. EPD and EPI represent the estimated proportion of $\boldsymbol{C_v}$ depleted or increased respectively as the case may be due to the variation.

As seen in Table 1, when $\boldsymbol{Q_v = 1.50}$, the density of vapour phase experiences severe depletion when compared to its corresponding value $\boldsymbol{C_v(old)}$ when $\boldsymbol{Q_v}$ is not varied. This depletion is quantified at 89.9946% approximately at day 571. Looking at Table 2, at $\boldsymbol{Q_v = 7.50}$, the depletion reduces to 49.9999% at day 571. In Tables 3-4, we have observed that for the model parameter values of $\boldsymbol{Q_v} = 13.50$ and 14.9985 respectively, the density of vapour phase experiences mild depletion. It decreases from 9.9999% in Table 3 to 0.0099% in Table 4. Meanwhile, in Tables 5-7, for $\boldsymbol{Q_v}$ =15.15, 15.75 and 16.50 respectively, the density of vapour molecules experiences an increase of 0.9999% for day 571 (Table 5), 4.9999% at day 571 (Table 6) and 9.9999% at day 571 (Table 7).

The practical application of this analysis as observed in this study is that a decrease in the rate at which vapour phase is formed in the atmosphere will ultimately lead to high instances of depletion on the density of vapour molecules. This will result to a high concentration of carbon dioxide in the atmosphere. Also, an increase in the rate at which vapour phase is formed in the atmosphere will lead to few instances of depletion on the density of vapour molecules thus resulting to a low concentration of carbon dioxide in the atmosphere.

We will like to mention here that Sundar et al (2009) worked on nonlinear mathematical modeling of the interaction of hot gases with cloud droplets and raindrops, examined in detail the mathematical ideas of equilibrium analysis and local stability analysis with a few numerical simulations of a specified variation of model parameter values and obtained a stable unique positive steady state solution. However, in this present work, we have successfully utilized the efficient computational method of ODE 45 numerical simulation to quantify the effect of varying the rate of formation of vapour phase on the concentration of carbon dioxide in the atmosphere. Thus, this paper has shown a cutting-edge contribution to knowledge in the scenario of modeling the effect of the rate of formation of vapour phase on the distribution of carbon dioxide in the atmosphere.

However, by the application of this same numerical method, the effects of the variation of other model parameters on the spread of carbon dioxide can be tackled.

REFERENCES

UCAR 2006: Centre for science education.

Sundar, S., Naresh, R., Mishra, A. K., Tripathy A. (2014): Modeling the dynamics os caron dioxide removal in the atmosphere. Computational Ecology and Software. 4(4): 248-268.

Overpeck J. T., Weiss, J. L. 2009: Projections of future sea level becoming direr. RNAS, 106 (51): 21461-21462

Douglas, H. E, Seldon, T. M. (1995). Stoking the fires? Carbon dioxide emission and economic growth. *Journal of Public Economy*, 57(1): 85-101.

Wu, S. H., Pan, T., He, S. F. (2012). Climate change risk research: A case study on flood disaster risk in China. Advances in Climate Change Research, 3(2): 92-98

Gao, K, Xu, J, Gao, G, Li, Y., Hutchins, D. A., Huang B., Wang, L., Zheng, Y, Jin, P, Cai, X, Hader, D. P., Li, W., Xu, K., Liu, N., Riebesell, U. (2012). Rising

Carbon dioxide and increased light exposure synergistically reduce marine primary productivity. *Nature Climate Change*, 2: 519-523.

World Health Organization – WHO (2019): Air pollution and health: Summary.

Sundar, S. (2013). An ecological type nonlinear model for the removal of carbon dioxide from the atmosphere by introducing liquid species. 3(2):33-34

Shukla, J. B., Misra, A. K, Naresh, R., Chandra, P. (2010): How artificial rain can be produced? A mathematical model, NNonlinear Anal. Real World Appl. 11: 2659-2668.

Shukla, J. B., Sundar, S., Misra A. K., Naresh, R. (2013). Modeling the effects of aerosols to increase rainfall in regions with shortage. *Meteorology. Atmos. Phys*. 120, 157-163.

Sundar, S., Naresh, R., Misra, A. K., Shukla, J. B. (2009): A nonlinear mathematical model to study the interactions of hot gases with cloud droplets and raindrops. Applied Mathematical Modeling, 33: 3015-3024.

Shukla J.B., Dubey, B. (1996). Simultaneous effect of two toxicants on biological species: A Mathematical Model. *Journal of Biological system*. 4(1):109-130

Naresh, R. (2003): Qualitative analysis of a nonlinear model for the removal of air pollutants. *Int. J. of Nonlinear Sciences and Numerical Simulation*, 4, 379-385.

Shukla, J. B., Sundar, S., Misra, A. K., Naresh, R. (2008): Modeling the removal of gaseous pollutants and particulate matters from the atmosphere of a city by rain: effect of cloud density. *Environ. Model. Assess*. 13(2): 255-263.

Naresh, R., Sundar, S. (2007). A nonlinear dynamical model to study the removal of gaseous and particulate pollutants in a rain system. Nonlinear Analysis: *Modeling and control*. 12(2): 227-243.

Udoh, N., Ekaka-a, E. N., Esekhaigbe, E., Alex, M. (2019): Numerical simulation of the differential effect of the rate of formation of vapour molecules on the density of vapour phase. International Journal of Innovation in Science and Mathematics. 7(2): 2347-9051.