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STABILIZING THE TRIVIAL STEADY STATE SOLUTION: PROBLEMS AND PROSPECT.

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ABSTRACT

Due to the inevitability of the characteristics of the Ecosystem in which the deterministic interaction between normal Agricultural assets, auxiliary Agricultural assets, Industrial assets and Ecospheric assets, it is important to test the numerical stability of the trivial steady state solution in which all the dependent variables are vulnerable to the Ecological risk of extinction, we have found that , irrespective of the decreased and increased variations of inter competition coefficient of normal Agriculture, the type of stability remains unstable. This observation can be further be explore by using a sophisticated numerical method in our next investigation.

Introduction:

The link between the Ecological System and the Mathematical theory of stability has a long standing history. However , there is as severe gap in the knowledge on how to test the type of stability of the trivial steady state solution due to a variation of a model parameter value, to the best of our knowledge , even in the saver research contributions of R.M May(1974, 1975), Neville et al (2010), Ekakaa(2009), Yubin Yan et al (2011) and several other researchers in the Ecological modelling and Mathematical Biology, the notion of using a numerical

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method to study the type of stability of the trivial steady state solution is not enough. It is against this background that we fully want to explore the application of a Matlab Algorithm to study the stability of the trivial steady state solution.

Mathematical Formulation

Following Ibrahim A and Freedom H.I (2009), we have consider the four system of nonlinear ordinary differential equation.

$$\frac{dx_1(t)}{dt} = \alpha_1 x_1 z - \beta_1 x_1^2 + \gamma_1 x_1 y - \rho_1 x_1 x_2 - \theta x_1 + \theta x_1 z$$
$$\frac{dx_2(t)}{dt} = \alpha_2 x_2 z - \beta_2 x_2^2 - \gamma_2 x_2 y - \rho_2 x_1 x_2$$
$$\frac{dy(t)}{dt} = -\xi y - \eta y^2 + \delta x_1 y$$
$$\frac{dz(t)}{dt} = -\kappa x_1 z + \kappa_1 z - \kappa_1 z^2 + \kappa_2 x_1 - \kappa_2 x_1 z$$

where all parameters are assumed to be positive constants except γ_1 which can be any real constant. $\alpha_1(\alpha_2)$ is inter competitive growth rate coefficient of normal (auxiliary) agriculture due to normal (auxiliary) agricultural activity for fixed *z*; $\beta_1(\beta_2)$ is the per asset diminishing returns rate coefficient for normal (auxiliary) agriculture in the absence of industry and auxiliary (normal) agriculture, $\gamma_1(\gamma_2)$ is the per asset terms of trade coefficient between normal (auxiliary) agriculture and industry, ξ is the constant depreciation rate coefficient of industry, η is the per asset (linear) depreciation rate coefficient of industry, δ is the per asset growth rate for industry in dealing with normal agriculture, $\rho_1(\rho_2)$ is the per asset competitive rate coefficient of auxiliary (normal) agriculture acting on normal (auxiliary) agriculture, κ is the per asset degradation rate coefficient of the ecosphere due to normal agricultural activities, κ_1 is the natural restoration rate coefficient for the ecosphere, κ_2 is the rate of effort input to restore the ecosphere by normal agriculture and $\boldsymbol{\theta}$ is the net cost rate to normal agriculture to restore the ecosphere.

With the following precise model parameter values from Agyemang and Freedman (2009)

$$\alpha_1 = 3, \alpha_2 = 1\,, \beta_1 = \frac{1}{10}, \beta_2 = \frac{1}{10}, \gamma_1 = -\frac{1}{49}, \gamma_2 = \frac{1}{10}, \rho_1 = \frac{1}{10}, \rho_2 = \frac{1}{5}, \delta = \frac{1}{4}, \quad \theta = \frac{6}{5}, \eta = \frac{1}{20}, \xi = 1, \kappa = 2, \kappa_1 = 2, \kappa_2 = 1$$

Method of Solution

The primary method that we have used in this work depends on the application of a Matlab Algorithm which evaluates the 16 Jacobian elements that we have obtained on the assumption that the four interacting functions are continuous and have first order partial derivatives. A Jacobian Matrix was defined from which four eigenvalues where calculated to determine the type of stability for this interacting environmental problem.

Results

The full results of this study are presented as shown in Table1, Table 2 and Table 3

Table 1: Quantifying the effect of decreasing the model parameter $\boldsymbol{\alpha_1 = 3}$ on the type of stability of a trivial steady state solution using Matlab Algorithm

Example	α_1	x_1	x_2	y	z	λ_1	λ_2	λ_3	λ_4	TOS
1	3	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
2	0.15	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
3	0.30	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
4	0.45	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
5	0.60	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
6	0.75	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
7	0.90	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
8	1.05	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable

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9	1.20	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
10	1.35	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable
11	1.50	0	0	0	0	2.00	-1.20	0.00	-1.00	Unstable

TOS= Type of Stability

Table 2: Quantifying the effect of increasing the model parameter $\boldsymbol{\alpha_1} = \mathbf{3}$ on the type of stability of a trivial steady state solution using Matlab Algorithm

Example	α_1	x_1	x_2	y	z	λ_1	λ_2	λ_3	λ_4	TOS
1	3	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
2	3.03	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
3	3.15	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
4	3.30	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
5	3.45	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
6	3.60	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
7	3.75	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
8	3.90	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
9	4.05	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
10	4.20	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
11	4.35	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
12	4.50	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable

TOS= Type of Stability

Table 3: Quantifying the effect of a severe semi-stochastic variation of the model parameter $\boldsymbol{\alpha_1} = \mathbf{3}$ on the type of stability of a trivial steady state solution using Matlab Algorithm

Example	α_1	x_1	x_2	y	z	λ_1	λ_2	λ_3	λ_4	TOS
1	3	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
2	3.75	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
3	6.65	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
4	3.70	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
5	5.17	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable

6	4.41	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
7	3.05	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
8	7.50	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
9	3.46	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
10	5.21	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
11	6.35	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable
12	4.92	0.00	0.00	0.00	0.00	2.00	-1.20	0.00	-1.00	Unstable

TOS= Type of Stability

Discussion of Results

Looking at Table 1, when the model parameter value $\boldsymbol{\alpha_1 = 3}$ is decreased deterministically from 5% to 50%, we have observed that the trivial steady state solution is dominantly unstable. In the same context , when the same parameter value is increased from 101% followed by 105% and in step length of 5% up to 150%, we have also found that the type of stability remain the same for the trivial steady state solution (Table 2).

Looking at Table 3 when the same parameter value is subjected to a semi-stochastic variation of the random noise intensity of 4.8, we have found a fluctuating pattern of the parameter value of $\boldsymbol{\alpha_1}$ in which the same steady state solution and its type of stability remains the same.

Having one real positive eigenvalue of 2 followed by two real negative eigenvalue of -1.2 and -1.0 with another eigenvalue having the value of 0.

In the context of stability, as the independent variable time(t) grows larger and larger, the two negative eigenvalues will decay to zero while the eigenvalue having the value of zero will have a constant solution trajectory whereas the eigenvalue of 2 will grow faster and positively exponential over time.

Conclusion

We have found that irrespective of decreasing, increasing and subjecting the $\boldsymbol{\alpha_1}$ parameter value to a semi-stochastic variation, the trivial steady state solution does not change its type of stability. Other possible extension of this present study will be explore in our next investigation.

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Reference

I. Agyemang, H.I. Freedman, J.W. Macki, An ecospheric recovery model for agriculture industry interactions, Diff. Eqns. Dyn. Syst. 15 (2007) 185-208.

I. Agyemang, H.I. Freedman, An environmental model for the interaction of industry with two competing agricultural resource, Elsevier. 49(2009) 1618-16

Neville J. Ford, Patricia M. Lumb and Enu Ekaka-a. (2011). Mathematical Modelling of plant species interactions in the harsh climate. Elsevier. 2732-2744

Yubin Yan, Enu-Obari Ekakaa.(2011). Stabilizing a Mathematical Model of population system. Elsevier. 2744-2758

R.M May and W.J. Leonard. (1975). Nonlinear Aspect of competition between three species. SIAM J. Applied Math. Vol 29, 243-253

R.M May. (1974). Stability and complexity in model Ecosystem. Princeton University Press, New Jersey, USA. 42-49