

ON THE NUMERICAL QUANTIFICATIONS OF BIODIVERSITY INTERVENTION USING THE METHOD OF POPULATION DYNAMICS

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ABSTRACT

To the best of our knowledge, the ecological concept of biodiversity is a challenging environmental problem that requires a sound mathematical reasoning. We have used the method of a numerical simulation that is indexed by ODE 45 numerical method to predict and obtain instances of biodiversity loss and biodiversity gain due to a decreasing and increasing variations of the intrinsic growth rates together. The novel results that we have obtained that we have not seen elsewhere, but do complement other similar numerical predictions of biodiversity are presented and discussed quantitatively.

***Keywords** – Ecological concept, biodiversity, environmental problem, numerical simulation, intrinsic growth rate.*

Introduction:

The ongoing debate between biodiversity, ecosystem stability, and its implications (Atsu and Ekaka-a 2017)¹ makes it imperative to examine the effects of varying the intrinsic growth rates together on biodiversity loss and biodiversity gain by using a computationally efficient numerical method called Matlab function ordinary differential equation of order 45 (ODE 45). Other related contributions on the link between biodiversity and ecosystem stability have been adequately sighted.

MATERIALS AND METHODS

If a variation of a model parameter value produces a new biomass which is

Smaller than the old biomass for any interacting legumes, such as cowpea and groundnut, then a biodiversity loss has occurred and can be quantified as we have done in this study.

On the other hand, if a variation of a model parameter value produces a new biomass which outweighs the old biomass irrespective of the type of legumes, then a biodiversity gain has occurred and can be similarly quantified.

Following Ekaka-a *et al* (2009), we have considered the following continuous dynamical system of non linear first order ordinary differential equation

$$\frac{d}{dt}C(t) = \alpha_1 C(t) - \beta_1 C^2(t) - r_1 C(t)G(t)$$

$$\frac{d}{dt}G(t) = \alpha_2 G(t) - \beta_2 G^2(t) - r_2 C(t)G(t)$$

With $C(0) = 0.12$ and $G(0) = 0.14$

For the purpose of clarity, the variables and the parameter values for these model equations are defined as follows

- $C_b(t)$ and $G_b(t)$ are called the biomass of cowpea and groundnut at time (t) in the unit of weeks
- α_1 and α_2 are called the intrinsic growth rates for populations $C_b(t)$ and $G_b(t)$ in the absence of self-interaction and inter-competition interaction
- β_1 and β_2 are called the intra-competition coefficients
- r_1 and r_2 are called the inter-competition coefficients

to analyze our propose problem,

$\alpha_1 = 0.0225$ & $\alpha_2 = 0.0446$; $\beta_1 = 0.0069$ & $\beta_2 = 0.0133$; $r_1 = 0.0018$ & $r_2 = 0.0012$.

The core numerical method that we have used in this present analysis is called ODE 45.

RESULTS AND DISCUSSIONS

The results of this study are displayed as shown in Tables 1.1, 1.2, 1.3, 1.4, 1.5, 2.1, 2.2, 2.3, 2.4 and 2.5.

DISCUSSION OF RESULTS

The results of this study are fully presented and discussed quantitatively as follows:

Table 1.1

Evaluating the effect of varying the intrinsic growth rates together by 10% on biodiversity loss using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1801	2.0021	0.3745	0.3598	3.9250
0.1876	0.1801	3.9591	0.3895	0.3596	7.6771
0.1914	0.1802	5.8719	0.4050	0.3594	11.2640
0.1954	0.1803	7.7415	0.4211	0.3592	14.6931
0.1994	0.1803	9.5689	0.4377	0.3590	17.9713
0.2035	0.1804	11.3549	0.4548	0.3588	21.1052
0.2077	0.1805	13.1004	0.4725	0.3587	24.1012
0.2119	0.1805	14.8063	0.4908	0.3585	26.9654
0.2162	0.1806	16.4735	0.5097	0.3583	29.7037
0.2206	0.1806	18.1029	0.5291	0.3581	32.3215
0.2250	0.1807	19.6951	0.5491	0.3579	34.8243
0.2296	0.1808	21.2511	0.5698	0.3577	37.2170
0.2342	0.1808	22.7716	0.5910	0.3575	39.5046
0.2388	0.1809	24.2573	0.6129	0.3574	41.6917
0.2436	0.1810	25.7092	0.6353	0.3572	43.7826
0.2484	0.1810	27.1278	0.6584	0.3570	45.7817
0.2533	0.1811	28.5139	0.6822	0.3568	47.6931
0.2583	0.1812	29.8682	0.7065	0.3566	49.5205
0.2634	0.1812	31.1914	0.7315	0.3565	51.2677
0.2685	0.1813	32.4843	0.7570	0.3563	52.9381

From Table 1.1, when all the model parameter values are fixed, the cowpea biomass data denoted $C_b(t)$ when the length of the growing season is twenty one weeks range from a low value of 0.18 grams/area to 0.2685

grams/area whereas $C_{bm}(t)$ data range from a low value 0.18 grams/area to 0.1813 grams/area due to a 10% variation of the intrinsic growth rates together. On the basis of this calculation, the new simulated cowpea data due to a joint variation of the intrinsic growth rates dominantly predicts a depletion which mimics biodiversity loss. The extent of biodiversity loss has been quantified to range from zero to 32.48 approximately providing an average of 17.4 which re-classifies the vulnerability of the cowpea biomass to biodiversity loss. A similar observation can be made from the groundnut biomass component. In summary, the groundnut biomass is about 1.73 approximately more vulnerable to biodiversity loss than the cowpea biomass. Statistically, the average of biomass vulnerability to biodiversity loss with respect to the groundnut legume is 28.9% approximately.

Table 1.2

Evaluating the effect of varying the intrinsic growth rates together by 15% on biodiversity loss using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1803	1.8919	0.3745	0.3606	3.7110
0.1876	0.1805	3.7433	0.3895	0.3612	7.2664
0.1914	0.1808	5.5549	0.4050	0.3618	10.6728
0.1954	0.1811	7.3275	0.4211	0.3624	13.9363
0.1994	0.1813	9.0620	0.4377	0.3630	17.0628
0.2035	0.1816	10.7590	0.4548	0.3636	20.0581
0.2077	0.1819	12.4194	0.4725	0.3642	22.9276
0.2119	0.1821	14.0439	0.4908	0.3648	25.6765
0.2162	0.1824	15.6332	0.5097	0.3654	28.3099
0.2206	0.1827	17.1880	0.5291	0.3660	30.8327
0.2250	0.1829	18.7090	0.5491	0.3666	33.2493
0.2296	0.1832	20.1969	0.5698	0.3671	35.5642
0.2342	0.1835	21.6524	0.5910	0.3677	37.7817

0.2388	0.1837	23.0761	0.6129	0.3683	39.9058
0.2436	0.1840	24.4687	0.6353	0.3689	41.9404
0.2484	0.1843	25.8308	0.6584	0.3695	43.8893
0.2533	0.1845	27.1631	0.6822	0.3700	45.7559
0.2583	0.1848	28.4660	0.7065	0.3706	47.5438
0.2634	0.1851	29.7404	0.7315	0.3712	49.2563
0.2685	0.1853	30.9866	0.7570	0.3717	50.8964

From Table 1.2, when all the model parameter values are fixed, the cowpea biomass data denoted $C_b(t)$ when the length of the growing season is twenty one weeks range from a low value of 0.18 grams/area to 0.2685 grams/area whereas $C_{bm}(t)$ data range from a low value 0.18 grams/area to 0.1853 grams/area due to a 15% variation of the intrinsic growth rates together. On the basis of this calculation, the new simulated cowpea data due to a joint variation of the intrinsic growth rates dominantly predicts a depletion which mimics biodiversity loss. The extent of biodiversity loss has been quantified to range from zero to 30.98 approximately providing an average of 16.6 which re-classifies the vulnerability of the cowpea biomass to biodiversity loss. A similar observation can be made from the groundnut biomass component. In summary, the groundnut biomass is about 1.74 approximately more vulnerable to biodiversity loss than the cowpea biomass. Statistically, the average of biomass vulnerability to biodiversity loss with respect to the groundnut legume is 28.7% approximately.

Table 1.3

Evaluating the effect of varying the intrinsic growth rates together by 20% on biodiversity loss using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1805	1.7816	0.3745	0.3614	3.4966
0.1876	0.1809	3.5270	0.3895	0.3628	6.8540
0.1914	0.1814	5.2368	0.4050	0.3642	10.0776

0.1954	0.1819	6.9117	0.4211	0.3656	13.1727
0.1994	0.1824	8.5523	0.4377	0.3670	16.1443
0.2035	0.1828	10.1592	0.4548	0.3684	18.9972
0.2077	0.1833	11.7332	0.4725	0.3698	21.7360
0.2119	0.1838	13.2747	0.4908	0.3712	24.3652
0.2162	0.1842	14.7845	0.5097	0.3726	26.8890
0.2206	0.1847	16.2630	0.5291	0.3740	29.3116
0.2250	0.1852	17.7110	0.5491	0.3754	31.6369
0.2296	0.1856	19.1289	0.5698	0.3768	33.8688
0.2342	0.1861	20.5174	0.5910	0.3782	36.0108
0.2388	0.1866	21.8769	0.6129	0.3796	38.0665
0.2436	0.1871	23.2080	0.6353	0.3810	40.0394
0.2484	0.1875	24.5113	0.6584	0.3823	41.9325
0.2533	0.1880	25.7873	0.6822	0.3837	43.7491
0.2583	0.1885	27.0365	0.7065	0.3851	45.4922
0.2634	0.1890	28.2594	0.7315	0.3865	47.1646
0.2685	0.1894	29.4566	0.7570	0.3878	48.7691

From Table 1.3, when all the model parameter values are fixed, the cowpea biomass data $C_b(t)$ when the length of the growing season is twenty one weeks range from a low value of 0.18 grams/area to 0.2685 grams/area whereas $C_{bm}(t)$ data range from a low value 0.18 grams/area to 0.1894 grams/area due to a 20% variation of the intrinsic growth rates together. On the basis of this calculation, the new simulated cowpea data due to a joint variation of the intrinsic growth rates dominantly predicts a depletion which mimics biodiversity loss. The extent of biodiversity loss has been quantified to range from zero to 29.45 approximately providing an average of 15.7 which re-classifies the vulnerability of the cowpea biomass to biodiversity loss. A similar observation can be made from the groundnut biomass component. In summary, the groundnut biomass is about 1.75 approximately more vulnerable to biodiversity loss than the cowpea

biomass. Statistically, the average of biomass vulnerability to biodiversity loss with respect to the groundnut legume is 28.5% approximately.

Table 1.4

Evaluating the effect of varying the intrinsic growth rates together by 25% on biodiversity loss using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1807	1.6712	0.3745	0.3622	3.2816
0.1876	0.1813	3.3103	0.3895	0.3644	6.4396
0.1914	0.1820	4.9177	0.4050	0.3666	9.4785
0.1954	0.1827	6.4940	0.4211	0.3689	12.4025
0.1994	0.1834	8.0397	0.4377	0.3711	15.2158
0.2035	0.1841	9.5554	0.4548	0.3733	17.9224
0.2077	0.1847	11.0416	0.4725	0.3755	20.5262
0.2119	0.1854	12.4987	0.4908	0.3778	23.0310
0.2162	0.1861	13.9273	0.5097	0.3800	25.4403
0.2206	0.1868	15.3279	0.5291	0.3822	27.7576
0.2250	0.1875	16.7009	0.5491	0.3845	29.9863
0.2296	0.1881	18.0468	0.5698	0.3867	32.1296
0.2342	0.1888	19.3661	0.5910	0.3889	34.1906
0.2388	0.1895	20.6592	0.6129	0.3912	36.1723
0.2436	0.1902	21.9267	0.6353	0.3934	38.0777
0.2484	0.1909	23.1688	0.6584	0.3957	39.9094
0.2533	0.1916	24.3861	0.6822	0.3979	41.6703
0.2583	0.1922	25.5791	0.7065	0.4001	43.3629
0.2634	0.1929	26.7481	0.7315	0.4024	44.9896
0.2685	0.1936	27.8935	0.7570	0.4046	46.5530

Table 1.5

Evaluating the effect of varying the intrinsic growth rates together by 95% on biodiversity loss using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1835	0.1123	0.3745	0.3737	0.2222
0.1876	0.1871	0.2242	0.3895	0.3878	0.4427
0.1914	0.1908	0.3356	0.4050	0.4024	0.6615
0.1954	0.1945	0.4466	0.4211	0.4174	0.8785
0.1994	0.1983	0.5570	0.4377	0.4329	1.0936
0.2035	0.2021	0.6671	0.4548	0.4489	1.3068
0.2077	0.2060	0.7766	0.4725	0.4654	1.5178
0.2119	0.2100	0.8855	0.4908	0.4823	1.7268
0.2162	0.2141	0.9940	0.5097	0.4998	1.9335
0.2206	0.2181	1.1018	0.5291	0.5178	2.1379
0.2250	0.2223	1.2091	0.5491	0.5363	2.3399
0.2296	0.2265	1.3158	0.5698	0.5553	2.5394
0.2342	0.2308	1.4219	0.5910	0.5748	2.7364
0.2388	0.2352	1.5274	0.6129	0.5949	2.9306
0.2436	0.2396	1.6321	0.6353	0.6155	3.1220
0.2484	0.2441	1.7363	0.6584	0.6366	3.3106
0.2533	0.2487	1.8397	0.6822	0.6583	3.4962
0.2583	0.2533	1.9424	0.7065	0.6805	3.6787
0.2634	0.2580	2.0443	0.7315	0.7032	3.8580
0.2685	0.2628	2.1455	0.7570	0.7265	4.0340

Table 2.1

Evaluating the effect of varying the intrinsic growth rates together by 105% on biodiversity gain using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1840	0.1124	0.3745	0.3753	0.2227

0.1876	0.1880	0.2247	0.3895	0.3912	0.4447
0.1914	0.1921	0.3367	0.4050	0.4077	0.6659
0.1954	0.1963	0.4485	0.4211	0.4248	0.8862
0.1994	0.2005	0.5601	0.4377	0.4425	1.1056
0.2035	0.2049	0.6715	0.4548	0.4609	1.3239
0.2077	0.2093	0.7826	0.4725	0.4798	1.5409
0.2119	0.2138	0.8933	0.4908	0.4994	1.7566
0.2162	0.2184	1.0038	0.5097	0.5197	1.9709
0.2206	0.2230	1.1139	0.5291	0.5407	2.1836
0.2250	0.2278	1.2237	0.5491	0.5623	2.3946
0.2296	0.2326	1.3330	0.5698	0.5846	2.6037
0.2342	0.2375	1.4420	0.5910	0.6076	2.8109
0.2388	0.2425	1.5505	0.6129	0.6314	3.0160
0.2436	0.2476	1.6585	0.6353	0.6558	3.2187
0.2484	0.2528	1.7660	0.6584	0.6809	3.4191
0.2533	0.2581	1.8731	0.6822	0.7068	3.6169
0.2583	0.2634	1.9795	0.7065	0.7334	3.8120
0.2634	0.2689	2.0854	0.7315	0.7607	4.0042
0.2685	0.2744	2.1907	0.7570	0.7888	4.1934

Table 2.2

Evaluating the effect of varying the intrinsic growth rates together by 110% on biodiversity gain using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1842	0.2250	0.3745	0.3762	0.4459
0.1876	0.1884	0.4498	0.3895	0.3930	0.8913
0.1914	0.1927	0.6745	0.4050	0.4105	1.3362
0.1954	0.1971	0.8991	0.4211	0.4286	1.7803
0.1994	0.2016	1.1234	0.4377	0.4474	2.2233
0.2035	0.2062	1.3475	0.4548	0.4670	2.6650
0.2077	0.2109	1.5712	0.4725	0.4872	3.1052

0.2119	0.2157	1.7946	0.4908	0.5082	3.5436
0.2162	0.2206	2.0175	0.5097	0.5300	3.9799
0.2206	0.2255	2.2400	0.5291	0.5525	4.4138
0.2250	0.2306	2.4620	0.5491	0.5757	4.8450
0.2296	0.2357	2.6835	0.5698	0.5998	5.2733
0.2342	0.2410	2.9043	0.5910	0.6247	5.6983
0.2388	0.2463	3.1244	0.6129	0.6504	6.1196
0.2436	0.2517	3.3438	0.6353	0.6769	6.5370
0.2484	0.2573	3.5623	0.6584	0.7042	6.9500
0.2533	0.2629	3.7801	0.6822	0.7323	7.3584
0.2583	0.2686	3.9969	0.7065	0.7613	7.7617
0.2634	0.2745	4.2127	0.7315	0.7911	8.1597
0.2685	0.2804	4.4274	0.7570	0.8218	8.5518

Table 2.3

Evaluating the effect of varying the intrinsic growth rates together by 115% on biodiversity gain using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1844	0.3376	0.3745	0.3770	0.6695
0.1876	0.1888	0.6755	0.3895	0.3947	1.3400
0.1914	0.1934	1.0135	0.4050	0.4132	2.0110
0.1954	0.1980	1.3516	0.4211	0.4324	2.6822
0.1994	0.2028	1.6898	0.4377	0.4524	3.3532
0.2035	0.2076	2.0279	0.4548	0.4731	4.0237
0.2077	0.2126	2.3659	0.4725	0.4947	4.6933
0.2119	0.2176	2.7038	0.4908	0.5171	5.3614
0.2162	0.2228	3.0413	0.5097	0.5404	6.0277
0.2206	0.2280	3.3785	0.5291	0.5645	6.6916
0.2250	0.2334	3.7153	0.5491	0.5895	7.3526
0.2296	0.2389	4.0515	0.5698	0.6154	8.0103
0.2342	0.2444	4.3871	0.5910	0.6422	8.6641

0.2388	0.2501	4.7221	0.6129	0.6699	9.3134
0.2436	0.2559	5.0562	0.6353	0.6986	9.9576
0.2484	0.2618	5.3894	0.6584	0.7282	10.5961
0.2533	0.2678	5.7216	0.6822	0.7587	11.2283
0.2583	0.2740	6.0527	0.7065	0.7902	11.8536
0.2634	0.2802	6.3825	0.7315	0.8227	12.4713
0.2685	0.2865	6.7111	0.7570	0.8561	13.0808

Table 2.4

Evaluating the effect of varying the intrinsic growth rates together by 120% on biodiversity gain using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1846	0.4504	0.3745	0.3779	0.8937
0.1876	0.1892	0.9017	0.3895	0.3965	1.7906
0.1914	0.1940	1.3536	0.4050	0.4159	2.6902
0.1954	0.1989	1.8062	0.4211	0.4362	3.5920
0.1994	0.2039	2.2594	0.4377	0.4574	4.4956
0.2035	0.2090	2.7129	0.4548	0.4794	5.4002
0.2077	0.2142	3.1668	0.4725	0.5023	6.3054
0.2119	0.2196	3.6210	0.4908	0.5262	7.2106
0.2162	0.2250	4.0752	0.5097	0.5510	8.1149
0.2206	0.2306	4.5294	0.5291	0.5768	9.0179
0.2250	0.2362	4.9836	0.5491	0.6036	9.9186
0.2296	0.2420	5.4374	0.5698	0.6314	10.8163
0.2342	0.2480	5.8909	0.5910	0.6602	11.7103
0.2388	0.2540	6.3438	0.6129	0.6901	12.5996
0.2436	0.2602	6.7961	0.6353	0.7210	13.4834
0.2484	0.2664	7.2476	0.6584	0.7530	14.3607
0.2533	0.2728	7.6982	0.6822	0.7860	15.2307
0.2583	0.2794	8.1476	0.7065	0.8202	16.0923
0.2634	0.2860	8.5958	0.7315	0.8554	16.9445

0.2685	0.2928	9.0425	0.7570	0.8917	17.7863
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Table 2.5

Evaluating the effect of varying the intrinsic growth rates together by 125% on biodiversity gain using ODE 45 numerical method.

$C_b(t)$	$C_{bm}(t)$	BL(%)	$G_b(t)$	$G_{bm}(t)$	BL(%)
0.1800	0.1800	0	0.3600	0.3600	0
0.1837	0.1848	0.5634	0.3745	0.3787	1.1184
0.1876	0.1897	1.1283	0.3895	0.3983	2.2432
0.1914	0.1947	1.6949	0.4050	0.4187	3.3739
0.1954	0.1998	2.2628	0.4211	0.4401	4.5099
0.1994	0.2051	2.8321	0.4377	0.4624	5.6504
0.2035	0.2104	3.4025	0.4548	0.4857	6.7947
0.2077	0.2159	3.9739	0.4725	0.5101	7.9421
0.2119	0.2215	4.5462	0.4908	0.5354	9.0916
0.2162	0.2273	5.1193	0.5097	0.5619	10.2425
0.2206	0.2331	5.6930	0.5291	0.5894	11.3937
0.2250	0.2391	6.2670	0.5491	0.6180	12.5442
0.2296	0.2453	6.8414	0.5698	0.6478	13.6930
0.2342	0.2515	7.4158	0.5910	0.6787	14.8390
0.2388	0.2579	7.9901	0.6129	0.7108	15.9808
0.2436	0.2645	8.5641	0.6353	0.7441	17.1174
0.2484	0.2711	9.1376	0.6584	0.7786	18.2475
0.2533	0.2779	9.7104	0.6822	0.8143	19.3696
0.2583	0.2849	10.2824	0.7065	0.8512	20.4823
0.2634	0.2920	10.8532	0.7315	0.8893	21.5844
0.2685	0.2992	11.4226	0.7570	0.9287	22.6742

CONCLUSION

By using ODE 45 we have found out that a biodiversity loss can be obtained due to a decreasing variation of the intrinsic growth rates together, whereas a dominant biodiversity gain can be obtained due to an increasing

variation of the intrinsic growth rates together. On the basis of this analysis, the decreasing variation of the intrinsic growth rates together has generally indicated a decrease in the yields of these two crops, whereas an increasing variation of the same parameter values has indicated an improvement in the yields of both cowpea and groundnut. In this context, an alarming rate of biodiversity loss of these quantified magnitude are a strong signal on lower food production, endemic poverty and a weak sustainable development scenario, whereas a biodiversity gain has the potential to alleviate poverty and sustain development. These two components of biodiversity as predicted in this work have their policy implications.

This present numerical idea can be extended to examine the effects of varying the intra and inter competition coefficients together in our future investigation.

RECOMMENDATIONS

We shall apply the same numerical idea that we have utilized in this present study to answer the following open research questions, which we did not solve and analyze in this present study.

1. How can we construct a time delay system of first order differential equations which can be used to predict the extent of biodiversity on the application of a numerical method?
2. How can we construct a continuous dynamical system of non-linear second order ordinary differential equations which can be used to quantify the extent of biodiversity and its implication for a social economic development?

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