03.31.2020

BERKELEY PUBLICATION AND RESEARCH INTERNATIONAL

Journal of Pure and Applied Science Vol. 12 No. 9

MATHEMATICAL STUDY OF THE SPREAD AND TREATMEANT OF LASSA FEVER USING ADOMIAN DECOMPOSITION METHOD

ENAGI, A. I AND MUHAMMED, I

Department of Mathematics, Federal University of Technology, P.M.B 65, Minna, Niger State, Nigeria.

ABSTRACT

In this study we carried out a mathematical study of the spread and treatment of lassa fever. The model equations were solved using Adomian Decomposition Method. The results obtained were used for numerical simulations using Matlab software. It was observed that at high treatment rate, the number of recovered individuals increases and the virus can be eradicated completely.

Key words: *Lassa Fever, Spread, Treatment, Adomian Decomposition Method, Simulation.*

Introduction:

Lassa fever is a viral infection belonging to arena-virus family (Centre for Disease Control and Prevention, 2013). The virus is transmitted to humans through exposure to foods or household items contaminated with infected rodents urine or faeces (World Health Organization, 2000). The virus is mostly common in West African countries such as Nigeria, Ghana, Guinea, Liberia, Mali, Sierra Leone, and Benin Republic (World Health organization, 2017). The virus is mostly common in West African countries such as Nigeria, Ghana, Guinea, Liberia, Mali, Sierra Leone, and Benin Republic (World Health organization, 2017). The natural reservoirs of the virus are the mastomyms

rats which are common in endemic areas (Eze, 2010). Person-to-person transmission of the virus also occur through direct contact with the blood, urine, faeces, or other bodily secretions or fluids of an infectious person (World Health Organization, 2017). Nosocomial transmission occurs in hospital lacking adequate prevention and control measures (World Health Organization, 2017).

Over the years Mathematical models have been used to study the dynamics lassa fever these includes Okuonghae et al., (2006), Bawa et al.,(2014), Mohammed et al., (2014), James et al., (2015), Onuorah et al., (2016), Akanni et al., (2018), Suleiman et al., (2018) e. t. c.

At the beginning of the 1980, George Adomian developed a very powerful method called Adomian decomposition method for solving linear and nonlinear functional equations.

The Adomian decomposition method (ADM) involves separating the equation under consideration into linear and nonlinear parts. The linear operator representing the linear part of the equation is inverted and the linear operator is then applied to the equation. Any given conditions are taken into consideration. The nonlinear part is decomposed into a series of what is known as Adomian Polynomials. The method generates a solution in the form of a series whose terms are obtained by a recursive relationship using the Adomian Polynomials.

In this study we carried out a mathematical study of the spread and treatment of Lassa fever. The model equations were solved using Adomian Decomposition Method.

Methodology

Model Equations

The model is represented by the following system of non-linear differential equations.

$$\frac{dS_H}{dt} = \wedge_1 - \beta(I_R + I_H + A_H + \delta T_H)S_H - \mu_1 S_H \quad (1)$$

$$\frac{dE_H}{dt} = \beta(I_R + I_H + A_H + \delta T_H)S_H - (\alpha + \mu_1)E_H \quad (2)$$

$$\frac{dA_H}{dt} = (1-\rho)\alpha E_H - (\eta + \phi + \mu_1)A_H \quad (3)$$

$$\frac{dI_H}{dt} = \rho\alpha E_H - (\gamma + \phi + \mu_1)I_H \quad (4)$$

$$\frac{dT_H}{dt} = \gamma I_H + \eta A_H - (\kappa + \phi + \mu_1)T_H \quad (5)$$

$$\frac{dR_H}{dt} = \kappa T_H - \mu_1 R_H \quad (6)$$

$$\frac{dS_R}{dt} = \wedge_2 - \lambda S_R I_R - (\nu + \mu_2)S_R \quad (7)$$

$$\frac{dI_R}{dt} = \lambda S_R I_R - (\nu + \mu_2)I_R \quad (8)$$

Description of Variables and Parameters

Variable/Parameter	Description
A_H	Number of asymptomatic infected humans
E_H	Number of exposed humans
I_H	Number of symptomatic infected humans
I_R	Number of infected humans
N_H	Total number of the human population
N_H	Total Population of the reservoir population
R_H	Number of recovered humans
S_H	Number of susceptible humans
S_R	Number of susceptible reservoirs
T_H	Number of humans undergoing treatment
$(1-\rho)$	Proportion of exposed humans that progress to asymptomatic infected humans
α	Progression rate from exposed humans to infected humans
β	Transmission rate from the infected reservoirs, asymptomatic infected, symptomatic infected and treatment or hospitalized humans to susceptible humans.
γ	Treatment rate of symptomatic infected humans

δ	Reduction rate in transmission due to treatment or isolation of infected humans
η	Treatment rate of asymptomatic infected humans
κ	Recovery rate due to treatment of infected humans
λ	Transmission rate from the infected reservoirs to susceptible reservoirs
Λ_1	Constant recruitment rate into susceptible humans' population
Λ_2	Constant recruitment rate into susceptible reservoirs' population
μ_1	Natural death rate of the humans' population
μ_2	Natural death rate of the reservoirs' population
ν	Hunting rate of the reservoirs
ρ	Proportion of exposed humans that progress to symptomatic infected humans
ϕ	Disease-induced death rate due to Lassa fever

Adomian Decomposition Method

A brief outline of the method is given as follows;

Consider a differential equation in general form

$$G(y) = g \tag{9}$$

This can be written in operator form as

$$Ly + Ry + Ny = g \tag{10}$$

Where L is a linear operator acting on y which is easily invertible, R is a linear operator for the remainder of the linear part, and N is a nonlinear operator representing the nonlinear term in G. For convenience, L is usually taken as the highest derivative.

Applying the inverse operator L^{-1} on both sides of equation (10) gives

$$L^{-1}Ly = L^{-1}g - L^{-1}Ry - L^{-1}Ny \tag{11}$$

L^{-1} Is integration since G is taken as a nonlinear differential operator and L is linear.

That is, L^{-1} is an nth integral of y for nth order differential equation, where $n \epsilon Z$.

Equation (11) becomes

$$y(t) = f(t) - L^{-1}Ry - L^{-1}Ny \quad (12)$$

Where f(t) is the function obtained by integrating g and applying the initial or boundary conditions.

The unknown function is assumed to be an infinite series of the form

$$y(t) = \sum_{n=0}^{\infty} y_n \quad (13)$$

We let

$$y_0 = f(t) \quad (14)$$

And the remaining terms are obtained by a recursive relationship. This relationship is found by decomposing the nonlinear terms into a series of what is called Adomian polynomial, P_n, (Biazar *et al*, 2005).

The nonlinear term is written as

$$Ny(t) = \sum_{n=0}^{\infty} P_n \quad (15)$$

In order to obtain P_n, a grouping parameter, λ is introduced. The following series are established

$$y(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n \quad (16)$$

$$Ny(t) = \sum_{n=0}^{\infty} \lambda^n P_n \quad (17)$$

Substituting equations (14), (16), (17) into equation (12) gives

$$y(t) = y_0 - L^{-1}\sum_{n=0}^{\infty} Ry_n - L^{-1}\sum_{n=0}^{\infty} P_n \quad (18)$$

Where P_n can be obtain from

$$P_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} Ny(\lambda)\right]_{\lambda=0} \quad (19)$$

The recursive relation is obtained to be

$$y_0 = f(t) \quad (20)$$

$$y_{n+1} = -L^{-1}\sum_{n=0}^{\infty} Ry_n - L^{-1}\sum_{n=0}^{\infty} P_n \quad (21)$$

The Adomian decomposition method (ADM) produces a series that is absolutely and uniformly convergent, (El-Kalla, 2008).

Semi-Analytical Solution of the Model Using Adomian Decomposition Method

Consider equations (1) through (8) with the following initial conditions

$S_H(0)=S_{h0}$, $E_H(0)=E_{h0}$, $A_H(0)=A_{h0}$, $I_H(0)=I_{h0}$, $T_H(0)=T_{h0}$, $R_H(0)=R_{h0}$, $S_R(0)=S_{r0}$, $I_R(0)=Ir_0$ (22)

Integrating both sides of (1) through (8) with respect to t and applying the initial conditions (22) gives

$$S_H(t) = S_{h0} + \wedge_1 t - \beta\int_0^t I_R S_H d\tau - \beta\int_0^t I_H S_H d\tau - \beta\int_0^t A_H S_H d\tau - \beta\delta\int_0^t T_H S_H d\tau - \mu_1\int_0^t S_H d\tau \quad (23)$$

$$E_H(t) = E_{h0} + \beta\int_0^t I_R S_H d\tau + \beta\int_0^t I_H S_H d\tau + \beta\int_0^t A_H S_H d\tau + \beta\delta\int_0^t T_H S_H d\tau - (\alpha+\mu_1)\int_0^t E_H d\tau \quad (24)$$

$$A_H(t) = A_{h0} + (1-\rho)\alpha\int_0^t E_H d\tau - (\eta+\phi+\mu_1)\int_0^t A_H d\tau \quad (25)$$

$$I_H(t) = I_{h0} + \rho\alpha\int_0^t E_H d\tau - (\gamma+\phi+\mu_1)\int_0^t I_H d\tau \quad (26)$$

$$T(t) = T_{h0} + \gamma\int_0^t I_H d\tau + \eta\int_0^t A_H d\tau - (\kappa+\phi+\mu_1)\int_0^t T_H d\tau \quad (27)$$

$$R_H(t) = R_{h0} + \kappa\int_0^t T_H d\tau - \mu_1\int_0^t R_H d\tau \tag{28}$$

$$S_R(t) = S_{r0} + \wedge_2 t - \lambda\int_0^t S_R I_R d\tau - (\nu + \mu_2)\int_0^t S_R d\tau \tag{29}$$

$$I_R(t) = I_{r0} + \lambda\int_0^t S_R I_R d\tau - (\nu + \mu_2)\int_0^t I_R d\tau \tag{30}$$

Using Adomian decomposition method, the solutions of equations (1) through (8) are given as the series of the form

$$\begin{bmatrix} S_H = \sum_{n=0}^{\infty} S_{Hn}, E_H = \sum_{n=0}^{\infty} E_{Hn}, A_H = \sum_{n=0}^{\infty} A_{Hn}, I_H = \sum_{n=0}^{\infty} I_{Hn}, \\ T_H = \sum_{n=0}^{\infty} T_{Hn}, R_H = \sum_{n=0}^{\infty} R_{Hn}, S_R = \sum_{n=0}^{\infty} S_{Rn}, I_R = \sum_{n=0}^{\infty} I_{Rn} \end{bmatrix} \tag{31}$$

And also, the nonlinear integrands in equations (21) through (30) are expressed as

$$\left[B = I_R S_H,\ C = I_H S_H,\ D = A_H S_H,\ F = T_H S_H, N = S_R I_R\right] \tag{32}$$

The nonlinear operators in equation (32) are decomposed in series form as

$$\left[B = \sum_{n=0}^{\infty} B_n, C = \sum_{n=0}^{\infty} C_n, D = \sum_{0}^{\infty} D_n, N = \sum_{0}^{\infty} N_n\right] \tag{33}$$

Where, B_n, C_n, D_n, F_n, N_n are the Adomian polynomials.

Substituting equations (31) through (33) into equations (21) through (30) gives

$$\sum_{n=0}^{\infty} S_{Hn} = S_{h0} + \wedge_1 t - \beta\int_0^t \sum_{n=0}^{\infty} B_n d\tau - \beta\int_0^t \sum_{n=0}^{\infty} C_n d\tau - \beta\int_0^t \sum_{n=0}^{\infty} D_n d\tau - \beta\delta\int_0^t \sum_{n=0}^{\infty} F_n d\tau - \mu_1\int_0^t \sum_{n=0}^{\infty} S_{Hn} d\tau \tag{34}$$

$$\sum_{n=0}^{\infty} E_{Hn} = E_{h0} + \beta\int_0^t \sum_{n=0}^{\infty} B_n d\tau + \beta\int_0^t \sum_{n=0}^{\infty} C_n d\tau + \beta\int_0^t \sum_{n=0}^{\infty} D_n d\tau + \beta\delta\int_0^t \sum_{n=0}^{\infty} F_n - (\alpha + \mu_1)\int_0^t \sum_{n=0}^{\infty} E_{Hn} d\tau \tag{35}$$

$$\sum_{n=0}^{\infty} A_{Hn} = A_{h0} + (1-\rho)\alpha\int_0^t \sum_{n=0}^{\infty} E_{Hn} d\tau - (\eta + \phi + \mu_1)\int_0^t \sum_{n=0}^{\infty} A_{Hn} d\tau \tag{36}$$

$$\sum_{n=0}^{\infty} I_{Hn} = I_{h0} + \rho\alpha\int_0^t \sum_{n=0}^{\infty} E_{Hn}d\tau - (\gamma+\phi+\mu_1)\int_0^t \sum_{n=0}^{\infty} I_{Hn}d\tau \quad (37)$$

$$\sum_{n=0}^{\infty} T_{Hn} = T_{h0} + \gamma\int_0^t \sum_{n=0}^{\infty} I_{Hn}d\tau + \eta\int_0^t \sum_{n=0}^{\infty} A_{Hn}d\tau - (\kappa+\phi+\mu_1)\int_0^t \sum_{n=0}^{\infty} T_{Hn}d\tau \quad (38)$$

$$\sum_{n=0}^{\infty} R_{Hn} = R_{h0} + \kappa\int_0^t \sum_{n=0}^{\infty} T_{Hn}d\tau - \mu_1\int_0^t \sum_{n=0}^{\infty} R_{Hn}d\tau \quad (39)$$

$$\sum_{n=0}^{\infty} S_{Rn} = S_{r0} + \wedge_2 t - \lambda\int_0^t \sum_{n=0}^{\infty} N_n d\tau - (\nu+\mu_2)\int_0^t \sum_{n=0}^{\infty} S_{Rn}d\tau \quad (40)$$

$$\sum_{n=0}^{\infty} I_{Rn} = I_{r0} + \lambda\int_0^t \sum_{n=0}^{\infty} N_n d\tau - (\nu+\mu_2)\int_0^t \sum_{n=0}^{\infty} I_{Rn}d\tau \quad (41)$$

Equations (34) through (41) can be written as

$$\sum_{n=0}^{\infty} S_{Hn} = S_{h0} + \wedge_1 t - \beta\sum_{n=0}^{\infty}\int_0^t B_n d\tau - \beta\sum_{n=0}^{\infty}\int_0^t C_n d\tau - \beta\sum_{n=0}^{\infty}\int_0^t D_n d\tau - \beta\delta\sum_{n=0}^{\infty}\int_0^t F_n d\tau - \mu_1\sum_{n=0}^{\infty}\int_0^t S_{Hn}d\tau \quad (42)$$

$$\sum_{n=0}^{\infty} E_{Hn} = E_{h0} + \beta\sum_{n=0}^{\infty}\int_0^t B_n d\tau + \beta\sum_{n=0}^{\infty}\int_0^t C_n d\tau + \beta\sum_{n=0}^{\infty}\int_0^t D_n d\tau + \beta\delta\sum_{n=0}^{\infty}\int_0^t F_n d\tau - (\alpha+\mu_1)\sum_{n=0}^{\infty}\int_0^t E_{Hn}d\tau \quad (43)$$

$$\sum_{n=0}^{\infty} A_{Hn} = A_{h0} + (1-\rho)\alpha\sum_{n=0}^{\infty}\int_0^t E_{Hn}d\tau - (\eta+\phi+\mu_1)\sum_{n=0}^{\infty}\int_0^t A_{Hn}d\tau \quad (44)$$

$$\sum_{n=0}^{\infty} I_{Hn} = I_{h0} + \rho\alpha\sum_{n=0}^{\infty}\int_0^t E_{Hn}d\tau - (\gamma+\phi+\mu_1)\sum_{n=0}^{\infty}\int_0^t I_{Hn}d\tau \quad (45)$$

$$\sum_{n=0}^{\infty} T_{Hn} = T_{h0} + \gamma\sum_{n=0}^{\infty}\int_0^t I_{Hn}d\tau + \eta\sum_{n=0}^{\infty}\int_0^t A_{Hn}d\tau - (\kappa+\phi+\mu_1)\sum_{n=0}^{\infty}\int_0^t T_{Hn}d\tau \quad (46)$$

$$\sum_{n=0}^{\infty} R_{Hn} = R_{h0} + \kappa\sum_{n=0}^{\infty}\int_0^t T_{Hn}d\tau - \mu_1\sum_{n=0}^{\infty}\int_0^t R_{Hn}d\tau \quad (47)$$

$$\sum_{n=0}^{\infty} S_{Rn} = S_{r0} + \wedge_2 t - \lambda\sum_{n=0}^{\infty}\int_0^t N_n d\tau - (\nu+\mu_2)\sum_{n=0}^{\infty}\int_0^t S_{Rn}d\tau \quad (48)$$

$$\sum_{n=0}^{\infty} I_{Rn} = I_{r0} + \lambda\sum_{n=0}^{\infty}\int_0^t N_n d\tau - (\nu+\mu_2)\sum_{n=0}^{\infty}\int_0^t I_{Rn}d\tau \quad (49)$$

From equations (42) through (49), we define the following scheme

$$\left.\begin{aligned} S_{H0} &= S_{h0} + \wedge_1 t \\ E_{H0} &= E_{h0} \\ A_{H0} &= A_{h0} \\ I_{H0} &= I_{h0} \\ T_{H0} &= T_{h0} \\ R_{H0} &= R_{h0} \\ S_{R0} &= S_{r0} + \wedge_2 t \\ I_{R0} &= I_{r0} \end{aligned}\right\} \quad (50)$$

$$S_{Hn+1} = -\beta\int_0^t B_n d\tau - \beta\int_0^t C_n d\tau - \beta\int_0^t D_n d\tau - \beta\delta\int_0^t F_n d\tau - \mu_1\int_0^t S_{Hn} \quad (51)$$

$$E_{Hn+1} = \beta\int_0^t B_n d\tau + \beta\int_0^t C_n d\tau + \beta\int_0^t D_n d\tau + \beta\delta\int_0^t F_n d\tau - (\alpha+\mu_1)\int_0^t E_{Hn} d\tau \quad (52)$$

$$A_{Hn+1} = (1-\rho)\alpha\int_0^t E_{Hn} d\tau - (\eta+\phi+\mu_1)\int_0^t A_{Hn} d\tau \quad (53)$$

$$I_{Hn+1} = \rho\alpha\int_0^t E_{Hn} d\tau - (\gamma+\phi+\mu_1)\int_0^t I_{Hn} d\tau \quad (54)$$

$$T_{Hn+1} = \eta\int_0^t A_{Hn} d\tau + \gamma\int_0^t I_{Hn} d\tau - (\kappa+\phi+\mu_1)\int_0^t T_{Hn} d\tau \quad (55)$$

$$R_{Hn+1} = \kappa\int_0^t T_{Hn} d\tau - \mu_1\int_0^t R_{Hn} d\tau \quad (56)$$

$$S_{Rn+1} = -\lambda\int_0^t N_n d\tau - (\nu+\mu_1)\int_0^t S_{Rn} d\tau \quad (57)$$

$$I_{Rn+1} = \lambda\int_0^t N_n d\tau - (\nu+\mu_1)\int_0^t I_{Rn} d\tau \quad (58)$$

Using the algorithm in (19), the Adomian polynomials in (33) are computed as

$$\left.\begin{array}{l} B_0 = S_{H0}I_{R0} \\ B_1 = S_{H1}I_{R0} + I_{R1}S_{H0} \\ B_2 = S_{H2}I_{R0} + S_{H1}I_{R1} + S_{H0}I_{R2} \\ \vdots \end{array}\right\} \quad (59)$$

$$\left.\begin{array}{l} C_0 = S_{H0}I_{H0} \\ C_1 = S_{H1}I_{H0} + S_{H0}I_{H1} \\ C_2 = S_{H2}I_{H0} + S_{H1}I_{H1} + S_{H0}I_{H2} \\ \vdots \end{array}\right\} \quad (60)$$

$$\left.\begin{array}{l} D_0 = S_{H0}A_{H0} \\ D_1 = S_{H1}A_{H0} + S_{H0}A_{H1} \\ D_2 = S_{H2}A_{H0} + S_{H1}A_{H1} + S_{H0}A_{H2} \\ \vdots \end{array}\right\} \quad (61)$$

$$\left.\begin{array}{l} F_0 = S_{H0}T_{H0} \\ F_1 = S_{H1}T_{H0} + S_{H0}T_{H1} \\ F_2 = S_{H2}T_{H0} + S_{H1}T_{H1} + S_{H0}T_{H2} \\ \vdots \end{array}\right\} \quad (62)$$

$$\left.\begin{array}{l} N_0 = S_{R0}I_{R0} \\ N_1 = S_{R1}I_{R0} + S_{R0}I_{R1} \\ N_2 = S_{R2}I_{R0} + S_{R1}I_{R1} + S_{R0}I_{R2} \\ \vdots \end{array}\right\} \quad (63)$$

For n=0, equation (51) gives

$$S_{H1} = -\beta\int_0^t B_0 d\tau - \beta\int_0^t C_0 d\tau - \beta\int_0^t D_0 d\tau - \beta\delta\int_0^t F_0 d\tau - \mu_1\int_0^t S_{H0} d\tau \quad (64)$$

Substituting equations (59) through (63) into equation (64) gives

$$S_{H1} = -\beta\int_0^t S_{H0}I_{R0} d\tau - \beta\int_0^t S_{H0}I_{H0} d\tau - \beta\int_0^t S_{H0}A_{H0} d\tau - \beta\delta\int_0^t S_{H0}T_{H0} d\tau - \mu_1\int_0^t S_{H0} d\tau \quad (65)$$

Substituting equation (50) into equation (65) gives

ISSN: 1399-8466

$$\left[\begin{array}{l} S_{H1}=-\beta\int_0^t (S_{h0}I_{r0}+\wedge_1 I_{r0}\tau)d\tau-\beta\int_0^t (S_{h0}I_{h0}+\wedge_1 I_{h0}\tau)d\tau-\beta\int_0^t (S_{h0}A_{h0}+\wedge_1 A_{h0}\tau)d\tau- \\ \beta\delta\int_0^t (S_{h0}T_{h0}+\wedge_1 T_{h0}\tau)d\tau-\mu_1\int_0^t (S_{h0}+\wedge_1\tau)d\tau \end{array}\right] \quad (66)$$

Integrating and collecting like terms gives

$$S_{H1}=-\beta\left(S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}+\frac{\mu_1}{\beta}S_{h0}\right)t-\frac{\beta\wedge_1}{2}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}+\frac{\mu_1}{\beta}\right)t^2 \quad (67)$$

For n=0, equation (52) gives

$$E_{H1}=\beta\int_0^t B_0 d\tau+\beta\int_0^t C_0 d\tau+\beta\int_0^t D_0 d\tau+\beta\delta\int_0^t F_0 d\tau-(\alpha+\mu_1)\int_0^t E_{H0}d\tau \quad (68)$$

Substituting equations (59) through (63) into equation (68) gives

$$E_{H1}=\beta\int_0^t S_{H0}I_{R0}d\tau+\beta\int_0^t S_{H0}I_{H0}d\tau+\beta\int_0^t S_{H0}A_{H0}d\tau+\beta\delta\int_0^t S_{H0}T_{H0}d\tau-(\alpha+\mu_1)\int_0^t E_{H0}d\tau \quad (69)$$

Substituting equation (22) into equation (69) gives

$$\left[\begin{array}{l} E_{H1}=\beta\int_0^t (S_{h0}I_{r0}+\wedge_1 I_{r0}\tau)d\tau+\beta\int_0^t (S_{h0}I_{h0}+\wedge_1 I_{h0}\tau)d\tau+\beta\int_0^t (S_{h0}A_{h0}+\wedge_1 A_{h0}\tau)d\tau+ \\ \beta\delta\int_0^t (S_{h0}T_{h0}+\wedge_1 T_{h0}\tau)d\tau-(\alpha+\mu_1)\int_0^t E_{h0}d\tau \end{array}\right] \quad (70)$$

Integrating and collecting like terms gives

$$E_{H1}=\beta\left(S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\frac{\alpha+\mu_1}{\beta}E_{h0}\right)t+\frac{\beta\wedge_1}{2}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)t^2 \quad (71)$$

For n=0, equation (53) gives

$$A_{H1}=(1-\rho)\alpha\int_0^t E_{H0}d\tau-(\eta+\phi+\mu_1)\int_0^t A_{H0}d\tau \quad (72)$$

Substituting equation (50) into (72) gives

$$A_{H1}=(1-\rho)\alpha\int_0^t E_{h0}d\tau-(\eta+\phi+\mu_1)\int_0^t A_{h0}d\tau \quad (73)$$

Integrating gives

$$A_{H1}=\left((1-\rho)\alpha E_{h0}-(\eta+\phi+\mu_1)A_{h0}\right)t \quad (74)$$

For n=0, equation (54) gives

$$I_{H1} = \rho\alpha\int_0^t E_{H0}d\tau - (\gamma+\phi+\mu_1)\int_0^t I_{H0}d\tau \tag{75}$$

Substituting equation (50) into (75) gives

$$I_{H1} = \rho\alpha\int_0^t E_{h0}d\tau - (\gamma+\phi+\mu_1)\int_0^t I_{h0}d\tau \tag{76}$$

Integrating gives

$$I_{H1} = \left(\rho\alpha E_{h0} - (\gamma+\phi+\mu_1)I_{h0}\right)t \tag{77}$$

For n=0, equation (55) gives

$$T_{H1} = \eta\int_0^t A_{H0}d\tau + \gamma\int_0^t I_{H0}d\tau - (\kappa+\phi+\mu_1)\int_0^t T_{H0}d\tau \tag{78}$$

Substituting equation (50) into equation (78) gives

$$T_{H1} = \eta\int_0^t A_{h0}d\tau + \gamma\int_0^t I_{h0}d\tau - (\kappa+\phi+\mu_1)\int_0^t T_{h0}d\tau \tag{79}$$

Integrating gives

$$T_{H1} = \left(\eta A_{h0} + \gamma I_{h0} - (\kappa+\phi+\mu_1)T_{h0}\right)t \tag{80}$$

For n=0, equation (56) gives

$$R_{H1} = \kappa\int_0^t T_{H0}d\tau - \mu_1\int_0^t R_{H0}d\tau \tag{81}$$

Substituting equation (50) into equation (81) gives

$$R_{H1} = \kappa\int_0^t T_{h0}d\tau - \mu_1\int_0^t R_{h0}d\tau \tag{82}$$

Integrating gives

$$R_{H1} = (\kappa T_{h0} - \mu_1 R_{h0})t \tag{83}$$

For n=0, equation (57) gives

$$S_{R1} = -\lambda\int_0^t S_{R0}I_{R0}d\tau - (\nu+\mu_1)\int_0^t S_{R0}d\tau \tag{84}$$

Substituting equation (50) into equation (84) gives

$$S_{R1} = -\lambda \int_0^t (S_{r0} I_{r0} + \wedge_2 I_{r0} \tau) d\tau - (\nu + \mu_1) \int_0^t (S_{r0} + \wedge_2 \tau) d\tau \tag{85}$$

Integrating and collecting like terms gives

$$S_{R1} = -S_{r0} (\lambda I_{r0} + (\nu + \mu_1)) t - \frac{\wedge_2}{2} \left(\lambda I_{r0} + (\nu + \mu_1) \right) t^2 \tag{86}$$

For n=0, equation (58) gives

$$I_{R1} = \lambda \int_0^t S_{R0} I_{R0} d\tau - (\nu + \mu_1) \int_0^t I_{R0} d\tau \tag{87}$$

Substituting equations (50) into equation (87) gives

$$I_{R1} = \lambda \int_0^t (S_{r0} I_{r0} + \wedge_2 I_{r0} \tau) d\tau - (\nu + \mu_1) \int_0^t I_{r0} d\tau \tag{88}$$

Integrating and collecting like terms gives

$$I_{R1} = I_{r0} (\lambda S_{r0} - (\nu + \mu_1)) t + \frac{\lambda \wedge_2}{2} I_{r0} t^2 \tag{89}$$

For n=1, equation (51) gives

$$S_{H2} = -\beta \int_0^t B_1 d\tau - \beta \int_0^t C_1 d\tau - \beta \int_0^t D_1 d\tau - \beta \delta \int_0^t F_1 d\tau - \mu_1 \int_0^t S_{H1} d\tau \tag{90}$$

Substituting equations (59) through (63) into equation (90) gives

$$S_{H2} = \begin{pmatrix} -\beta \int_0^t (S_{H1} I_{R0} + I_{R1} S_{H0}) d\tau - \beta \int_0^t (S_{H1} I_{H0} + I_{H1} S_{H0}) d\tau - \\ \beta \int_0^t (S_{H1} A_{H0} + A_{H1} S_{H0}) d\tau - \beta \delta \int_0^t (S_{H1} T_{H0} + T_{H1} S_{H0}) d\tau - \mu_1 \int_0^t S_{H1} d\tau \end{pmatrix} \tag{91}$$

Substituting equations (50), (67), (74), (77), (80) and (89) into equation (91) gives

$$S_{H2}=\left[\begin{array}{l}
\beta^2\int_0^t\left(S_{h0}(I_{r0})^2+S_{h0}I_{h0}I_{r0}+S_{h0}A_{h0}I_{r0}+\delta S_{h0}T_{h0}I_{r0}+\frac{\mu_1}{\beta}S_{h0}I_{r0}\right)\tau d\tau\\
+\frac{\beta^2\wedge_1}{2}\int_0^t\left((I_{r0})^2+I_{h0}I_{r0}+A_{h0}I_{r0}+\delta T_{h0}I_{r0}+\frac{\mu_1}{\beta}I_{r0}\right)\tau^2 d\tau\\
-\beta\int_0^t\left(I_{r0}(\lambda S_{r0}-(\nu+\mu_1))\tau+\frac{\lambda\wedge_2}{2}I_{r0}\tau^2\right)(S_{h0}+\wedge_1\tau)d\tau\\
+\beta^2\int_0^t\left(S_{h0}(I_{h0})^2+S_{h0}I_{h0}I_{r0}+S_{h0}A_{h0}I_{h0}+\delta S_{h0}T_{h0}I_{h0}+\frac{\mu_1}{\beta}S_{h0}I_{h0}\right)\tau d\tau\\
+\frac{\beta^2\wedge_1}{2}\int_0^t\left((I_{h0})^2+I_{h0}I_{r0}+A_{h0}I_{h0}+\delta T_{h0}I_{h0}+\frac{\mu_1}{\beta}I_{h0}\right)\tau^2 d\tau\\
-\beta\int_0^t\left((\rho\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0})\tau\right)(S_{h0}+\wedge_1\tau)d\tau\\
+\beta^2\int_0^t\left(S_{h0}(A_{h0})^2+S_{h0}A_{h0}I_{r0}+S_{h0}A_{h0}I_{h0}+\delta S_{h0}T_{h0}A_{h0}+\frac{\mu_1}{\beta}S_{h0}A_{h0}\right)\tau d\tau\\
+\frac{\beta^2\wedge_1}{2}\int_0^t\left((A_{h0})^2+A_{h0}I_{r0}+A_{h0}I_{h0}+\delta T_{h0}A_{h0}+\frac{\mu_1}{\beta}A_{h0}\right)\tau^2 d\tau\\
-\beta\int_0^t\left(((1-\rho)\alpha E_{h0}-(\eta+\phi+\mu_1)A_{h0})\tau\right)(S_{h0}+\wedge_1\tau)d\tau\\
+\beta^2\delta\int_0^t\left(\delta S_{h0}(T_{h0})^2+S_{h0}T_{h0}I_{r0}+S_{h0}A_{h0}T_{h0}+S_{h0}T_{h0}I_{h0}+\frac{\mu_1}{\beta}S_{h0}T_{h0}\right)\tau d\tau\\
+\frac{\beta^2\delta\wedge_1}{2}\int_0^t\left(\delta(T_{h0})^2+T_{h0}I_{r0}+A_{h0}T_{h0}+T_{h0}I_{h0}+\frac{\mu_1}{\beta}T_{h0}\right)\tau^2 d\tau\\
-\beta\delta\int_0^t\left((\eta E_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0})\tau\right)(S_{h0}+\wedge_1\tau)d\tau\\
+\beta\mu_1\int_0^t\left(S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}+\frac{\mu_1}{\beta}S_{h0}\right)\tau d\tau\\
+\frac{\beta\wedge_1\mu_1}{2}\int_0^t\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}+\frac{\mu_1}{\beta}\right)\tau^2 d\tau
\end{array}\right] \quad (92)$$

Integrating and collecting like terms gives

ISSN: 1399-8466

International Journal of Pure and Applied Science

$$S_{H2}=\left[\begin{array}{l}\left(\begin{array}{l}\beta^2 S_{h0}\left(\begin{array}{l}\left((I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2\right)+2\beta^2\begin{pmatrix}I_{r0}I_{h0}+I_{r0}A_{h0}+A_{h0}I_{h0}+\\ \delta I_{r0}T_{h0}+\delta I_{h0}T_{h0}+\delta A_{h0}T_{h0}\end{pmatrix}+\\ \frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right)\\ -\beta S_{h0}\begin{pmatrix}I_{r0}(\lambda S_{r0}-(\nu+\mu_1))+(\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0}-\\ (\eta+\phi+\mu_1)A_{h0}+\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0}))\end{pmatrix}+\beta\mu_1 S_{h0}\\ \begin{pmatrix}I_{r0}+I_{h0}+A_{h0}\\ +\delta T_{h0}+\frac{\mu_1}{\beta}\end{pmatrix}\end{array}\right)\frac{t^2}{2}\\ \left(\begin{array}{l}\frac{\beta^2\wedge_1}{2}\begin{pmatrix}(I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2+2(I_{r0}I_{h0}+I_{r0}A_{h0}+A_{h0}I_{h0}+\\ \delta I_{r0}T_{h0}+\delta I_{h0}T_{h0}+\delta A_{h0}T_{h0})+\frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{pmatrix}\\ -\beta\left(\begin{array}{l}\wedge_1 I_{r0}(\lambda S_{r0}-(\nu+\mu_2))+\frac{\lambda\wedge_2}{2}S_{h0}I_{r0}+\wedge_1\alpha E_{h0}-\wedge_1(\gamma+\phi+\mu_1)I_{h0}-\\ \wedge_1(\eta+\phi+\mu_1)A_{h0}+\wedge_1\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0})+\\ \frac{\beta\wedge_1\mu_1}{2}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}+\frac{\mu_1}{\beta}\right)\end{array}\right)\end{array}\right)\frac{t^3}{3}\\ -\frac{\beta\lambda\wedge_1\wedge_2}{8}I_{r0}t^4\end{array}\right] \qquad (93)$$

$$S_{H2}=\left[\begin{array}{l}\left(\begin{array}{l}\beta^2 S_{h0}\left(\begin{array}{l}\left((I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2\right)+2\beta^2\begin{pmatrix}I_{r0}I_{h0}+I_{r0}A_{h0}+A_{h0}I_{h0}+\\ \delta I_{r0}T_{h0}+\delta I_{h0}T_{h0}+\delta A_{h0}T_{h0}\end{pmatrix}+\\ \frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right)\\ -\beta S_{h0}\begin{pmatrix}I_{r0}(\lambda S_{r0}-(\nu+\mu_1))+(\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0}-\\ (\eta+\phi+\mu_1)A_{h0}+\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0}))\end{pmatrix}+\beta\mu_1 S_{h0}\\ \begin{pmatrix}I_{r0}+I_{h0}+A_{h0}\\ +\delta T_{h0}+\frac{\mu_1}{\beta}\end{pmatrix}\end{array}\right)\frac{t^2}{2}\\ \left(\begin{array}{l}\frac{\beta^2\wedge_1}{2}\begin{pmatrix}(I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2+2(I_{r0}I_{h0}+I_{r0}A_{h0}+A_{h0}I_{h0}+\\ \delta I_{r0}T_{h0}+\delta I_{h0}T_{h0}+\delta A_{h0}T_{h0})+\frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{pmatrix}\\ -\beta\left(\begin{array}{l}\wedge_1 I_{r0}(\lambda S_{r0}-(\nu+\mu_2))+\frac{\lambda\wedge_2}{2}S_{h0}I_{r0}+\wedge_1\alpha E_{h0}-\wedge_1(\gamma+\phi+\mu_1)I_{h0}-\\ \wedge_1(\eta+\phi+\mu_1)A_{h0}+\wedge_1\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0})+\\ \frac{\beta\wedge_1\mu_1}{2}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}+\frac{\mu_1}{\beta}\right)\end{array}\right)\end{array}\right)\frac{t^3}{3}\\ -\frac{\beta\lambda\wedge_1\wedge_2}{8}I_{r0}t^4\end{array}\right] \qquad (94)$$

For n=1, equation (52) gives

$$E_{H2}=\beta\int_0^t B_1 d\tau+\beta\int_0^t C_1 d\tau+\beta\int_0^t D_1 d\tau+\beta\delta\int_0^t F_1 d\tau-(\alpha+\mu_1)\int_0^t E_{H1} d\tau \qquad (95)$$

Substituting equations (59) through (63) into equation (95) gives

$$E_{H2}\left(\begin{array}{l}\beta\int_0^t (S_{H1}I_{R0}+I_{R1}S_{H0})d\tau+\beta\int_0^t (S_{H1}I_{H0}+I_{H1}S_{H0})d\tau+\beta\int_0^t (S_{H1}A_{H0}+A_{H1}S_{H0})d\tau+\\ \beta\delta\int_0^t (S_{H1}T_{H0}+T_{H1}S_{H0})d\tau-(\alpha+\mu_1)\int_0^t E_{H1}d\tau\end{array}\right) \qquad (96)$$

Substituting equations (67), (71), (74), (77), (80) into equation (96) gives

$$E_{H2}=\left[\begin{array}{l}
-\beta^2\int_0^t\left(S_{h0}(I_{r0})^2+S_{h0}I_{h0}I_{r0}+S_{h0}A_{h0}I_{r0}+\delta S_{h0}T_{h0}I_{r0}+\frac{\mu_1}{\beta}S_{h0}I_{r0}\right)\tau d\tau\\
-\frac{\beta^2\wedge_1}{2}\int_0^t\left((I_{r0})^2+I_{h0}I_{r0}+A_{h0}I_{r0}+\delta T_{h0}I_{r0}+\frac{\mu_1}{\beta}I_{r0}\right)\tau^2 d\tau\\
+\beta\int_0^t\left(I_{r0}(\lambda S_{r0}-(\nu+\mu_1))\tau+\frac{\lambda\wedge_2}{2}I_{r0}\tau^2\right)(S_{h0}+\wedge_1\tau)d\tau\\
-\beta^2\int_0^t\left(S_{h0}(I_{h0})^2+S_{h0}I_{h0}I_{r0}+S_{h0}A_{h0}I_{h0}+\delta S_{h0}T_{h0}I_{h0}+\frac{\mu_1}{\beta}S_{h0}I_{h0}\right)\tau d\tau\\
-\frac{\beta^2\wedge_1}{2}\int_0^t\left((I_{h0})^2+I_{h0}I_{r0}+A_{h0}I_{h0}+\delta T_{h0}I_{h0}+\frac{\mu_1}{\beta}I_{h0}\right)\tau^2 d\tau\\
+\beta\int_0^t\left((\rho\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0})\tau\right)(S_{h0}+\wedge_1\tau)d\tau\\
-\beta^2\int_0^t\left(S_{h0}(A_{h0})^2+S_{h0}A_{h0}I_{r0}+S_{h0}A_{h0}I_{h0}+\delta S_{h0}T_{h0}A_{h0}+\frac{\mu_1}{\beta}S_{h0}A_{h0}\right)\tau d\tau\\
-\frac{\beta^2\wedge_1}{2}\int_0^t\left((A_{h0})^2+A_{h0}I_{r0}+A_{h0}I_{h0}+\delta T_{h0}A_{h0}+\frac{\mu_1}{\beta}A_{h0}\right)\tau^2 d\tau\\
+\beta\int_0^t\left(((1-\rho)\alpha E_{h0}-(\eta+\phi+\mu_1)A_{h0})\tau\right)(S_{h0}+\wedge_1\tau)d\tau\\
-\beta^2\delta\int_0^t\left(\delta S_{h0}(T_{h0})^2+S_{h0}T_{h0}I_{r0}+S_{h0}A_{h0}T_{h0}+S_{h0}T_{h0}I_{h0}+\frac{\mu_1}{\beta}S_{h0}T_{h0}\right)\tau d\tau\\
-\frac{\beta^2\delta\wedge_1}{2}\int_0^t\left(\delta(T_{h0})^2+T_{h0}I_{r0}+A_{h0}T_{h0}+T_{h0}I_{h0}+\frac{\mu_1}{\beta}T_{h0}\right)\tau^2 d\tau\\
+\beta\delta\int_0^t\left((\eta E_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0})\tau\right)(S_{h0}+\wedge_1\tau)d\tau\\
-\beta(\alpha+\mu_1)\int_0^t\left(S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\frac{\alpha+\mu_1}{\beta}E_{h0}\right)\tau d\tau\\
-\frac{\beta\wedge_1(\alpha+\mu_1)}{2}\int_0^t(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0})\tau^2 d\tau
\end{array}\right] \qquad (97)$$

Integrating and collecting like terms gives

$$E_{H2} =$$

$$\left(\begin{array}{l} -\beta^2 S_{h0}\left(\begin{array}{l}\left((I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2\right)+2\beta^2\left(\begin{array}{l} I_{r0}I_{h0}+I_{r0}A_{h0} \\ +A_{h0}I_{h0}+\delta I_{r0}T_{h0}+ \\ \delta I_{h0}T_{h0}+\delta A_{h0}T_{h0}\end{array}\right)+ \\ \frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right) \\ +\beta S_{h0}\left(\begin{array}{l} I_{r0}(\lambda S_{r0}-(\nu+\mu_1))+(\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0}- \\ (\eta+\phi+\mu_1)A_{h0}+\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0}))\end{array}\right)- \\ \beta(\alpha+\mu_1)S_{h0}\left(\begin{array}{l} I_{r0}+I_{h0}+ \\ A_{h0}+\delta T_{h0} \\ -(\alpha+\mu_1)E_{h0}\end{array}\right)\end{array}\right)\frac{t^2}{2}$$

$$\left(\begin{array}{l} -\frac{\beta^2\wedge_1}{2}\left(\begin{array}{l}(I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2+2(I_{r0}I_{h0}+I_{r0}A_{h0}+A_{h0}I_{h0}+ \\ \delta I_{r0}T_{h0}+\delta I_{h0}T_{h0}+\delta A_{h0}T_{h0})+\frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right) \\ +\beta\left(\begin{array}{l}\wedge_1 I_{r0}(\lambda S_{r0}-(\nu+\mu_2))+\frac{\lambda\wedge_2}{2}S_{h0}I_{r0}+\wedge_1\alpha E_{h0}- \\ \wedge_1(\gamma+\phi+\mu_1)I_{h0}-\wedge_1(\eta+\phi+\mu_1)A_{h0} \\ +\wedge_1\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0})- \\ \frac{\beta\wedge_1(\alpha+\mu_1)}{2}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right)\end{array}\right)\frac{t^3}{3}$$

$$+\frac{\beta\lambda\wedge_1\wedge_2}{8}I_{r0}t^4 \qquad (98)$$

For n=1, equation (53) gives

$$A_{H2}=(1-\rho)\alpha\int_0^t E_{H1}d\tau-(\eta+\phi+\mu_1)\int_0^t A_{H1}d\tau \qquad (99)$$

Substituting equations (69) and (75) into (99) gives

$$A_{H2}=\begin{bmatrix}\beta(1-\rho)\alpha\left(S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\dfrac{\alpha+\mu_1}{\beta}E_{h0}\right)\int_0^t\tau d\tau\\ +\dfrac{\beta\wedge_1(1-\rho)\alpha}{2}(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0})\int_0^t\tau^2d\tau-\\ (\eta+\phi+\mu_1)((1-\rho)\alpha E_{h0}-(\eta+\phi+\mu_1)A_{h0})\int_0^t\tau d\tau\end{bmatrix} \quad (100)$$

Integrating and collecting like terms gives

$$A_{H2}=\begin{bmatrix}\beta(1-\rho)\alpha\begin{pmatrix}S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\dfrac{\alpha+\mu_1}{\beta}E_{h0}-\\(\eta+\phi+\mu_1)((1-\rho)\alpha E_{h0}-(\eta+\phi+\mu_1)A_{h0})\end{pmatrix}\dfrac{t^2}{2}\\ +\dfrac{\beta\wedge_1(1-\rho)\alpha}{2}(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0})\dfrac{t^3}{3}\end{bmatrix} \quad (101)$$

For n=1, equation (54) gives

$$I_{H2}=\rho\alpha\int_0^t E_{H1}d\tau-(\gamma+\phi+\mu_1)\int_0^t I_{H1}d\tau \quad (102)$$

Substituting equations (71) and (78) into (102) gives

$$I_{H2}=\begin{bmatrix}\beta\rho\alpha\left(S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\dfrac{\alpha+\mu_1}{\beta}E_{h0}\right)\int_0^t\tau d\tau+\\ \dfrac{\beta\wedge_1\rho\alpha}{2}(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0})\int_0^t\tau^2d\tau-\\ (\gamma+\phi+\mu_1)(\rho\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0})\int_0^t\tau d\tau\end{bmatrix} \quad (103)$$

Integrating and collecting like terms gives

$$I_{H2}=\begin{bmatrix}\beta\rho\alpha\begin{pmatrix}S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\dfrac{\alpha+\mu_1}{\beta}E_{h0}-\\(\gamma+\phi+\mu_1)(\rho\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0})\end{pmatrix}\dfrac{t^2}{2}\\ +\dfrac{\beta\wedge_1\rho\alpha}{2}(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0})\dfrac{t^3}{3}\end{bmatrix} \quad (104)$$

For n=1, equation (55) gives

$$T_{H2} = \eta\int_0^t A_{H1} d\tau + \gamma\int_0^t I_{H1} d\tau - (\kappa+\phi+\mu_1)\int_0^t T_{H1} d\tau \quad (105)$$

Substituting equations (75), (78) and (81) into equation (105) gives

$$T_{H2} = \begin{pmatrix} \eta((1-\rho)\alpha E_{h0} - (\eta+\phi+\mu_1)A_{h0})\int_0^t \tau d\tau + \gamma(\rho\alpha E_{h0} - (\gamma+\phi+\mu_1)I_{h0})\int_0^t \tau d\tau \\ -(\kappa+\phi+\mu_1)T_{H1}\left(\eta E_{h0} + \gamma I_{h0} - (\kappa+\phi+\mu_1)T_{h0}\right)\int_0^t \tau d\tau \end{pmatrix} \quad (106)$$

Integrating gives

$$T_{H2} = \begin{pmatrix} \eta((1-\rho)\alpha E_{h0} - (\eta+\phi+\mu_1)A_{h0}) + \gamma(\rho\alpha E_{h0} - (\gamma+\phi+\mu_1)I_{h0}) - \\ (\kappa+\phi+\mu_1)T_{H1}\left(\eta E_{h0} + \gamma I_{h0} - (\kappa+\phi+\mu_1)T_{h0}\right) \end{pmatrix}\frac{t^2}{2} \quad (107)$$

For n=1, equation (57) gives

$$R_{H2} = \kappa\int_0^t T_{H1} d\tau - \mu_1\int_0^t R_{H1} d\tau \quad (108)$$

Substituting equation (81) and (84) into equation (108) gives

$$R_{H2} = \kappa\left(\eta E_{h0} + \gamma I_{h0} - (\kappa+\phi+\mu_1)T_{h0}\right)\int_0^t \tau d\tau - \mu_1(\kappa T_{h0} - \mu_1 R_{h0})\int_0^t \tau d\tau \quad (109)$$

integrating gives

$$R_{H2} = \left(\kappa\left(\eta E_{h0} + \gamma I_{h0} - (\kappa+\phi+\mu_1)T_{h0}\right) - \mu_1(\kappa T_{h0} - \mu_1 R_{h0})\right)\frac{t^2}{2} \quad (110)$$

For n=1, equation (57) gives

$$S_{R2} = -\lambda\int_0^t (S_{R1}I_{R0} + S_{R0}I_{R1}) d\tau - (\nu+\mu_1)\int_0^t S_{R1} d\tau \quad (111)$$

Substituting equations (50), (88) and (90) into equation (111) gives

$$S_{R2} = \begin{pmatrix} \lambda S_{r0}I_{r0}(\lambda I_{r0} - (\nu+\mu_2))\int_0^t \tau d\tau + \frac{\lambda\wedge_2}{2}I_{r0}(\lambda I_{r0} + (\nu+\mu_2))\int_0^t \tau^2 \\ -\lambda\int_0^t (S_{r0} + \wedge_2\tau)\left(I_{r0}(\lambda S_{r0} - (\nu+\mu_2))\tau + \frac{\lambda\wedge_2}{2}I_{r0}\tau^2\right)d\tau \\ +(\nu+\mu_2)S_{r0}(\lambda I_{r0} + (\nu+\mu_2))\int_0^t \tau d\tau + \frac{\wedge_2}{2}(\nu+\mu_2)(\lambda I_{r0} + (\nu+\mu_2))\int_0^t \tau^2 d\tau \end{pmatrix} \quad (112)$$

Integrating and collecting like terms gives

$$S_{R2}=\begin{pmatrix}\begin{pmatrix}\lambda S_{r0}I_{r0}(\lambda I_{r0}-(\nu+\mu_2))-\lambda S_{r0}I_{r0}(\lambda S_{r0}-(\nu+\mu_2))+S_{r0}(\nu+\mu_2)\\(\lambda I_{r0}+(\nu+\mu_2))\end{pmatrix}\frac{t^2}{2}\\+\begin{pmatrix}\frac{\lambda\wedge_2}{2}I_{r0}(\lambda I_{r0}+(\nu+\mu_2))-\lambda\left(\wedge_2 I_{r0}(\lambda S_{r0}-(\nu+\mu_2)+\frac{\lambda\wedge_2}{2}S_{r0}I_{r0}\right)+\\\frac{\wedge_2}{2}(\nu+\mu_2)(\lambda I_{r0}+(\nu+\mu_2))\end{pmatrix}\frac{t^3}{3}-\\\frac{\lambda^2(\wedge_2)^2}{8}I_{r0}t^4\end{pmatrix} \quad (113)$$

For n=1, equation (58) gives

$$I_{R2}=\lambda\int_0^t(S_{R1}I_{R0}+S_{R0}I_{R1})d\tau-(\nu+\mu_1)\int_0^t I_{R1}d\tau \quad (114)$$

Substituting equations (3.50), (3.87) and (3.90) into equation (3.114) gives

$$I_{R2}=\begin{pmatrix}-\lambda S_{r0}I_{r0}(\lambda I_{r0}-(\nu+\mu_2))\int_0^t\tau d\tau-\frac{\lambda\wedge_2}{2}I_{r0}(\lambda I_{r0}+(\nu+\mu_2))\int_0^t\tau^2\\+\lambda\int_0^t(S_{r0}+\wedge_2\tau)\left(I_{r0}(\lambda S_{r0}-(\nu+\mu_2))\tau+\frac{\lambda\wedge_2}{2}I_{r0}\tau^2\right)d\tau\\-(\nu+\mu_2)I_{r0}(\lambda S_{r0}-(\nu+\mu_1))\int_0^t\tau d\tau-(\nu+\mu_2)\frac{\lambda\wedge_2}{2}I_{r0}\int_0^t\tau^2 d\tau\end{pmatrix} \quad (115)$$

Integrating and collecting like terms gives

$$I_{R2}=\begin{pmatrix}\begin{pmatrix}-\lambda S_{r0}I_{r0}(\lambda I_{r0}-(\nu+\mu_2))+\lambda S_{r0}I_{r0}(\lambda S_{r0}-(\nu+\mu_2))\\-I_{r0}(\nu+\mu_2)(\lambda S_{r0}-(\nu+\mu_2))\end{pmatrix}\frac{t^2}{2}\\\begin{pmatrix}-\frac{\lambda\wedge_2}{2}I_{r0}(\lambda I_{r0}+(\nu+\mu_2))+\lambda\left(\wedge_2 I_{r0}(\lambda S_{r0}-(\nu+\mu_2))\right)\\+\frac{\lambda\wedge_2}{2}S_{r0}I_{r0}-(\nu+\mu_2)\frac{\lambda\wedge_2}{2}I_{r0}\end{pmatrix}\frac{t^3}{3}\\+\frac{\lambda^2(\wedge_2)^2}{8}I_{r0}t^4\end{pmatrix} \quad (116)$$

Substituting equations (50), (67) and (94) into (31) gives

$$S_H(t)=\left[\begin{array}{l} S_{h0}+\left(\wedge_1-\beta\left(S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}+\frac{\mu_1}{\beta}S_{h0}\right)\right)t+ \\ \left(\begin{array}{l} \beta^2 S_{h0}\left(\begin{array}{l}\left((I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2\right)+2\beta^2\left(\begin{array}{l}I_{r0}I_{h0}+I_{r0}A_{h0}+\\ A_{h0}I_{h0}+\delta I_{r0}T_{h0}+\\ \delta I_{h0}T_{h0}+\delta A_{h0}T_{h0}\end{array}\right)+\\ \frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right) \\ -\beta S_{h0}\left(\begin{array}{l}I_{r0}(\lambda S_{r0}-(\nu+\mu_1))+(\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0}-\\ (\eta+\phi+\mu_1)A_{h0}+\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0}))\end{array}\right)+ \\ (\beta\mu_1 S_{h0}-\beta\wedge_1)\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}+\frac{\mu_1}{\beta}\right) \end{array}\right)\frac{t^2}{2} \\ \left(\begin{array}{l} \frac{\beta^2\wedge_1}{2}\left(\begin{array}{l}(I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2+2(I_{r0}I_{h0}+I_{r0}A_{h0}+A_{h0}I_{h0}+\\ \delta I_{r0}T_{h0}+\delta I_{h0}T_{h0}+\delta A_{h0}T_{h0})+\frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right) \\ -\beta\left(\begin{array}{l}\wedge_1 I_{r0}(\lambda S_{r0}-(\nu+\mu_2))+\frac{\lambda\wedge_2}{2}S_{h0}I_{r0}+\wedge_1\alpha E_{h0}-\wedge_1(\gamma+\phi+\mu_1)I_{h0}-\\ \wedge_1(\eta+\phi+\mu_1)A_{h0}+\wedge_1\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0})+\frac{\beta\wedge_1\mu_1}{2}\\ \left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}+\frac{\mu_1}{\beta}\right)\end{array}\right) \end{array}\right)\frac{t^3}{3} \\ -\frac{\beta\lambda\wedge_1\wedge_2}{8}I_{r0}t^4 \end{array}\right] \quad (117)$$

Substituting equations (50), (71) and (98) into (31) gives

$$E_H(t)=\left(\begin{array}{l} E_{h0}+\beta\left(S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\frac{\alpha+\mu_1}{\beta}E_{h0}\right)t+ \\ \left(\begin{array}{l} -\beta^2 S_{h0}\left(\begin{array}{l}\left((I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2\right)+2\beta^2\left(\begin{array}{l}I_{r0}I_{h0}+I_{r0}A_{h0}+\\ A_{h0}I_{h0}+\delta I_{r0}T_{h0}+\\ \delta I_{h0}T_{h0}+\delta A_{h0}T_{h0}\end{array}\right)+\\ \frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right) \\ +\beta S_{h0}\left(\begin{array}{l}I_{r0}(\lambda S_{r0}-(\nu+\mu_1))+(\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0}-\\ (\eta+\phi+\mu_1)A_{h0}+\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0}))\end{array}\right)- \\ \beta(\alpha+\mu_1)S_{h0}\left(\begin{array}{l}I_{r0}+I_{h0}+A_{h0}\\ +\delta T_{h0}-\\ (\alpha+\mu_1)E_{h0}\end{array}\right) \\ +\beta\wedge_1\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right)\frac{t^2}{2} \\ \left(\begin{array}{l} -\frac{\beta^2\wedge_1}{2}\left(\begin{array}{l}(I_{r0})^2+(I_{h0})^2+(A_{h0})^2+\delta(T_{h0})^2+2(I_{r0}I_{h0}+I_{r0}A_{h0}+A_{h0}I_{h0}+\\ \delta I_{r0}T_{h0}+\delta I_{h0}T_{h0}+\delta A_{h0}T_{h0})+\frac{\mu_1}{\beta}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right) \\ +\beta\left(\begin{array}{l}\wedge_1 I_{r0}(\lambda S_{r0}-(\nu+\mu_2))+\frac{\lambda\wedge_2}{2}S_{h0}I_{r0}+\wedge_1\alpha E_{h0}-\wedge_1(\gamma+\phi+\mu_1)I_{h0}\\ -\wedge_1(\eta+\phi+\mu_1)A_{h0}+\wedge_1\delta(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0})-\\ \frac{\beta\wedge_1(\alpha+\mu_1)}{2}\left(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0}\right)\end{array}\right)\end{array}\right)\frac{t^3}{3} \\ +\frac{\beta\lambda\wedge_1\wedge_2}{8}I_{r0}t^4 \end{array}\right) \quad (118)$$

Substituting equations (50), (75) and (101) into (31) gives

$$A_H(t)=\left[\begin{array}{l} A_{h0}+\left((1-\rho)\alpha E_{h0}-(\eta+\phi+\mu_1)A_{h0}\right)t+ \\ \beta(1-\rho)\alpha\left(\begin{array}{l}S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\frac{\alpha+\mu_1}{\beta}E_{h0}-\\ (\eta+\phi+\mu_1)((1-\rho)\alpha E_{h0}-(\eta+\phi+\mu_1)A_{h0})\end{array}\right)\frac{t^2}{2} \\ +\frac{\beta\wedge_1(1-\rho)\alpha}{2}(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0})\frac{t^3}{3} \end{array}\right] \quad (119)$$

Substituting equations (50), (78) and (104) into (31) gives

$$I_H(t)=\left[\begin{array}{l} I_{h0}+\left(\rho\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0}\right)t+ \\ \beta\rho\alpha\left(\begin{array}{l} S_{h0}I_{r0}+S_{h0}I_{h0}+S_{h0}A_{h0}+\delta S_{h0}T_{h0}-\dfrac{\alpha+\mu_1}{\beta}E_{h0}- \\ (\gamma+\phi+\mu_1)(\rho\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0}) \end{array}\right)\dfrac{t^2}{2} \\ +\dfrac{\beta\wedge_1\rho\alpha}{2}(I_{r0}+I_{h0}+A_{h0}+\delta T_{h0})\dfrac{t^3}{3} \end{array}\right] \quad (120)$$

Substituting equations (50), (81) and (108) into (31) gives

$$T_H(t)=\left(\begin{array}{l} T_{h0}+\left(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0}\right)t+ \\ \left(\begin{array}{l} \eta((1-\rho)\alpha E_{h0}-(\eta+\phi+\mu_1)A_{h0})+\gamma(\rho\alpha E_{h0}-(\gamma+\phi+\mu_1)I_{h0})- \\ (\kappa+\phi+\mu_1)\left(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0}\right) \end{array}\right)\dfrac{t^2}{2} \end{array}\right) \quad (121)$$

Substituting equations (50), (84) and (110) into (31) gives

$$R_H(t)=R_{h0}+(\kappa T_{h0}-\mu_1 R_{h0})t+\left(\kappa\left(\eta A_{h0}+\gamma I_{h0}-(\kappa+\phi+\mu_1)T_{h0}\right)-\mu_1(\kappa T_{h0}-\mu_1 R_{h0})\right)\frac{t^2}{2} \quad (122)$$

Substituting equations (50), (87) and (113) into (31) gives

$$S_R(t)=\left(\begin{array}{l} S_{r0}+\left(\wedge_2-S_{r0}(\lambda I_{r0}+(\nu+\mu_1))\right)t+ \\ \left(\begin{array}{l} \lambda S_{r0}I_{r0}(\lambda I_{r0}-(\nu+\mu_2))-\lambda S_{r0}I_{r0}(\lambda S_{r0}-(\nu+\mu_2))+S_{r0}(\nu+\mu_2) \\ (\lambda I_{r0}+(\nu+\mu_2))-\wedge_2\left(\lambda I_{r0}+(\nu+\mu_1)\right) \end{array}\right)\dfrac{t^2}{2} \\ +\left(\begin{array}{l} \dfrac{\lambda\wedge_2}{2}I_{r0}(\lambda I_{r0}+(\nu+\mu_2))-\lambda\left(\begin{array}{l} \wedge_2 I_{r0}(\lambda S_{r0}-(\nu+\mu_2)+ \\ \dfrac{\lambda\wedge_2}{2}S_{r0}I_{r0} \end{array}\right)+ \\ \dfrac{\wedge_2}{2}(\nu+\mu_2)(\lambda I_{r0}+(\nu+\mu_2)) \end{array}\right)\dfrac{t^3}{3} \\ -\dfrac{\lambda^2(\wedge_2)^2}{8}I_{r0}t^4 \end{array}\right) \quad (123)$$

Substituting equations (50), (90) and (116) into (31) gives

$$I_R(t)=\begin{pmatrix} I_{ro}+I_{r0}(\lambda S_{r0}-(\nu+\mu_1))t \\ +\begin{pmatrix} -\lambda S_{r0}I_{r0}(\lambda I_{r0}-(\nu+\mu_2))+\lambda S_{r0}I_{r0}(\lambda S_{r0}-(\nu+\mu_2))- \\ I_{r0}(\nu+\mu_2)(\lambda S_{r0}-(\nu+\mu_2))+\frac{\lambda\wedge_2}{2}I_{r0} \end{pmatrix}\frac{t^2}{2} \\ +\begin{pmatrix} -\frac{\lambda\wedge_2}{2}I_{r0}(\lambda I_{r0}+(\nu+\mu_2))+\lambda\left(\wedge_2 I_{r0}(\lambda S_{r0}-(\nu+\mu_2))\right)+ \\ \frac{\lambda\wedge_2}{2}S_{r0}I_{r0}-(\nu+\mu_2)\frac{\lambda\wedge_2}{2}I_{r0} \end{pmatrix}\frac{t^3}{3} \\ +\frac{\lambda^2(\wedge_2)^2}{8}I_{r0}t^4 \end{pmatrix} \quad (124)$$

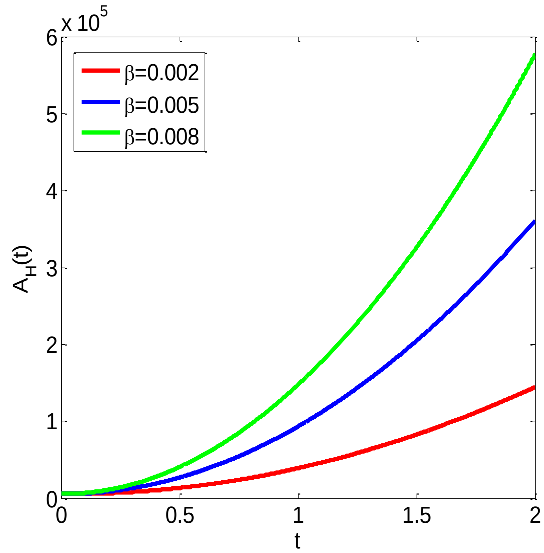
Numerical Simulations

In this section, we plot the graphs of semi-analytical solutions of our model equations using Matlab software.

Table 4.1; Initial conditions and parameter values

Parameters and State Variables	Value	Source
$S_H(0)$	10000	Assumed
$E_H(0)$	7000	Assumed
$A_H(0)$	5600	Calculated
$I_H(0)$	1400	Calculated
$T_H(0)$	6500	Assumed
$R_H(0)$	5500	Assumed
$S_R(0)$	3000	Assumed
$I_R(0)$	700	Assumed
$\wedge_1$	1200	Assumed
$\wedge_2$	400	Assumed
μ_1	0.000047	Mohammed et. al. (2015)
μ_2	0.08	Assumed
β	0.002	Assumed
λ	0.03	Assumed
δ	0.2	Assumed
α	0.05	Assumed
ρ	0.2	WHO (2017)
ϕ	0.01	WHO (2017)
η	0.5	Assumed
γ	0.8	Assumed
κ	0.8	Assumed
ν	0.02	Assumed

International Journal of Pure and Applied Science



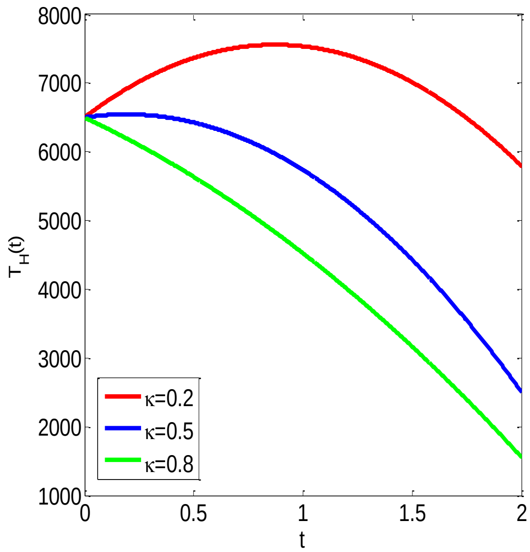
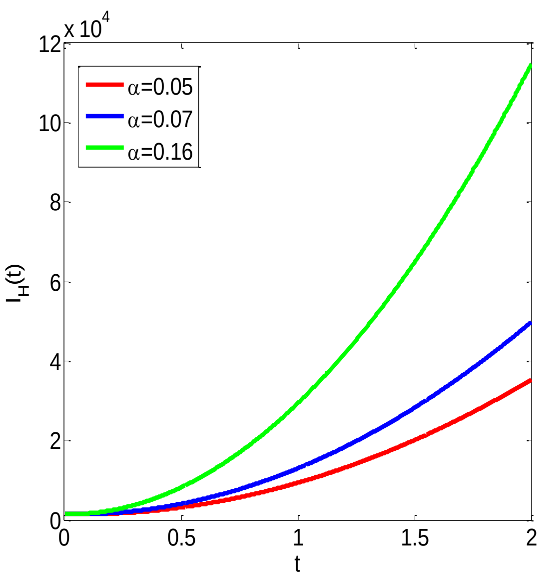


Figure 4.1: Simulation of Asymptomatic infected individuals against time for different values of Infection rate β

Figure 4.2: Simulation of treated individuals against time for different values of recovery rate κ.



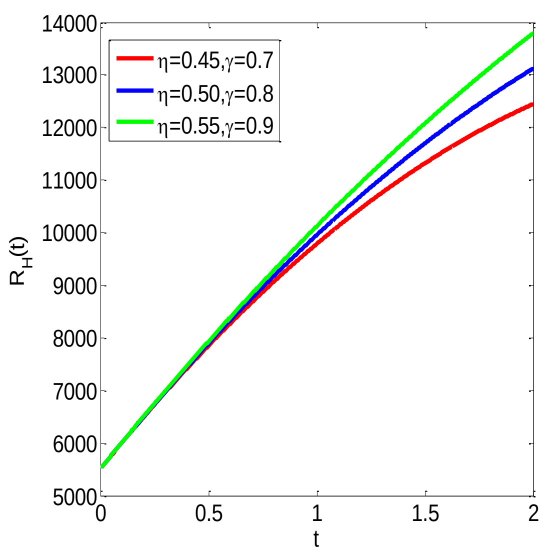


Figure 4.3: Simulation of Symptomatic infected individuals against time for different values disease-incubation rate α.

Figure 4.4: Simulation of Recovered individuals against time for different values of treatment rates of asymptomatic and symptomatic infected individuals .

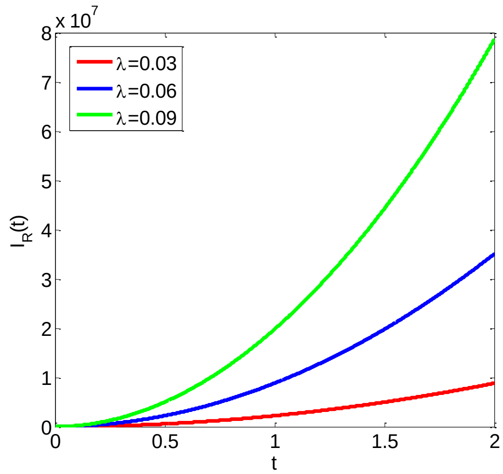


Figure 4.5: Simulation of infected reservoir against time for different values of Infection rate λ.

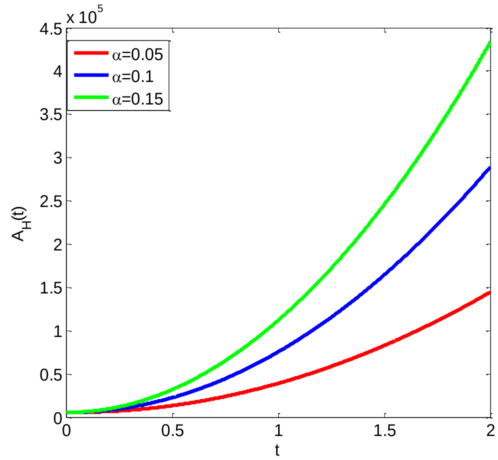


Figure 4.6: Simulation of Asymptomatic infected individuals against time for different values of incubation rate

Discussion of Results

From figure 4.1, it is observed that the number of asymptomatic individuals increases with increase in infection rate β.

From figure 4.2, it is observed that the number of treated individuals decreases as the recovery rate κ increases.

It is observed from figure 4.3 that the number of symptomatic infected individuals increases as the disease-incubation rate α increase.

From Figure 4.4, it is observed that the number of recovered individuals increases as the treatment rates η and γ of asymptomatic and symptomatic individuals increase respectively.

Figure 4.5 shows that the number of infected reservoirs increases as the infection rate parameter λ increases.

From figure 4.6, it is observed that the number of asymptomatic infected individuals increases with increase in incubation rate α.

Conclusion.

Presented a a mathematical model for the spread and treatment of lassa fever. The model equations were solved using Adomian Decomposition Method. Graphical profiles of each compartment were generated using

Matlab software. It was observed that at high treatment rate, the number of recovered individuals increases and the virus can be eradicated completely.

REFERENCES

Akanni, J. O. and Adediipo, A. D. (2018). Sensitivity Analysis of the Dynamical Transmission and Control of Lassa Fever Virus, *Asian Research Journal of Mathematics* 9(3): 1-11, doi: 10.9734/ARJOM/2018/37441.

Bawa, M., Abdulrahman, S., Jimoh, O., R., Adabara, N., U., (2013). stability analysis of the disease-free equilibrium state of lassa fever disease. *Journal of Science and Technology, Mathematics and Education (JOSTMED),* 9(2), 115-123.

Biazar, J., Babalion, Montazeri, R., (2005).A computational method for solutionof the prey and predator problem.*Applied Mathematics and Computation, 163, 841–847*

Centre for Disease Control and Prevention, Lassa fever sheet, 2013.

El-Kalla, I., L., (2008). Convergence of the Adomian method applied to a class of nonlinear integral equations. *Applied Mathematics Letters,* 21, 372–376.

Eze K.C., Salami T.A.T, Eze I.C., Pgoson A.E, OmdiaN., & Ugochukwu M. (2010). High Lassa Fever Activity in Northern Part of Nigeria: Re Analysis of Confirmatory Test Result, African Journal of Health Sciences, 16 , 52-56.

James, T.O., Abdlrahman. S., Akinyemi.S and Akinwande, N.I. (2015). Dynamics Transmission of Lassa Fever Disease.*International Journal of Innovation and Research in Educational Sciences Volume 2,* Issue 1, ISSN (Online): 2349-5219.

Mohammed B.A., Umar C.D., &Mamman M. (2015). International Conference On Mathematics, Engineering and Industrial Applications. doi:10.1063/1.4915683

Ogabi, C.O., Olusa, T.V., & Madufor, M.A. (2012*). Controlling Lassa Fever in Northernn Part of Edo State, Nigeria using SIR Model.* New Science Journal 5(12), 115-121.

Ogbu,O. E., Ajuluchukwu, C. J.,& Uneke, C.J. (2007). *Lassa fever in West Africa sub-region:an Overview*. Journal of Vector Borne Diseases 44, 1-11.

Omale, D., & Edibo T., E., (2014). Mathematical modeels of Lassa Fever transmission with control strategies, computing, information systems. *Development Mathematics & Allied Research Journal.6(4),25-32.*

Onuorah Martins .O., Ojo Moses .S.,Usman Dahiru . J.,Ademu Abdulkadir (2016). Basic Reproductive Number for the Spread and Control of Lassa fever.*International Journal of Mathematics Trends and Technology (IJMTT) – Volume30, 1-16.*

Onuorah, M. O., Akinwande, N. I., Nasir, M. O. and Ojo, M. S. (2016) Sensitivity Analysis of Lassa FeverModel. *International Journal of Mathematics and Statistics Studies*, Vol. 4, Number 1, 30-49.

Okuonghae, D. & Okuonghae, R. (2006). *A Mathematical model for Lassa fever.* Journal of the Nigerian Association of Mathematical Physics, 10,457-464.

Omilabu S.A, Badaru S.O, Okokhere P, Asogun D, Drosten C and Emmerich .P (2005). Lassa Fever, Nigeria 2003 and 2004. Emerg Infect Dis. 11:1642-4.

Sulaiman Usman and Ibrahim Isa Adamu (2018). Modelling the Transmission Dynamics of the Lassa Fever Infection, *Mathematical Theory and Modeling* Vol.8, No.5, 2018, ISSN 2224-5804 (Paper)

World Health Organization, WHO Lassa Fever Sheet No 179. Geneva: WHO, 2000. World Health Organization, WHO. (2017). Lassa Fever. *WHO Factsheets*. Retrieved fromhttp://www.who.int/mediacentre/factsheets/fs179/en/