Numerical analysis treatment of the investment option of predation as a type of interaction between two stock exchange investors

N. M. Nafo
Department of Mathematics, Rivers State University, Port Harcourt, Rivers State, Nigeria.

Abstract
The stock exchange investors with a relatively higher initial investment tend to observe consistent growth of their dividends than the investors with a lower investment scenario. In terms of the quantified increase in the dividends, the dividends of the investors with a lower initial investment (0.2, 0.2) in the units of millions of naira do better than the dividends of the investors with the initial investment values (0.3, 0.3) and (1.2, 1.4) under the simplifying assumption of the predation interaction between two stock exchange investors. We utilized the ODE 45 numerical method to investigate this daunting investment problem. The novel results that we have obtained have not been seen elsewhere; these are presented and discussed in this present pioneering numerical simulation study.

Keywords: *Stock exchange, investment, dividends, predation, numerical method*

Introduction
In our earlier research contribution, we have found that the bigger the initial investment into the stock exchange business the better the expected dividends for the chosen length of the trading period in the unit of months when there is a predation between two stock exchange investors. This key observation is associated with a linear prediction numerical analysis idea which may be counter-intuitive in terms of an effective stock exchange business planning. For example, we have also found in our earlier analysis that the initial investment (0.2, 02) is associated with a small local minimum error which can suggest a lower risk option whereas the initial investment (1.2, 1.4) with a relatively bigger p-norms error does suggest a higher risk option. These two numerical analysis-stock exchange ideas do not implicitly infer that the higher initial investment will always have a better dividend than the smaller initial investment. While this conclusion may be feasible in a deterministic stock exchange simulation modelling, however it can change since the stock exchange model specification is inherently more stochastic and can predict more realistic dividends

than the deterministic predation interaction between two stock exchange investors. In this present study, we would consider a predation that is vulnerable to a relatively random noise intensity value of 0.2 and how this random economic recession can affect the distribution of the dividends.

Model Equations

To investigate our proposed problem, we have used a system of continuous non-linear first order ordinary differential equations of the Lotka-Volterra type (Cushing (1977), Tang and Zhang (2005), Cajueior et al (2009), Chiang et al (2008), Khodabin and Shekarabi (2016), Lee et al (2005), Modis (1999), Shiller (1981), Lakka et al (2013), Sikder and Roy (1994), May (1982), Tsai (2015), Moris and Pratt (2003), Verhulst (1990), He and Gopulsamy (1997), Nafo (2016)). This system of equations, for the predation type of interaction, is given by

$$\frac{dw_1(t)}{dt} = w_1(t)(\alpha_1 - \beta_1 w_1(t) + \gamma_1 w_2(t)$$

$$\frac{dw_2(t)}{dt} = w_2(t)(\alpha_2 - \beta_2 w_2(t) - \gamma_2 w_1(t)$$

$$w_1(0) > 0 \ , \ w_2(0) > 0$$

where:

$w_1(t)$ is the dividend of the first of investor at time t in the unit of months

$w_2(t)$ is the dividend of the second of investor at time t in the unit of months.

α_1 is the intrinsic growth rate of the dividend of the first investor

α_2 is the intrinsic growth rate of the dividend of the second investor

β_1 is the intra-competition coefficient which is the inhibiting factor on the dividend of the first investor due to its interaction with itself.

β_2 is the intra-competition coefficient which is the inhibiting factor on the growth of the dividend of the second investor due to its interaction with itself.

γ_1 is the inter-competition coefficient which is the inhibiting factor on the growth of the dividend of the first investor due to the interaction of the second investor.

γ_2 is another inter-competition coefficient which is the inhibiting factor on the growth of the dividend of the second investor to the interaction of the first investor.

$w_1(0)$ and $w_2(0)$ are the initial dividends of the first and second investor respectively.

Method of Analysis

The model equations we have used here do not have closed form solutions. We have, therefore, applied a numerical simulation method to analyze the investment option of predation as a type of interaction between two stock exchange investors, using the

starting investment values (0.2, 0.2), (0.3, 0.3) and (1.2, 1.4) in millions of naira in some specified trading periods in months. The parameter values used are $\alpha_1 = 0.037$, $\alpha_2 = 0.03$, $\beta_1 = 0.0014$, $\beta_2 = 0.001$, $\gamma_1 = 0.0012$, $\gamma_2 = 0.0009$. Our results are presented and discussed below.

Results

Table 1.1: Calculating the dividends for two investors in a predation interaction $\gamma_1 = 0.001212$, initial investment (1.2, 1.4), random economic recession value of 0.2

Dividends (1) [old]	Dividends (1) [new]	Effect 1 (%)	Dividends (2) [old]	Dividends (2) [news]	Effect 2 (%)
1.2000	1.2000	0.00	1.4000	1.4000	0.00
4.9221	12.8780	161.64	3.7384	8.6175	130.52
5.8405	15.3359	162.58	4.1568	9.3785	125.62
6.9002	18.2473	164.45	4.5918	10.2738	123.74
8.1090	20.9451	158.29	5.0354	10.7508	113.50
9.4696	23.5833	149.04	5.4779	11.4126	108.34
10.9767	26.1030	137.80	5.9079	11.5143	94.90
12.6158	28.2738	124.12	6.3135	11.6429	84.41

Table 1.2: Calculating the dividends for two investors in a predation interaction $\gamma_1 = 0.001212$, initial investment (0.2, 0.2), random economic recession value of 0.2

Dividends (1) [old]	Dividends (1) [new]	Effect 1 (%)	Dividends (2) [old]	Dividends (2) [news]	Effect 2 (%)
0.2000	0.2000	0.00	0.2000	0.2000	0.00
0.8409	8.9799	967.92	0.6252	6.5356	945.39
1.0093	11.0305	992.89	0.7209	7.5589	948.51
1.2106	13.1896	989.47	0.8302	8.4672	919.89
1.4509	15.6764	980.43	0.9546	9.1896	862.70
1.7371	18.1162	942.90	1.0955	10.0613	818.41
2.0770	20.6968	896.45	1.2546	10.7098	753.66
2.4796	23.5263	848.80	1.4331	11.2059	681.93

Table 1.3: Calculating the dividends for two investors in a predation interaction $\gamma_1 = 0.001212$, initial investment (0.3, 0.3), random economic recession value of 0.2

Dividends (1) [old]	Dividends (1) [new]	Effect 1 (%)	Dividends (2) [old]	Dividends (2) [news]	Effect 2 (%)
0.3000	0.3000	0.00	0.3000	0.3000	0.00
1.2568	9.7019	671.95	0.9239	6.3876	591.40

1.5066	11.7253	6778.25	1.0615	7.3211	589.72
1.8042	14.1447	683.97	1.2172	8.3418	585.34
2.1579	16.6797	672.96	1.3925	9.1062	553.94
2.5768	19.3036	649.14	1.5889	9.5672	502.12
3.0710	21.9792	615.71	1.8074	9.9689	451.56
3.6513	24.3557	567.05	2.0486	10.4593	410.55

Discussion of Results

In the scenario of the initial investment (1.2, 1.4), in the absence of a zero random noise intensity (that is when the economic environment is relatively not vulnerable to a severe economic recession), the dividends of the first investor range from the deterministic value of 1.2 million naira to 12.62 million naira monotonically increasing whereas the dividends of the second investor range from 1.4 million naira to 6.3 million naira monotonically increasing. With the inclusion of a random economic recession value of 0.2, the dividends of the first investor range from 1.2 million naira to 28.3 million naira whereas the dividends of the second investor range from 1.4 million naira to 11.64 million naira.

When we considered the initial investment (0.2, 0.2), in the absence of a zero random noise intensity (that is when the economic environment is relatively not vulnerable to a severe economic recession), the dividends of the first investor range from the deterministic value of 0.2 million naira to 2.48 million naira monotonically increasing whereas the dividends of the second investor range from 0.2 million naira to 1.43 million naira monotonically increasing. With the inclusion of a random economic recession value of 0.2, the dividends of the first investor range from 0.2 million naira to 23.53 million naira whereas the dividends of the second investor range from 0.2 million naira to 11.21 million naira.

In the scenario of the initial investment (0.3, 0.3), in the absence of a zero random noise intensity (that is when the economic environment is relatively not vulnerable to a severe economic recession), the dividends of the first investor range from the deterministic value of 0.3 million naira to 3.65 million naira monotonically whereas the dividends of the second investor range from 0.3 million naira to 2.05 million naira monotonically. With the inclusion of a random economic recession value of 0.2, the dividends of the second investor range from 0.3 million naira to 10.46 million naira.

In summary, in the absence of a zero random economic recession and in the presence of a random economic recession, the stock exchange investors having the initial investment value of (1.2, 1.4) is more associated with a higher growth of their dividends than the initial investment scenarios of (0.2, 0.2) and (0.3, 0.3). However, in terms of the continuous growth of their dividends, the consistent increase in the dividends of the investors is favoured by the initial investment of (0.2, 0.2), (0.3, 0.3)

and (1.2, 1.4). Therefore, in a harsh economic recession, it is the small initial investment (the least expected in terms of general common knowledge) that is associated with a relatively improved benefit in their dividends.

Conclusion

In a harsh economic recession, stock exchange investment and its benefits evolve over the trading period. However, the investors with the smaller initial investment tend to benefit higher than the other initial investment scenarios provided the inter-competition coefficient of the first investor is 0.001212 and the inter-competition coefficient of the second investor is 0.001 when the random economic recession value of 0.2 is considered. This same numerical analysis idea can be extended to other types of interaction such as competition, commensalism, and mutualism. Similarly, in a future investigation, we can consider some variations of higher random economic recession values which we did not analyze in the present study.

Recommendation

From our results, it is recommended that in harsh economic recession, for a predation type of interaction between stock market investors, a low initial investment should be encouraged for greater and sustained benefit.

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