LEVELING NETWORK ADJUSTMENT USING MATLAB

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ABSTRACT
We currently live in what is often termed the technological age. Aided by new and emerging technologies, data are being collected at unprecedented rates in all walks of life. For example, in the field of surveying, digital level instrument, total station instruments, global positioning system (GPS) equipment, Geographic Information System (GIS), digital metric cameras, and satellite imagery systems are only some of the new instruments that are now available for rapid generation of vast quantities of measured data. . In this research the leveling network consist of six stations was analyzed, using weighted and unweighted observation equations of LSE. The aim of this analysis is to find out, if leveling network can be realized and the reliability of the methods via MATLAB. The results obtained from these methods are compared. The results showed that an accuracy of sub-centimeter level could be obtained from the two methods. Conclusion was drawn concerning the suitability of the methods for leveling network adjustment via MATLAB program.

Keywords: *Leveling Network, LSE, MATLAB, Weighted, Unweighted.*

INTRODUCTION
Leveling is the general term applied to any of the various processes by which elevations of points or differences in elevation are determined. it is a vital operation in producing necessary data for mapping, engineering design, and construction. Leveling results are use to (1) design highways, railroads, canals, sewers, water supply systems, and other facilities having grade lines that best conform to existing topography; (2) lay out construction projects according to planned elevations; (3) calculate volume of earthwork and other materials; (4) investigate drainage characteristics of an area; (5) develop maps showing general ground configurations; and (6) study earth subsidence and crustal motion.(Paul R. Wolf and Charles D. Ghilani, 2006.)
Precision in leveling is increased by repeating measurements, making frequent ties to established bench marks, using high-quality equipment, keeping it in good adjustment, and performing the measurements carefully. Since permissible misclosures are based

on the lengths of lines leveled, or number of setups, it is logical to adjust elevations in proportion to this value.

Levelling is the identification of points on the earth surface in their vertical distance relationship. The information on the relative positions of the various points on the surface of the earth in their vertical distance relationship (that leveling determines) is prerequisite to the successful planning and execution of any engineering project, such as, construction of roads, railways, bridges, canals, dams, sewerage and drainage works, water distribution networks or any other construction on the surface of the earth, and also for mining and also for mining and metallurgical activities e.t.c. (Alak De 2007.)

In surveying, observations must often satisfy established numerical relationships known as geometric constraints. As examples, in closed polygon traverse, horizontal angle and distance measurements should conform to the geometric constraints, and in a differential leveling loop, the elevation differences should sum to a given quantity. However, the geometric constraints meet perfectly rarely, if ever, the data are adjusted. Adjustment of observations is a model for the solution of an overdetermined system of equations based on the principles of least squares. It is used extensively in the disciplines of surveying, geodesy, photogrammetry (the field of geomatics, collectively).

Since the nineteenth century, applied scientists have adopted the principle of Least Squares to select the best estimates of parameters from a series of observations. In fact when we accept the arithmetic mean of a number of observations as the best estimate of an observed parameter, we are making use of this principle. Most of the theory and procedures developed by Legendre and Gauss have been applied for over a century, but it has not been until the development of computers that Least Squares has become commonplace for every applications.

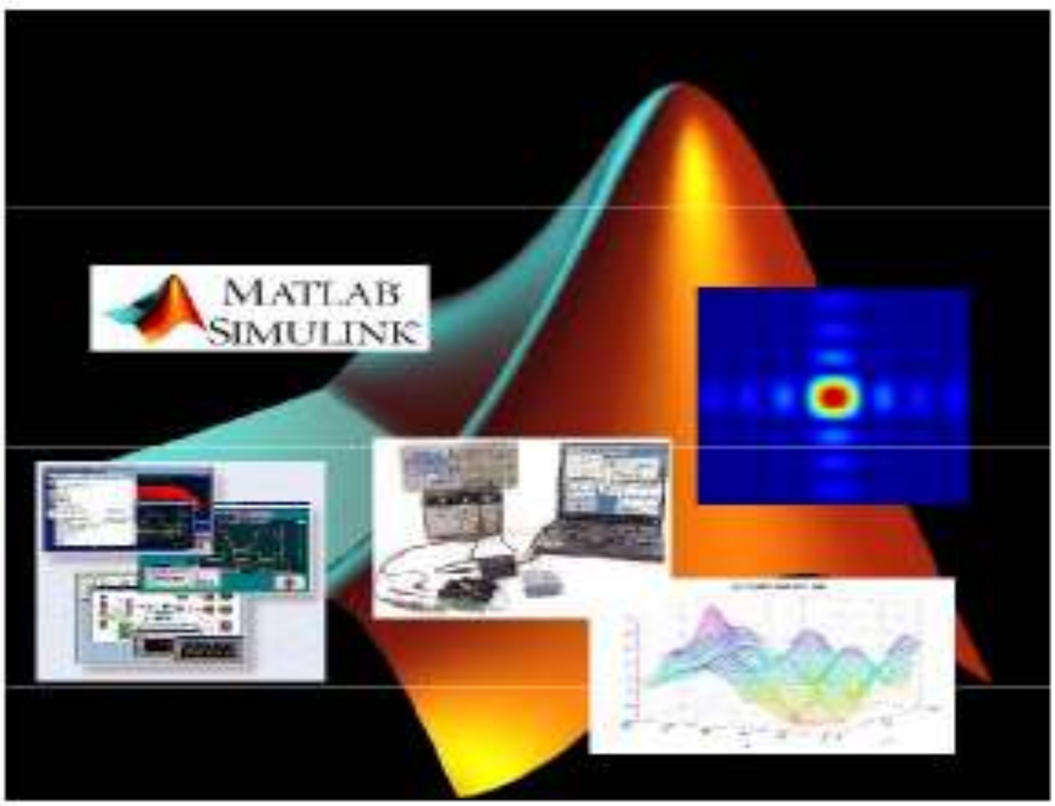
The principle provides a method of making the best use of observations when more than the minimum number has been observed, and also a way of combining different types of observations to yield best estimates of parameters. It should be noted that the application of Least Squares does not imply any statistical distribution of observed variables. However, when an attempt is made to infer quality from them and derive results, it is usual to assume that a Normal distribution is involved. Most of the procedures used to analyse data can also be used without data as a pre-analysis tool when designing control networks, before making any observations, thus saving much time and money.

There are three forms of adjustments of observations: parametric, conditional, and combined. In parametric adjustment, one can find an observation equation $h(X)=Y$ relating observations Y explicitly in terms of parameter X. in conditional adjustment, there exists a condition equation $g(Y)=0$ involving only observations Y with no parameters X at all. Finally, in the combined adjustment, parameters X and observations Y are involved implicitly in a mixed model equation f(X, Y) =0. Clearly, parametric and conditional adjustments corresponds to the more general combined case when f(X, Y) =h(X)-Y and f(X, Y) =g(Y) respectively.

For decades almost every community or city around the globe has relied on two-dimensional (E, N) maps to represent their living environment. These maps have help a lot in solving host of problems such as; performing analysis on flight and communication signals, landslide analysis, calculating cut and fill volumes in earth works, and finding height and volume of features which relies on the accuracy of the vertical control (elevation) were found impossible to be solved using 2D maps. The aforementioned problems will not be solved in the absence of LSE of leveling network.

From the above overview on leveling network adjustment, it is clear that leveling network has to do with points and space geometry. The points and space geometry which are basic fundamental for map can crop up though the process of least square adjustment using MATLAB.

MATLAB



MATLAB is a high-level technical computing language and interactive environment for: Algorithm development, Data visualization, Data analysis, Numerical computation. Using MATLAB, we can solve- technical computing- problems faster than with traditional programming languages, such as C, C++, and Fortran. MATLAB contains hundreds of commands to do mathematics. It can be used to: Graph functions, Solve equations, Perform statistical tests, and much more…. It is high-level programming language that can communicate with other most widely used languages such as Fotran and C. Can prepare materials for export to the World Wide Web. We can do simulations and modeling (especially if you have access not just to basic MATLAB but also to its accessory Simulink@). We can use MATLAB to combine mathematical computations with text and graphics in order to produce a polished, integrated, interactive document. MATLAB is more than a fancy calculator; it is an extremely useful and versatile tool. MATLAB prints the answer and assigns the value to a variable called ans. If you want to perform further calculation with the answer, we can use the variable ans rather than retype the answer. MATLAB uses double precision floating point arithmetic, which is accurate to approximately 15 digits; however, MATLAB displays only 5 digits by default. MATLAB was written to allow mathematicians, scientists, and engineers to handle the tools of linear algebra- that is, vectors and matrices- as effortlessly as possible.

CONCEPT OF WEIGHTS IN MEASUREMENTS

Weight is the worth or reliability of one measurement relative to a standard or another measurement

Measurements with higher precision (smaller std. deviation) should be assigned higher weights

Measurements with higher weights should receive smaller corrections after an adjustment

Weights are relative, and therefore, determined by comparing with another measurement

Weights are inversely proportional to some precision index of measurements

Weight of an uncorrelated measurement is inversely proportional to square of std. deviation (variance)

Since weights are relative, any error estimate, similar to std. deviation, could be used, e.g. E95

Weight of a mean value computed from repeated measurements is proportional to the number of repetitions

Weight assigned to an elevation difference, determined by differential leveling, is inversely proportional to the number of setups or the length of the level line.

METHODOLOGY

The leveling network consists of six stations (A, B, C, D, E, and F). Station A has a known height of 434.576m. There are ten lines of levels connecting the six stations. Since only five such lines of levels would be sufficient to give relative heights between the points, five lines are extra to requirements. Therefore there is five degree of freedom in the problem.

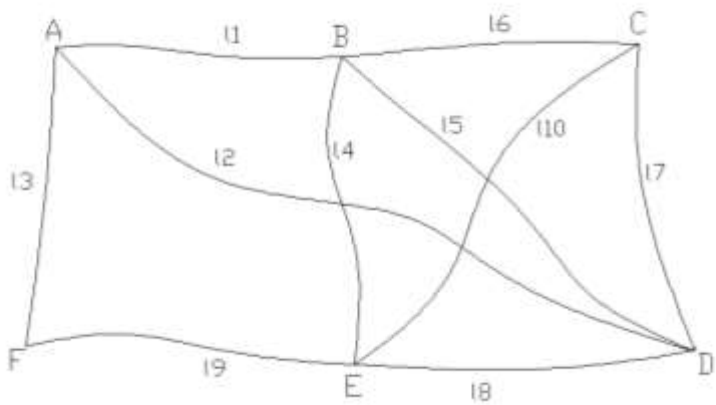


Figure 1: Leveling Network

Since the height of A is fixed. We know the heights differences in level, and the respective distances. The standard deviation of the measurements (unit m) is $0.007\sqrt{d}$, where d is the distance in kilometers, leaving the five parameters h_B, h_C, h_D, h_E and h_F to be estimated.

A=434.576m, it's a fix station

Then, Observation equation was formed $V=AX-L^b$.

$v_1=h_B-h_A-l_1;$ $v_6=h_C-h_B-l_6;$

$v_2=h_D-h_A-l_2;$ $v_7=h_D-h_C-l_7;$

$v_3=h_F-h_A-l_3;$ $v_8=h_E-h_D-l_8;$

$v_4=h_E-h_B-l_4;$ $v_9=h_F-h_E-l_9;$

$v_5=h_D-h_B-l_5;$ $v_{10}=h_C-h_E-l_{10};$

Where: A is the matrix of coefficients for the unknowns, X the matrix of unknowns, L the matrix of the observations, and V the matrix of the residuals. The detailed structures of these matrices are:

v_1	1	0	0	0	0	213.980
v_2	0	0	1	0	0	227.055

v_3		0	0	0	0	1		h_B		228.310
v_4		-1	0	0	1	0		h_C		065.544
v_5	=	-1	0	1	0	0	*	h_D	-	013.008
v_6		-1	1	0	0	0		h_E		162.577
v_7		0	-1	1	0	0		h_F		-147.510
v_8		0	0	-1	1	0				052.543
v_9		0	0	0	-1	1				-051.265
v_{10}		0	1	0	-1	0				097.011

$${}_{10}V_1 = {}_{10}A_5 \; * \; {}_5X_1 - {}_{10}L_1^b$$

Then;

Normal equations that results from a set of weighted and unweighted observation equations were formed as; $A^TPAX=A^TPL^b$ or NX=U, and $A^TAX=A^TL^b$ or NX=U respectively.

Weight (P) was computed weights of differential leveling lines are inversely proportional to their lengths, and since any course length is proportional to its number of setups, weights are also inversely proportional to the number of setups.

The parameter (X) and the Residual (V) were computed as $X=N^{-1}U$, and $V=AX-L^b$. Then, the Unit variance ($б^2_0$) = $V^TPV/n-u$ and Adjusted observations (L^a) $=L^b+V$ were computed.

Variance for parameters: diagonal elements of Σ_x, were Computed as $\Sigma_x=N^{-1}$. Standard deviations of parameters: square root of variances.

Summary of adjusted output observations (weighted)

A Matrix					Observation Matrix (L)	Weight Matrix									
A =					L =	P = 1.0e-003 *									
1	0	0	0	0	213.9800	-0.5390	0	0	0	0	0	0	0	0	0
0	0	1	0	0	227.0550	0	-0.8918	0	0	0	0	0	0	0	0
0	0	0	0	1	228.3100	0	0	-0.3185	0	0	0	0	0	0	0
-1	0	0	1	0	65.5440	0	0	0	-0.2499	0	0	0	0	0	0
-1	0	1	0	0	13.0080	0	0	0	0	-0.4459	0	0	0	0	0
-1	1	0	0	0	162.5770	0	0	0	0	0	-0.3871	0	0	0	0
0	-1	1	0	0	-149.5100	0	0	0	0	0	0	-0.1960	0	0	0
0	0	-1	1	0	52.5430	0	0	0	0	0	0	0	-0.2695	0	0
0	0	0	-1	1	-51.2650	0	0	0	0	0	0	0	0	-0.5733	0
0	1	0	-1	0	97.0110	0	0	0	0	0	0	0	0	0	-0.3234

Unknowns Matrix(X)

X =

214.0055
376.5744
227.0412
279.5677
228.3053

Adjusted Standard Deviation Matrix (SD) Adjusted Observation Matrix (adj L) Adjusted Height difference (m)

SD =	adjL =	
0.0151	214.0055	AB = -220.5705
0.0206	227.0412	AD = -207.5348
0.0134	228.3053	AF = -206.2707
0.0185	65.5622	BE = 65.5622
0.0201	13.0357	BD = 13.0357
	162.5689	BC = 162.5689
	-149.5332	CD = -149.5332
	52.5265	DE = 52.5265
	-51.2624	EF = -51.2624
	97.0067	EC = 97.0067

Summary of adjusted output observations (without weighted)

A Matrix A =	Observation Matrix (L) L =	Weight Matrix P = I	Unknowns Matrix(X) X =
1 0 0 0 0	213.9800		214.0045
0 0 1 0 0	227.0550		376.5674
0 0 0 0 1	228.3100		227.0363
-1 0 0 1 0	65.5440		279.5633
-1 0 1 0 0	13.0080		228.3042
-1 1 0 0 0	162.5770		
0 -1 1 0 0	-149.5100		
0 0 -1 1 0	52.5430		
0 0 0 -1 1	-51.2650		
0 1 0 -1 0	97.0110		

Adjusted Standard Deviation Matrix (SD) Adjusted Observation Matrix (adj L) Adjusted Height difference (m)

SD =	adjL =	
0.0168	214.0045	AB = -220.5715
0.0202	227.0363	AD = -207.5397

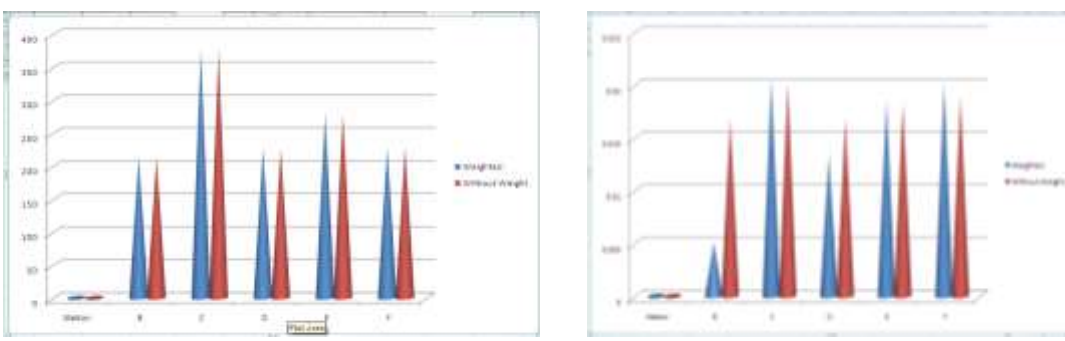
0.0168	228.3042	AF = -206.2718
0.0183	65.5588	BE = 65.5588
0.0189	13.0318	BD = 13.0318
	162.5629	BC = 162.5629
	-149.5311	CD = -149.5311
	52.5270	DE = 52.5270
	-51.2592	EF = -51.2592
	97.0040	EC =97.00400

RESULTS AND DISCUSSION

The differences between the heights of the points and standard deviations computed using the two different methods is shown in the above table. The minimum and maximum differences between the adjusted heights and standard deviation of the weighted and unweighted observation are 0.001 and 0.007, and 0.0002 and 0.0034 respectively. See Table 1 below. It is shown in graphical form in figure 2 below.

Table 1: Height and Standard Deviation of Adjusted Results

	Weighted		Without Weight		Differences	
Station	Height (m)	Std. Dev	Height (m)	Std. Dev	Height (m)	Std. Dev
B	214.0055	0.0051	214.0045	0.0168	0.001	0.0017
C	376.5744	0.0206	376.5674	0.0202	0.007	0.0004
D	227.0412	0.0134	227.0363	0.0168	0.0049	0.0034
E	279.5677	0.0185	279.5633	0.0183	0.0044	0.0002
F	228.3053	0.0201	228.3042	0.0189	0.0011	0.0012



Heights   Standard Deviation

Figure 2: Weighted and Unweighted Adjusted Results

The differences between the Adj L are; 0.0010, 0.0049, 0.0011, 0.0034, 0.0039, 0.0060, 0.0021, 0.0005, 0.0032 and 0.0027 for AB, AD, AF, BE, BD, BC, CD, DE, EF and EC respectively. Adjusted L and their differences between the two observations are shown in Table 2 below. It is shown in graphical form in Figure 3 below. The minimum and maximum differences are 0.001 and 0.006 respectively.

Table 2: Adjusted L and Differences

side	Weighted	Without weight	Diff
l1	214.0055	214.0045	0.001
l2	227.0412	227.0363	0.0049
l3	228.3053	228.3042	0.0011
l4	65.5622	65.5588	0.0034
l5	13.0357	13.0318	0.0039
l6	162.5689	162.5629	0.006
l7	-149.5332	-149.5311	0.0021
l8	52.5265	52.527	0.0005
l9	-51.2624	-51.2592	0.0032
l10	97.0067	97.004	0.0027

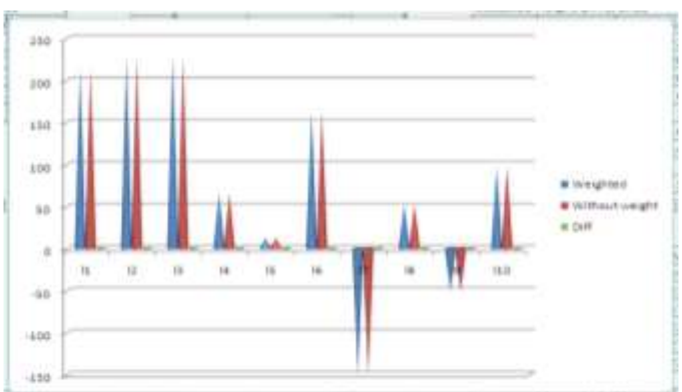


Figure 3: Adjusted L

It also reveals that the difference between the adjusted height differences between the two methods are; 0. 0010, 0.0049, 0.0011, 0.0034, 0.0039, 0.0060, 0.0021, 0.0005, 0.0032, 0.0027 for the respective lines, see Table 3 below. It is shown in graphical form in Figure 4 below. The minimum and maximum adjusted differences are 0.001 and 0.006 respectively.

Station	Weighted	Without weight	Diff.
AB	-220.5705	-220.5715	0.001
AD	-207.5348	-207.5397	0.0049

AF	-206.2707	-206.2718	0.0011
BE	65.5622	65.5588	0.0034
BD	13.0357	13.0318	0.0039
BC	162.5689	162.5629	0.006
CD	-149.5332	-149.5311	0.0021
DE	52.5265	52.527	0.0005
EF	-51.2624	-51.2592	0.0032
EC	97.0067	97.004	0.0027

Table 3: Difference between the adjusted Heights

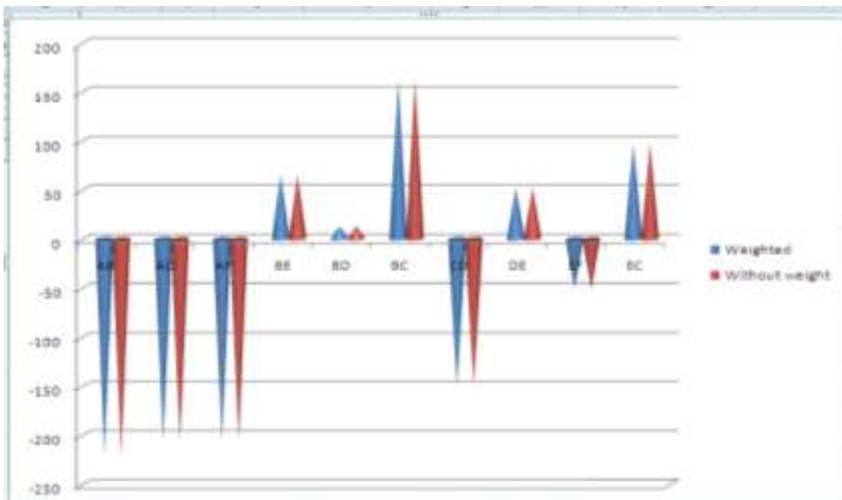


Figure 4: Difference between the Adjusted Heights

CONCLUSION

This study focuses on the applications of MATLAB, a powerful and popular windows based commercial LSE software, for 1D, 2D and 3D network adjustment. MATLAB provides special setting for generating the M-files, which contain the necessary information (especially variance covariance matrix) for leveling network adjustment purposes. The results computed heights, the adjusted L and adjusted height difference from the two methods are not exactly the same; it is in sub=centimeter level of accuracy, and it didn't show much significant difference. It is hoped that surveyors might be beneficial from MATLAB for leveling network adjustment in surveying applications.

RECOMMENDATION

Even though the method is fast, it required deep thinking, calm and devotion. One good thing about software is that if there is a problem in running your program it will alert to the particular line where you need to debug and re-run over and over, until all conditions are satisfied. Constant/regular use of the program makes it easier to be used.

The hard part, however, is figuring out which of the; hundreds of commands, scores of help pages, and thousands of items of documentation we need to look at to start using it quickly and effectively.
As a way of improving beyond the scope of this study, I would like to recommend that future research be done in this area "leveling network adjustment" based on two or more software. So that comparism can be made to determine which one is better for a preference to be made.
This research was carried out using six stations, for more study and understanding, I do recommend that future research to be carried out in the same manner, and however, it should be expanded further by using ten or more stations.

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