

NUMERICAL ANALYSIS OF BIODIVERSITY OF TWO COMPETING PLANT SPECIES DUE TO CHANGING PLANT DISEASES CONDITION

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ABSTRACT

***T**his paper has discussed the numerical analysis of biodiversity gain of two interacting plant species with changing plant diseases conditions in a harsh climate like the Arctic. MATLAB ODE 45 was used to quantify the effect of plant diseases conditions on the two interacting plant species. It was observed that as the plant disease condition was decreased, the crop yields increased while as the plant disease condition increased, the crop yields decreased.*

Introduction

The two intrinsic growth rate increase the growth of the two plant species on the simplified assumption that the birth rate outweighs the death rate. This key hypothesis support the mathematical ideology that the two plant species will grow positively exponentially over time.

Introduction:

A plant becomes diseased when it is continuously disturbed by agents that produce abnormal physiological process that disrupts the plant's normal structure, growth and function (Encyclopedia Britannica (2002)). Plant disease is a state in which the normal state and nature of a plant is disrupted. The effect of plant disease on plant growth can be visible according to the percentage effect of the diseases. An increased in percentage effect of the diseases decrease the crop yield while decrease in percentage effect of the disease increases the crop yield.

This standard mathematical result is a tractable mathematical principle but does not offer a meaningful biological implication in the sense that plant species does not grow indefinitely over time because there is an harvesting time and other factors such as the intra competition coefficient, inter competition coefficient and the plant disease factors which act to inhibit the growth of the two plant species

The increased growth rate of plant production enhances food security. The plant species can be affected by intrinsic growth rate, intra competition coefficient, inter competition coefficient, initial condition and plant diseases. Plant disease is one of the major factors that inhibits growth rate which in turn affect crop production.

A severe decrease in the plant disease condition has the potential to predict a bigger volume of biodiversity gain while a severe increase in the plant disease condition has the potential to predict a bigger volume of biodiversity loss. Biodiversity loss influences ecosystem functioning and its impact on goods and services. Biodiversity loss decreases the stability of ecosystem function through time and the impact of biodiversity is nonlinear and saturating in any single ecosystem.

Mathematical Formulations

Following Ekaka a (2009), we have considered the following system of continuous nonlinear first order differential equation.

$$\frac{dx(t)}{dt} = \alpha_1 x - \beta_1 x^2 - r_1 xy - k_1 x^2 y \quad (1)$$

$$\frac{dy(t)}{dt} = \alpha_2 y - \beta_2 y^2 - r_2 xy - k_2 xy^2 \quad (2)$$

With the initial condition $x(0) = x_0 > 0$ and $y(0) = y_0 > 0$.

Following Ekaka-a (2009), we have considered the following parameters values;

$\alpha_1 = 0.168$, $\alpha_2 = 0.002$, $\beta_1 = 0.0020339$, $\beta_2 = 0.000015$, $r_1 = 0.0005$, $r_2 = 0.00002$, with the assumption that the plant disease are $k_1 = k_2 = 0.1$.

For the purpose of clarity, the variables for this model equations are defined as follows:

$x(t)$ represents plant species 1 biomass at time t in the unit of months

$y(t)$ represents plant species 2 biomass at time t in the unit of months

t represents time in the unit of months

α_1 represents the intrinsic growth rate of the plant species 1 biomass

α_2 represents the intrinsic growth rate of the plant species 2 biomass

β_1 represents the intra competition coefficient due to the interaction of the population x to inhibit the growth of x plant species biomass

β_2 represents the intra competition coefficient due to the interaction of the population y to inhibit the growth of y plant species biomass

r_1 represents the inter competition coefficient due to the interaction of the population y to inhibit the growth of the x population.

r_2 represents the inter competition coefficient due to the interaction of the population x to inhibit the growth of the y population

k_1 represents plant diseases affecting plant species 1 biomass

k_2 represents plant diseases affecting plant species 2 biomass

Notations

The key method of analyzing the proposed problem is defined as follows:

PB1 (old) column represents the data values of the first plant species biomass when all the model parameters values are fixed indexed by the unit of time

PB1 (new) column represents the data values of the first plant species biomass when parameter values of k_1 changes and all other model parameter values fixed.

PB2 (old) column represents the data values of the second plant species biomass when all the model parameter values are fixed.

PB2 (new) column represents the data values of the second plant species biomass where all the model parameters are fixed except k_2

g/A represents grams per area of plant cover.

EBG(%) represents the estimated biodiversity gain in percentage for the first plant species biomass and the second plant species biomass due to the effect of the plant disease scenario.

When the new biomass of the first plant species and the new biomass of the second plant species with the satisfying length of the growing season outweighs, then a biodiversity gain has occurred.

Method of Analysis

The key method of analyzing the proposed problem in these work is defined as:

$$EBG(\%) = \left[\frac{PB1(\text{new}) - PB1(\text{old})}{PB1(\text{old})} \right] (100)$$

Results

Table 1: Calculating the effect of equal plant disease condition on the yields of two plant species and its implication on biodiversity: $k_1 = k_2 = 0.005$

PB1(old) g/A	PB1(new) g/A	EBG1(%)	PB2(old) g/A	PB2(new) g/A	EBG2(%)
1.2000	1.2000	0.00	1.4000	1.4000	0.00
1.2110	1.4020	15.77	1.2004	1.3901	15.81
1.2447	1.6350	31.36	1.0482	1.3783	31.49
1.2955	1.9027	46.87	0.9270	1.3643	47.18
1.3602	2.2089	62.40	0.8270	1.3481	63.02
1.4375	2.5576	77.92	0.7427	1.3293	79.00
1.5267	2.9526	93.40	0.6703	1.3080	95.13
1.6275	3.3975	108.76	0.6074	1.2838	111.37
1.7401	3.8956	123.88	0.5521	1.2569	127.65
1.8647	4.4499	138.64	0.5031	1.2271	143.91
2.0018	5.0624	152.89	0.4594	1.1945	160.03
2.1521	5.7348	166.47	0.4202	1.1594	175.94
2.3163	6.4677	179.23	0.3848	1.1219	191.51
2.4950	7.2611	191.02	0.3529	1.0823	206.67
2.6893	8.1141	201.72	0.3239	1.0409	221.33
2.9001	9.0252	211.21	0.2976	0.9983	235.42
3.1284	9.9923	219.41	0.2736	0.9547	248.91
3.3753	11.0127	226.27	0.2518	0.9108	261.77
3.6421	12.0836	231.78	0.2318	0.8668	273.99
3.9298	13.2017	235.94	0.2135	0.8231	285.61

Table 2: Calculating the effect of equal plant disease condition on the yields of two plant species and its implication on biodiversity: $k_1 = k_2 = 0.02$

PB1(old) g/A	PB1(new) g/A	EBG1(%)	PB2(old) g/A	PB2(new) g/A	EBG2(%)
1.2000	1.2000	0.00	1.4000	1.4000	0.00
1.2110	1.3658	12.78	1.2004	1.3541	12.80
1.2447	1.5487	24.42	1.0482	1.3052	24.52
1.2955	1.7494	35.04	0.9270	1.2538	35.25
1.3602	1.9686	44.73	0.8270	1.2003	45.14
1.4375	2.2068	53.52	0.7427	1.1452	54.19
1.5267	2.4648	61.45	0.6703	1.0891	62.48
1.6275	2.7432	68.55	0.6074	1.0327	70.02
1.7401	3.0425	74.85	0.5521	0.9763	76.83
1.8647	3.3636	80.38	0.5031	0.9206	82.99
2.0018	3.7072	85.19	0.4594	0.8659	88.48
2.1521	4.0744	89.32	0.4202	0.8128	93.43
2.3163	4.4660	92.81	0.3848	0.7615	97.90
2.4950	4.8833	95.72	0.3529	0.7122	101.81
2.6893	5.3274	98.10	0.3239	0.6651	105.34
2.9001	5.7997	99.98	0.2976	0.6204	108.47
3.1284	6.3014	101.43	0.2736	0.5782	111.33
3.3753	6.8339	102.47	0.2518	0.5383	113.78
3.6421	7.3987	103.14	0.2318	0.5009	116.09
3.9298	7.9971	103.50	0.2135	0.4659	118.22

Table 3: Calculating the effect of equal plant disease condition on the yields of two plant species and its implication on biodiversity: $k_1 = k_2 = 0.05$

PB1(old) g/A	PB1(new) g/A	EBG1(%)	PB2(old) g/A	PB2(new) g/A	EBG2(%)
1.2000	1.2000	0.00	1.4000	1.4000	0.00
1.2110	1.3010	7.43	1.2004	1.2898	7.45
1.2447	1.4105	13.32	1.0482	1.1884	13.38
1.2955	1.5290	18.02	0.9270	1.0950	18.12

1.3602	1.6573	21.84	0.8270	1.0091	22.02
1.4375	1.7961	24.95	0.7427	0.9300	25.22
1.5267	1.9462	27.48	0.6703	0.8572	27.88
1.6275	2.1086	29.56	0.6074	0.7902	30.10
1.7401	2.284	31.26	0.5521	0.7285	31.95
1.8647	2.4734	32.64	0.5031	0.6717	33.51
2.0018	2.6779	33.77	0.4594	0.6194	34.83
2.1521	2.8986	34.69	0.4202	0.5712	35.94
2.3163	3.1365	35.41	0.3848	0.5269	36.93
2.4950	3.3928	35.98	0.3529	0.4861	37.74
2.6893	3.6688	36.42	0.3239	0.4485	38.47
2.9001	3.9659	36.75	0.2976	0.4139	39.08
3.1284	4.2852	36.98	0.2736	0.3821	39.66
3.3753	4.6282	37.12	0.2518	0.3528	40.11
3.6421	4.9964	37.18	0.2318	0.3258	40.55
3.9298	5.3911	37.19	0.2135	0.3009	40.94

Table 4: Calculating the effect of equal plant disease condition on the yields of two plant species and its implication on biodiversity: $k_1 = k_2 = 0.095$

PB1(old) g/A	PB1(new) g/A	EBG1(%)	PB2(old) g/A	PB2(new) g/A	EBG2(%)
1.2000	1.2000	0.00	1.4000	1.4000	0.00
1.2110	1.2191	0.67	1.2004	1.2085	0.67
1.2447	1.2587	1.12	1.0482	1.0601	1.14
1.2955	1.3141	1.44	0.9270	0.9404	1.45
1.3602	1.3829	1.67	0.8270	0.8409	1.68
1.4375	1.4640	1.84	0.7427	0.7565	1.86
1.5267	1.5569	1.98	0.6703	0.6837	2.00
1.6275	1.6614	2.08	0.6074	0.6202	2.11
1.7401	1.7777	2.16	0.5521	0.5643	2.21
1.8647	1.9062	2.23	0.5031	0.5145	2.27
2.0018	2.0474	2.28	0.4594	0.4701	2.33
2.1521	2.2019	2.31	0.4202	0.4302	2.38
2.3163	2.3705	2.34	0.3848	0.3942	2.44

2.4950	2.5540	2.36	0.3529	0.3616	2.47
2.6893	2.7534	2.38	0.3239	0.3320	2.50
2.9001	2.9695	2.39	0.2976	0.3051	2.52
3.1284	3.2036	2.40	0.2736	0.2806	2.56
3.3753	3.4566	2.41	0.2518	0.2582	2.54
3.6421	3.7298	2.41	0.2318	0.2377	2.55
3.9298	4.0244	2.41	0.2135	0.2190	2.58

Discussion of Results

In Table 1, within the length of a growing season of twenty months, it was observed that as the plant disease condition were decreased equally from 0.1 to 0.005, the predicted old biomass of the first and second plant species specified as 3.93 grams per area and 0.21 grams per area increased to new plant species biomass of 13.20 grams per area and 0.82 grams per area respectively. A biodiversity gain has occurred as a result of this variations having the percentage quantified values of 235.94 and 285.61 for the first and second plant species for the first and second plant species respectively. In Table 2, as the plant disease condition were decreased equally from 0.1 to 0.02, the predicted old biomass of the first and second plant species specified as 3.93 grams per area and 0.21grams per area increased to new biomass of the first and second plant species of 8.00 grams per area and 0.47 grams per area respectively. This variations resulted to biodiversity gain with percentage quantified values of 103.50 and 118.22 respectively for the first and second plant species within the length of twenty months growing season.

In Table 3, the predicted old biomass of the first and second plant species, specified as 3.93grams per area and 0.21grams per area are increased to new plant species biomass specified as 5.39 grams per area and 0.30 grams per area respectively due to the effect of the decrease in plant disease condition from 0.1 to 0.05. The result of this variations caused biodiversity gain having percentage quantified values of 37.19 and 40.94 respectively for the first and second plant species when the length of the growing season of twenty months.

In Table 4, the predicted old biomass of the first and second plant species are specified by 3.93 grams per area of plant cover and 0.21 grams per area of plant cover respectively are increased slightly to 4.02g/A and 0.22g/A as the new plant species biomass due to the effect of a plant disease. As a result of this variations, a biodiversity gain has occurred, having the percentage quantified values of 2.41 and 2.58 respectively for the first and second plant species when the length of the growing season is twenty months.

Conclusion

We have applied the method of ODE 45 simulation analysis to predict the effect of equal plant disease effect on two competing plant species and obtain novel results that we have not seen elsewhere which are in tandem with the crop science disease control policy. In this study it was observed that the biodiversity gain increased as the plant disease condition decreased. As to have higher crop yield emphasizes should be made on having minimal disease condition.

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